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A Proof Toward Optimality of a Combined Rate, Power, and Cell Control Algorithm Employed in a Cellular CDMA Network

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***Abstract*—Hanly and Yates proposed algorithms for combined power and cell (P/C) control that proved to be an optimal solution to P/C control if such a solution exists. Anpalagan and Sousa have proposed a combined rate, power, and cell (R/P/C) control algorithm with the main idea of interference balancing between base stations. They proposed an algorithm which was empirically proved to yield the optimal solution in terms of the average transmit bit energy of the system. In this paper, a mathematical proof is given that shows the optimality of the R/P/C algorithm. The R/P/C algorithm gives flexibility in setting the rate in response to the congestion level in the network, and hence, it has applications in the areas of delay-tolerant data services.**

***Index Terms*—Cellular code-division multiple-access (CDMA) systems, combined rate, interference balancing, optimal transmit bit energy, power and cell (P/C) control.**

I. Introduction

This paper deals with the proof of a combined rate, power, and cell (R/P/C) control algorithm used in a cellular code-division multiple-access (CDMA) network. Hanly [1] proposed a combined power and cell (P/C) control algorithm. At the same time, and independently, Yates and Huang proposed a similar scheme in [2]. It can be shown that the two approaches are basically the same in terms of minimum transmission power. Anpalagan and Sousa [3] proposed an algorithm for R/P/C control for delay-insensitive applications, where the data rate and the power of transmission for each user are adjusted according to the congestion (or level of interference) at the corresponding base station and the average congestion in the network. It was shown that the proposed algorithm results in congestion balance of the network. For a number of delay-insensitive applications, such as the ones on the Internet, the algorithm is an ideal solution. In [3], an extensive simulation study was performed that empirically proved the superior performance of the algorithm, as compared to P/C control schemes. As the proof of the optimality of the technique, the authors heavily relied on [1], which proved the optimality of the P/C control algorithm.

This paper addresses the details of the proofs given in [1] adapted to the R/P/C control scheme. To this end, the same steps toward the proof of P/C control algorithm are followed. Most of the lemmas and theorems presented in this paper are modifications of the lemmas and theorems given in [1] toward the proof of existence, characterization, and obtaining of the optimal P/C control. The iterations in R/P/C control aim to adjust transmit power and data rate with the goal of interference balancing in the background. We do not address the issue of interference balancing in this paper. Rather, we consider the algorithm and its proof toward optimality in terms of minimizing the average transmit bit energy.

This paper is organized as follows. Section II states the formal problem of the R/P/C control. In Section III, the algorithm for R/P/C control is very briefly presented. Section IV is devoted to the mathematical proof of the optimality and convergence of the algorithm, followed by concluding remarks in Section V.

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II. Problem Statement

Let us consider a cellular CDMA system consisting of N transmitters and B base stations. The uplink gain between mobile-user transmitter i and base-station receiver j is denoted by G_{ij}. Transmitter i broadcasts with transmit power P_i. Thermal noise at base station j has a power-spectral density of $\eta_j/2$. The channel is assumed to have a spectral bandwidth of W. It is assumed that transmitter i communicates with base station $b(i)$. With these notations, we get (1) for the total interference power at base station $b(i)$. On the other hand, the total received power at base station $b(i)$ and the mean value of the total received power at all base stations are given by (2) and (3), respectively

$$I_{b(i)} = \sum_{k \neq i} G_{kb(i)} P_k + \eta_{b(i)} W \quad (1)$$

$$Q_{b(i)} = \sum_{k=1}^{N} G_{kb(i)} P_k + \eta_{b(i)} W \quad (2)$$

$$\overline{Q} = \frac{1}{B} \sum_{j=1}^{B} Q_j. \quad (3)$$

The required energy per bit to interference density ratio (E_b/I_0) and the required data rate for user i are denoted by γ_i^{req} and R_i^{req}, respectively. Similarly, γ_i and R_i are the instantaneous E_b/I_0 and data rate, respectively, for user i when communicating with home base station $b(i)$. Hence, $\gamma_i = (WG_{ib(i)}P_i)/(R_i I_{b(i)})$. For notational convenience, let us define $\Gamma_i = (\gamma_i R_i)/W$. The transmit bit energy (as defined in [3]) of user i is denoted by $\theta_i = P_i/R_i$. Thus, the average transmit bit energy is given by

$$\theta = \frac{1}{N} \sum_{i=1}^{N} \theta_i. \quad (4)$$

The problem of R/P/C control can be stated as finding $b(i)$'s and θ_i's $\forall i$ such that θ is minimized. The constraints are the required energy to interference density ratio $(\gamma_i = \gamma_i^{\mathrm{req}}, \forall i)$ and a positive data rate and transmission power $(P_i \geq 0, R_i \geq 0, \forall i)$. Formal formulation relating transmit power, data rate, energy-per-bit to interference-density ratio, link gains, and noise power level in this system can be simplified to vector notation, as given by

$$(\mathbf{I} - \mathbf{H})\underline{P} = \underline{U} \quad (5)$$

where $\mathbf{I}$ is the $N \times N$ identity matrix

$$\mathbf{H} = [H_{ij}] \text{ with } H_{ij} = \frac{\Gamma_i G_{jb(i)}}{G_{ib(i)}}, \qquad i,j = 1,\ldots,N \tag{6}$$

$$\underline{U} = [U_1 U_2 \ldots U_N]^{\mathrm{T}} \text{ with } U_i = \frac{W\Gamma_i \eta_{b(i)}}{G_{ib(i)}} \tag{7}$$

and $\underline{P} = [P_1 P_2 \ldots P_N]^{\mathrm{T}}$. Note that, in (6), $\mathbf{H}$ represents the $N \times N$ matrix, where row i of $\mathbf{H}$ represents (normalized) reverse link gain of user i with its home base station and also with all the other users' base stations. Equation (5) is the basic equation of the network that is fixed in all control schemes and is solved for a snapshot of the network condition.

III. R/P/C Control

In [3], an algorithm (called Algorithm III in [3] and R/P/C Algorithm in this paper) was proposed for R/P/C control in a cellular CDMA system. It is an iterative algorithm in which user i computes the required transmit bit energy $\theta_{ij}(n)$ to each base station j in each iteration n and picks up the (home) base station for which this quantity is minimum. $\theta_{ij}(n+1)$ is calculated by

$$\theta_{ij}(n+1) = \frac{P_{ij}(n+1)}{R_{ij}(n+1)} \tag{8}$$

where

$$P_{ij}(n+1) = \frac{\gamma_i^{\mathrm{req}} R_i^{\mathrm{req}} \overline{Q}(n)}{W G_{ij}} \tag{9}$$

$$R_{ij}(n+1) = R_i^{\mathrm{req}} \frac{\overline{Q}(n)}{Q_j(n)} \tag{10}$$

where $P_{ij}(n)$ and $R_{ij}(n)$ are the transmit power and the transmit data rate of user i in iteration n when user i connects to base station j. From the above equations, it can be seen that the transmit power and data rate are dependent on the average congestion level in the network. We drop the index j when a mobile is communicating with its home base station, which is defined as $b(i) = \arg\min_{\langle j \rangle}\{\theta_{ij}(n)\}$. Similarly, index j is dropped from P_{ij}, R_{ij}, and θ_{ij} in the following whenever it is appropriate.

IV. Existence, Characterization, and Capture of the Optimal Solution

In this section, a proof is given for the existence of the solution of the problem stated in Section II. In (5), $\mathbf{H}$ is a nonnegative primitive matrix. Perron–Frobenius theory [4] guarantees a dominant positive eigenvalue for $\mathbf{H}$ that will be denoted by r. The main theorem to be used in this section is the following.

Theorem 1: Equation (5) has a positive solution if and only if $r < 1$, where r is the dominant positive eigenvalue of $\mathbf{H}$. In this case, the solution is unique.

Definition 1: D_i is the set of allowable base stations to which user i can connect. $\{b(i)\}_{i=1}^N$ shows the allocation of the mobiles to the (home) base stations. Due to geometric restrictions dictated by D_i's

$$\{b(i)\}_{i=1}^N \in B(D) \equiv \left\{ \{b(i)\}_{i=1}^N \in \{1,2,\ldots,B\}^N : b(i) \in D_i, \forall i \right\}.$$

In this paper, $D = \{D_i\}_{i=1}^N$ is considered as fixed. In other words, the change in the geometry of the system due to mobility of users is considered slow. This is one of the basic assumptions on which the proofs in later sections are based. Everywhere in the proofs, we are looking at a snapshot of the system. The more detailed analysis of the networks considering mobility of the transmitters at a speed greater than system convergence speed is a challenge.

Definition 2: The triple $(N, \Gamma, \underline{\gamma}) \in F(D)$, if and only if there exists an allocation $\{b(i)\}_{i=1}^N$ of base stations and a vector $\underline{P} > 0$ (componentwise) of transmitter powers such that $(W/R_i)\Gamma_i = \gamma_i$, $\forall i$. In the above discussion, $\underline{\gamma} = [\gamma_1 \gamma_2 \ldots \gamma_N]$, and $F(D)$ is the collection of all possible configurations. Since Γ_i can be considered as the service characteristic of user i, a row of matrix Γ represents all possible service characteristics that can be achieved by connecting to B possible base stations. There are N users in the system and, hence, Γ is an $N \times B$ matrix representing all possible service characteristics that can be achieved instantaneously from the network.

A. Existence of the Optimal Cell Allocation

If there is more than one feasible network configuration such as $(N, \Gamma, \underline{\gamma}) \in F(D)$, then we can search for the optimal solution for which the θ given by (4) is minimized. In this section, it is shown that such an optimal solution exists. In the following sections, this solution is characterized, and it is captured with the help of the algorithm given in Section III.

Definition 3: $F(N, \Gamma, \underline{\gamma}, D)$ has the set of solutions $(\underline{P}, \underline{R}, \{b(i)\}_{i=1}^N)$ such that $\{b(i)\}_{i=1}^N \in B(D)$. Let us define $\underline{P}^{(c)}$, $\underline{R}^{(c)}$, and $\underline{\gamma}^{(c)}$ to be the transmit power, the rate, and the E_b/I_0 vector, respectively, in configuration c. In addition, $\underline{\theta} = [\theta_1 \theta_2 \ldots \theta_N]$, and $\underline{\theta}^{(c)}$ is defined as $\underline{P}^{(c)}/\underline{R}^{(c)}$, where componentwise division takes place, and $b(i)^{(c)}$ denotes the home-base-station assignment for user i in configuration c. Correspondingly, $P_i^{(c)}$,$R_i^{(c)}$, and $\gamma_i^{(c)}$ are instantaneous power, rate, and E_b/I_0, respectively, for user i in configuration c.

Lemma 1: Suppose that $(N, \Gamma, \underline{\gamma}^{(1)}) \in F(D)$ and, therefore, $(\underline{P}^{(1)}, \underline{R}^{(1)}, \{b(i)^{(1)}\}_{i=1}^N)$ are the corresponding power, rate, and cell allocation. If $\underline{\gamma}^{(2)} \le \underline{\gamma}^{(1)}$ (componentwise), then there exists a configuration $(\underline{P}^{(2)}, \underline{R}^{(2)}, \{b(i)^{(2)}\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}^{(2)}, D)$ such that $\underline{\theta}^{(2)} = (\underline{P}^{(2)}/\underline{R}^{(2)}) \le \underline{\theta}^{(1)} = (\underline{P}^{(1)}/\underline{R}^{(1)})$ (componentwise).

Proof: Since there is a configuration for $(N, \Gamma, \underline{\gamma}^{(1)}) \in F(D)$, Theorem 1 implies that $r^{(1)} < 1$. It is given that $\underline{\gamma}^{(2)} \le \underline{\gamma}^{(1)}$, so if we fix the rates to be $\underline{R}^{(2)} = \underline{R}^{(1)}$(componentwise), then $\mathbf{H}^{(2)} \le \mathbf{H}^{(1)}$, thus, $r^{(2)} < 1$, which guarantees the existence of a solution for $\underline{P}^{(2)} \le \underline{P}^{(1)}$. In the above discussion, a componentwise comparison is made in $\mathbf{H}$ and $\underline{P}$ with a superscript in r and $\mathbf{H}$ indicating the configuration number. This is based in [4, Th. 1.4]. This implies that there exists a solution with $\underline{\theta}^{(2)} \le \underline{\theta}^{(1)}$, which means that $\theta^{(2)} \le \theta^{(1)}$. ■

Corollary 1: If $(N, \Gamma, \underline{\gamma}^{(1)}) \in F(D)$ and $\underline{\gamma}^{(2)} \le \underline{\gamma}^{(1)}$, then $(N, \Gamma, \underline{\gamma}^{(2)}) \in F(D)$. In other words, if there is a solution for the system for a certain required level-of-energy to interference-density ratio, a solution exists for lower required level-of-energy to interference-density ratios, as expected.

Lemma 2: Suppose $(N, \Gamma, \underline{\gamma}) \in F(D)$ and $(\underline{P}^{(1)}, \underline{R}^{(1)}, \{b^{(1)}(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$ and $(\underline{P}^{(2)}, \underline{R}^{(2)}, \{b^{(2)}(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$. Let us define $I^* = \{i : P_i^{(2)} < P_i^{(1)}\}$. In addition, define for $i \in I^*, b^{(3)}(i) = b^{(2)}(i)$, $P_i^{(3)} = \min(P_i^{(1)}, P_i^{(2)})$, and $R_i^{(3)} = R_i^{(2)}$ if $R_i^{(2)} \ge R_i^{(1)}$ and $R_i^{(3)} = R_i^{(2)}(\underline{I}_{b(i)}^{(2)}/\underline{I}_{b(i)}^{(3)})$ otherwise, and for $i \notin I^*$, $b^{(3)}(i) = b^{(1)}(i)$, $P_i^{(3)} = \min(P_i^{(1)}, P_i^{(2)})$, $R_i^{(3)} = R_i^{(1)}$ if $R_i^{(1)} \ge R_i^{(2)}$ and $R_i^{(3)} = R_i^{(1)}(\underline{I}_{b(i)}^{(1)}/\underline{I}_{b(i)}^{(3)})$ otherwise. Here, $\underline{I}_{b(i)}^{(c)}$ is the total interference-power vector at home base station of each user, as defined by (1), with c being the configuration number. Then, there is an allocation $\{b^{(3)}(i)\}_{i=1}^N$ such that $\underline{\gamma}^{(3)} \ge \underline{\gamma}$ with $(\underline{P}^{(3)}, \underline{R}^{(3)}, \{b^{(3)}(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$.

As there are four cases (in the comparison) with $R_i^{(1)}$, $R_i^{(2)}$, $P_i^{(1)}$, and $P_i^{(2)}$, we needed to separate them in such a way as to show that we can construct a new configuration that provides for better $\underline{\gamma}^{(3)}$. Accordingly, data rates are set differently for difference cases.

Proof: Since $b^{(3)}(i)$ is either equal to $b^{(1)}(i)$ or $b^{(2)}(i)$, it is a feasible cell allocation. For user $i \in I^*$, there are two cases, as discussed below.

Case 1) $R_i^{(2)} \geq R_i^{(1)}$

It can be shown that using the definitions for transmit power and data rate assignment in Lemma 2 and with $b^{(3)}(i) = b^{(2)}(i)$ that we have (11), shown at the bottom of this page, where $P_i^{(3)} = \min(P_i^{(1)}, P_i^{(2)})$. Since $P_k^{(3)} \leq P_k^{(2)}$, the denominator in (11) is smaller (or equal to) than what would be in $\gamma_i^{(2)}$. Therefore, $\gamma_i^{(3)} \geq \gamma_i^{(2)} = \gamma_i$.

Case 2) $R_i^{(2)} < R_i^{(1)}$

It can be shown that using the definitions for transmit power and data rate assignment in Lemma 2 and with $b^{(3)}(i) = b^{(2)}(i)$ and $R_i^{(3)} = R_i^{(2)}(\underline{I}_{b(i)}^{(2)}/\underline{I}_{b(i)}^{(3)})$ that

$$\gamma_i^{(3)} = \frac{WG_{ib^{(2)}(i)}P_i^{(2)}}{R_i^{(3)}I_{b(i)}^{(3)}} = \frac{WG_{ib^{(2)}(i)}P_i^{(2)}}{R_i^{(2)}I_{b(i)}^{(2)}} = \gamma_i^{(2)} = \gamma_i. \quad (12)$$

From (11) and (12), it can be seen that $\gamma_i^{(3)} \geq \gamma_i$ for $i \in I^*$. Note that $R_i^{(3)} \geq R_i^{(2)}$, as $I_{b(i)}^{(3)} \leq I_{b(i)}^{(2)}$, with $P_k^{(3)} = \min(P_k^{(1)}, P_k^{(2)})$, $\forall k$ in a fixed network.

For user $i \notin I^*$, there are also two cases. Following the similar steps discussed above, it can be shown that $\gamma_i^{(3)} \geq \gamma_i$ for $i \notin I^*$. As a result, we can conclude that $\gamma_i^{(3)} \geq \gamma_i$, $\forall i$ and $(\underline{P}^{(3)}, \underline{R}^{(3)}, \{b^{(3)}(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}^{(3)}, D)$.

The interpretation of Lemma 2 is that, whenever two possible configurations for transmit power and data rate are available, it is possible to pick a new configuration, as constructed in Lemma 2, that preserves the energy to interference-density condition and has less transmit power as compared to the previous configurations.

We summarize the power and rate allocations as follows for the new configuration: $\underline{P} = \min(\underline{P}^{(1)}, \underline{P}^{(2)})$ and $\underline{R} = \{\{R_i\}_{i=1}^N$: for $i \in I^*, R_i = R_i^{(2)}$ if $R_i^{(2)} \geq R_i^{(1)}$ and $R_i = R_i^{(2)}(\underline{I}_{b(i)}^{(2)}/\underline{I}_{b(i)}^{(3)})$, otherwise; and for $i \notin I^*, R_i = R_i^{(1)}$ if $R_i^{(1)} \geq R_i^{(2)}$ and $R_i = R_i^{(1)}(\underline{I}_{b(i)}^{(1)}/\underline{I}_{b(i)}^{(3)})$, otherwise$\}$. ■

Corollary 2: Suppose $(N, \Gamma, \underline{\gamma}) \in F(D)$ and $(\underline{P}^{(1)}, \underline{R}^{(1)}, \{b^{(1)}(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$ and $(\underline{P}^{(2)}, \underline{R}^{(2)}, \{b^{(2)}(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$. Then, there exists an allocation $\{b(i)\}_{i=1}^N \in B(D)$ of user cells with $\underline{P}$ and $\underline{R}$ such that $(\underline{P}, \underline{R}, \{b(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$.

Proof: By Lemma 2, there is an allocation $\{b^{(3)}(i)\}_{i=1}^N$ such that $\underline{\gamma}^{(3)} \geq \underline{\gamma}$ and $(\underline{P}^{(3)}, \underline{R}^{(3)}, \{b^{(3)}(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}^{(3)}, D)$. Since $\underline{\gamma}^{(3)} \geq \underline{\gamma}$ by Lemma 1, there exists a solution $(\underline{P}, \underline{R}, \{b(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$ such that $\theta \leq \theta^{(3)}$.

By Corollary 2, one can reduce any pair of solutions to an optimal one. Given any pair of possible configurations, it is possible to find a third configuration that is feasible and for which the goal function θ is minimized. Therefore, one can begin by reducing the possible configurations to the optimal one in $F(N, \Gamma, \underline{\gamma}, D)$. Theorem 2 states this fact, which proves the existence of the optimal solution. ■

Relationship Between R/P/C Algorithm and Lemma 2: In Lemma 2, a user chooses a new base station (if necessary) with minimum power and with data rate set in such a way as not to decrease γ from the other two configurations. In the base station, a new configuration is constructed for user i with P_i and R_i as set in the definition, and the ration of these parameters is as follows: $\theta_i() = (\gamma_i^{(3)} I_{b(i)}())/(WG_{ib(i)})$. The constraint on E_b/I_0 is that $\gamma_i^{(3)} = \gamma_i^{\text{req}}$ at the optimal point. In the R/P/C algorithm, user i chooses a base station that directly gives minimum θ_i, which is given as $\theta_i(n+1) = (\gamma_i^{\text{req}} Q_{b(i)}(n))/(WG_{ib(i)})$ from (9) and (10). Therefore, the difference between the θ_i's is on the numerator with I (total interference power) and Q (total receive power) used in Lemma 2 and R/P/C algorithm, respectively. In a large-capacity CDMA system, the difference between Q and I is very small, and hence, both in effect do the same.

Theorem 2: If $(N, \Gamma, \underline{\gamma}) \in F(D)$, then there exists a solution $(\underline{P}^*, \underline{R}^*, \{b^*(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$ such that if $(\underline{P}, \underline{R}, \{b(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$, then $\theta^* \leq \theta$.

B. Characterization of the Optimal Solution

Lemma 3: Suppose $(\underline{P}, \underline{R}, \{b(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$ has the property that for any $i, k \in D_i, ((W\theta_i G_{ik})/(\sum_{j\neq i} P_j G_{jk} + \eta_k W)) \leq \gamma_i$, then $(\underline{P}, \underline{R}, \{b(i)\}_{i=1}^N) \in F(N, \Gamma, \underline{\gamma}, D)$ is the optimal solution for $F(N, \Gamma, \underline{\gamma}, D)$. ■

Proof: See [1] for the proof.

C. Proof of Convergence of R/P/C Algorithm to the Optimal Solution

To show that algorithm given in Section III converges to the optimal solution, the action of the algorithm is considered to be a mapping on $R_+^N \times R_+^N$.

Definition 4: $T : R_+^N \times R_+^N \rightarrow R_+^N \times R_+^N$ is the mapping, taking any pair $(\underline{P}(n), \underline{R}(n))$ such that $\underline{P} = \{P_i(n)\}_{i=1}^N$ and $\underline{R} = \{R_i(n)\}_{i=1}^N$ to the next pair $(\underline{P}(n+1), \underline{R}(n+1))$, according to (9) and (10).

Lemma 4: If $(N, \Gamma, \underline{\gamma}) \notin F(D)$, then T has no fixed point. On the other hand, if $(N, \Gamma, \underline{\gamma}) \in F(D)$, then $(\underline{P}^*, \underline{R}^*)$ is the fixed point of T.

Proof: First, note that if $(\underline{P}, \underline{R})$ is a fixed point of T, then there exists an allocation $\{b(i)\}_{i=1}^N \in B(D) \equiv \{\{b(i)\}_{i=1}^N \in \{1, 2, \ldots, B\}^N : b(i) \in D_i, \forall i\}$ such that $((WG_{ib(i)}P_i)/(R_i(\sum_{k\neq i} G_{kb(i)}P_k + \eta_{b(i)}W))) = \gamma_i$ for $\forall i$. Then, for $i = 1, 2, \ldots, N$, one has $((WG_{ib(i)}P_i)/(R_i(\sum_{k\neq i} G_{kb(i)}P_k + \eta_{b(i)}W))) \leq \gamma_i$, where, together with the characterization of optimal solution, one concludes that $(\underline{P}, \underline{R}) = (\underline{P}^*, \underline{R}^*)$.

Now, if $(N, \Gamma, \underline{\gamma}) \in F(D)$, then Theorem 2 implies the existence of a solution $(\underline{P}^*, \underline{R}^*)$, which is a fixed point of T. Iteration on T and application of T iteratively on the obtained solutions is equivalent to finding the fixed point of T. As indicated in Lemma 4, if there exists a solution, T converges to it. This concludes the proof of the convergence of the algorithm given in Section III to the optimal solution. ■

V. Conclusion

For delay-tolerant applications, such as file video streaming or file downloading from the wireless Internet, the R/P/C control algorithm

$$\gamma_i^{(3)} = \frac{WG_{ib^{(2)}(i)}P_i^{(2)}}{R_i^{(2)}\left(\sum\limits_{k\in I^*, k\neq i,} G_{kb^{(2)}(k)}P_k^{(2)} + \sum\limits_{k\notin I^*} G_{kb^{(2)}(k)}P_k^{(3)} + \eta_{b^{(2)}(i)}W\right)} \quad (11)$$

proposed in [3] can be effectively used. The proponent of R/P/C algorithm performed extensive simulation study and empirically proved the superior performance of the algorithm, as compared to P/C schemes. In this paper, the problem of R/P/C control is presented in correct mathematical notations. Existence, characterization, and capture of the optimal solution by using the R/P/C algorithm have been studied in detail by means of linear-algebraic techniques. A proof has been outlined toward the optimality of the R/P/C control algorithm.

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A Gaussian Symbol-Level Model for CDMA Forward Link Intracell Interference

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***Abstract*—This paper presents a Gaussian approximation for intracell interference on the forward link of a code division multiple access system with fast power control. A probability density function (PDF) is derived for the transmit power variations of each user due to power control adjustments. The PDF is then used to determine the variance of a Gaussian random variable that approximates intracell interference. Simulation demonstrates that this approximation is very accurate even for small spreading factors and low traffic loading levels. The accuracy of the approximation is evaluated for convolutional and turbo coding. The effect of finite power control step sizes and update rates is also investigated.**

***Index Terms*—Channel coding, code division multiple access (CDMA), Gaussian approximation, intracell interference, power control.**

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I. INTRODUCTION

Starting with the study of Pursley [1], there has been considerable work into the investigation of whether the overall interference plus noise process at the input to the detection block in a code division multiple access (CDMA) receiver can be approximated as Gaussian. This approximation is known as the standard Gaussian approximation (SGA). It has been demonstrated for large spreading factors that bit error rate (BER) in an asynchronous CDMA system in the additive white Gaussian noise channel can be determined using the SGA (see [2] and the references therein). The result in [2] can be applied to the synchronous CDMA forward link to show that forward link intracell interference is also Gaussian for large spreading factors. For very low spreading factors, the actual intracell interference probability density function (PDF) for the CDMA forward link has been derived in [3].

This paper extends the investigation of using a Gaussian random process to approximate CDMA forward link intracell interference by making three new contributions. First, it examines how to apply the Gaussian approximation to the forward link intracell interference in a CDMA system that uses fast power control. The systems considered in [2] and [3] assume that all users have fixed transmit gains. In the following, a PDF for the random transmit power variations in the signal of each interfering user due to power control adjustments is derived. This PDF is used to calculate the variance of the forward link intracell interference process in a CDMA system with a fast forward power control. That variance is then incorporated in the Gaussian intracell interference assumption.

The second contribution is to demonstrate that the Gaussian intracell interference approximation provides very accurate results even for very low spreading factors and small traffic loads. The results in [2] are generated only for large spreading factors, and Fan *et al.* [3] derive an exact expression for intracell interference rather than an approximation. The advantage of using the Gaussian approximation rather than the exact intracell interference process generated at the chip level is that the Gaussian approximation can be implemented at the symbol level, greatly improving simulation efficiency.

Finally, the third contribution is to evaluate the effect of channel coding and realistic power control algorithms on the Gaussian approximation. It will be shown that the approximation can be applied equally well to systems using convolutional and turbo coding. It will also be shown that a power control algorithm with finite step size and update rate can degrade the accuracy of the Gaussian approximation but only at high mobile velocities.

Section II presents a model for the synchronous CDMA forward link with fast forward power control and defines a Gaussian intracell interference approximation for that model. The accuracy of the approximation is verified in Section III by comparing a full chip-level CDMA forward link simulation to a symbol-level forward link simulation that uses the Gaussian intracell interference approximation. Concluding remarks are made in Section IV.

II. CDMA FORWARD-LINK MODEL

By following the notation in [1], the aggregate signal transmitted by a CDMA base station to K mobiles is

$$s(t) = \sum_{k=0}^{K-1} \sqrt{2P_k(t)}\, b_k(t) a_k(t) \cos(\omega_c t + \theta_k) \tag{1}$$

where $P_k(t)$ is the transmitted signal power, $b_k(t)$ is the data signal, $a_k(t)$ is the spreading waveform, ω_c is the carrier frequency, and θ_k is the carrier phase of the kth user's spread signal. The data