Performance Analysis of a Threshold-based Group-adaptive Modulation Scheme with Adaptive Subcarrier Allocation in OFCDM Systems

Lamiaa Khalid and Alagan Anpalagan

***Abstract*—This paper proposes an adaptive modulation algorithm for orthogonal frequency and code division multiplexing (OFCDM) system to increase the spectral efficiency without sacrificing the BER performance under different spreading factors. The proposed algorithm is used with an adaptive subcarrier allocation technique which assigns users to subcarriers to minimize the overall BER of the system. A fixed threshold is used to switch between modulation levels depending on the estimated SINR in each group. A spectral efficiency of 3.2 bits per symbol is obtained for a target BER of 10^{-2}. BCH (511, 385) coding with rate 3/4 used to accommodate a lower target BER of 10^{-3} yields a spectral efficiency of 2.8 bits per symbol. The proposed algorithm provides an increase in spectral efficiency than using BPSK only, without increasing the total transmit power.**

***Index Terms*—OFDM, CDMA, subcarrier allocation, group-adaptive scheme, time and frequency domain spreading.**

I. Introduction

In order to deal with the high data rates the fourth generation (4G) wireless system is expected to deliver [1], a number of multiuser systems has been proposed. One of the most recently proposed systems is orthogonal frequency and code division multiplexing (OFCDM) [2]. OFCDM uses data spreading where each data stream is segmented into multiple substreams and spread over multiple subcarriers and several OFCDM symbols, exploiting additional frequency and time diversity. OFCDM outperforms both MC-CDMA and MC-DS-CDMA because of the utilization of two dimensional spreading.

An adaptive subcarrier allocation technique has been proposed in [3] to further improve the bit error rate (BER) performance of OFCDM with various spreading factors. The proposed algorithm allocates subcarriers to users such that the mean BER performance of the cell is maximized. Adaptive modulation helps maximize the data rates in wireless transmission over fading channels [4] by adapting to the changing channel conditions and by making use of spectrally efficient modulation schemes. Adaptive subcarrier allocation with adaptive modulation for OFDM was investigated in [5]; the results show a significant improvement in spectral efficiency. Motivated by this, the goal of this paper is to apply adaptive modulation combined with the subcarrier allocation technique proposed for OFCDM in [3] to increase the data rate, thus increasing the spectral efficiency without increasing the total transmit power. This letter is organized as follows: section II gives a brief overview of the adaptive subcarrier allocation technique in OFCDM. A proposed adaptive modulation algorithm in OFCDM is presented in section III followed by performance analysis in section IV. Numerical results are presented in section V. Finally, the paper is concluded in section VI.

Manuscript received March 20, 2007; revised July 31, 2007 and January 7, 2008; accepted January 16, 2008. The associate editor coordinating the review of this paper and approving it for publication was S. Shen. A part of this paper was presented in IEEE Vehicular Technology Conference-Fall in September 2006 in Montreal, Canada. This work was in part supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

L. Khalid is a Ph.D. student associated with WINCORE Lab at Ryerson University, Canada (e-mail: lamiaa_khalid@yahoo.com).

A. Anpalagan is an Associate Professor at Ryerson University, Canada (e-mail: alagan@ee.ryerson.ca).

Digital Object Identifier 10.1109/TWC.2008.070294.

II. Adaptive Subcarrier Allocation

Various adaptive subcarrier allocation algorithms were proposed for MC-CDMA and MC-DS-CDMA systems [6]. Many of these algorithms outperform the respective non-adaptive systems in terms of the BER performance. In OFCDM systems, an adaptive subcarrier allocation algorithm was developed to maximize the mean BER performance under different spreading factors [3]. This algorithm proposes separating the total spectrum into small groups of non-contiguous subcarriers that are equally spaced throughout the spectrum to maximize frequency diversity gains and minimize multiple access interference (MAI). At the base station, the subcarrier group for each user substream was assigned based on the instantaneous SINR characteristics of each subcarrier such that the average SINR of the system is maximized while minimizing the interference caused to other users simultaneously.

III. Proposed Adaptive Modulation Algorithm

Various selection algorithms have been proposed for adaptive systems to decide between modulation schemes based on channel conditions [7]. In multicarrier systems, subcarriers are often grouped together and adaptation is performed on the entire subcarrier group to reduce the computational complexity and signaling overhead [8]. Since the algorithm in [3] is implemented on subcarrier groups, it lends itself to group-adaptive modulation.

A. The Adaptive Modulation Algorithm

We propose to use a threshold-based adaptive modulation algorithm for downlink OFCDM systems that switches between the different modulation levels depending on the estimated SINR for each group. The SINR is estimated at the receiver and is reported to the transmitter through a feedback channel and rate selection is done at the transmitter. The modulation level is selected such that it maintains the BER below a desired performance threshold. To have a constant estimated channel SINR for all the OFCDM symbol durations we require a slow fading channel [9] which is necessary to ensure that channel conditions do not change drastically. In the proposed algorithm, the same power is allocated to each subcarrier under a given total transmit power.

The steps for the algorithm are as follows:

Step 1: For each group, determine the substream with minimum SINR to ensure that the level of modulation used will be suitable for transmitting all the substreams in this group.

Step 2: Compare the minimum SINR in each group with the threshold level for different modulation techniques; these thresholds are calculated according to the desired BER.

Step 3: The highest modulation level with a threshold value lower than the minimum SINR calculated for a certain group is chosen and all substreams in this group are transmitted using this modulation level.

IV. Performance Analysis

In our analysis, we consider two performance metrics: spectral efficiency and BER. The modulation schemes chosen for adaptation in this work are BPSK, QPSK, 8PSK and 16QAM. We consider maximal ratio combining (MRC) receiver to give more weight to subcarriers that experience more favorable fading characteristics. Without loss of generality, if we aim to recover the data transmitted to user 1, the SINR $\gamma^{(1)}$ is calculated based on the analysis in [3] and [10]

$$\gamma^{(1)} = \frac{2NST_c \sum_{m \in G_y} (\alpha_m^{(1)})^2}{(K_y - 1)ST_c E[(\alpha_m^{(k)})^2] + N_o}, \tag{1}$$

where S is the transmitted power of a user with respect to one subcarrier; T_c is the chip duration; N is the length of the time domain PN code; N_o is the noise power density; G_y represents the set of subcarriers in group y; $\alpha_m^{(k)}$ is the Rayleigh fading gain coefficients for the k^{th} user on the m^{th} subcarrier which are independent identically distributed for different subcarriers. $E[(\alpha_m^{(k)})^2]$ is the average fading gain squared for the K_y users. Based on the law of large numbers, $E[(\alpha_m^{(k)})^2]$ can be estimated as follows:

$$E[(\alpha_m^{(k)})^2] = \frac{1}{K_y M_y} \sum_{k \in G_y} \sum_{m \in G_y} (\alpha_m^{(k)})^2, \tag{2}$$

where M_y is the number of subcarriers in group y.

The BER using BPSK, QPSK, 8PSK and 16QAM modulations for each user k is given by

$$P_{BPSK}(\gamma^{(k)}) = \mathrm{Q}\left(\sqrt{2\gamma^{(k)}}\right), \tag{3}$$

$$P_{QPSK}(\gamma^{(k)}) = \mathrm{Q}\left(\sqrt{\gamma^{(k)}}\right), \tag{4}$$

$$P_{8PSK}(\gamma^{(k)}) = \frac{2}{3}\mathrm{Q}\left(\sin\frac{\pi}{8}\sqrt{2\gamma^{(k)}}\right), \tag{5}$$

$$P_{16QAM}(\gamma^{(k)}) = \frac{3}{4}\mathrm{Q}\left(\sqrt{0.2\gamma^{(k)}}\right). \tag{6}$$

To evaluate the mean BER of adaptive modulation $P_{adaptive}(\gamma)$ we use

$$\begin{aligned} P_{adaptive}(\gamma) &= \frac{P_{BPSK}(\gamma)\ N_{BPSK} + P_{QPSK}(\gamma)\ N_{QPSK}}{N_{total}} \\ &+ \frac{P_{8PSK}(\gamma)\ N_{8PSK} + P_{16QAM}(\gamma)\ N_{16QAM}}{N_{total}}, \end{aligned} \tag{7}$$

where γ is the SNR. N_{BPSK}, N_{QPSK}, N_{8PSK} and N_{16QAM} are the number of substreams for different users transmitted using BPSK, QPSK, 8PSK and 16QAM respectively and N_{total} is the total number of substreams for all users. $P_{BPSK}(\gamma)$, $P_{QPSK}(\gamma)$, $P_{8PSK}(\gamma)$, $P_{16QAM}(\gamma)$ are the mean bit error rates for BPSK, QPSK, 8PSK and 16QAM calculated from (3)-(6) by averaging over all users, subcarriers and transmitted symbols.

In order to accommodate a lower target BER while keeping the threshold values attainable, we apply error correcting coding. We use BCH codes in our analysis which has a total of n encoded symbols and k original information symbols and is capable of correcting up to $l(< n)$ errors. A good approximation for BER after decoding, P_b, is given by [11]

$$P_b = \frac{1}{n} \sum_{i=l+1}^{n} i \binom{n}{i} P_e^i (1 - P_e)^{n-i}. \tag{8}$$

Further approximation of this formula is given by [12]

$$P_b = P_e Q\left(\frac{l - nP_e}{\sqrt{nP_e}}\right), \tag{9}$$

where P_e is the BER calculated from (3)-(6) and $Q(.)$ is the standard Q function. We will use the approximation given by (9) in our analysis.

In our system, we define the spectral efficiency to be the expected value of $\log_2 L$ (bits per symbol), where L is the modulation level. In this analysis, we have set the target BER to a value that we want the system to operate under and the adaptation system will try to achieve this level of performance so we do not take into account whether the transmitted bits are received correctly or not. Therefore, the spectral efficiency η of the uncoded system is calculated as follows:

$$\begin{aligned} \eta &= \frac{n_{BPSK}\ N_{BPSK} + n_{QPSK}\ N_{QPSK}}{N_{total}} \\ &+ \frac{n_{8PSK}\ N_{8PSK} + n_{16QAM}\ N_{16QAM}}{N_{total}} \\ &= 1 + \frac{N_{QPSK} + 2N_{8PSK} + 3N_{16QAM}}{N_{total}}, \end{aligned} \tag{10}$$

where n_{BPSK}, n_{QPSK}, n_{8PSK} and n_{16QAM} are the number of bits per symbol for BPSK, QPSK, 8PSK and 16QAM respectively.

Equation (10) shows that using uncoded adaptive modulation with BPSK, QPSK, 8PSK and 16QAM, we can obtain spectral efficiency between 1 and 4 bits per symbol, as the number of substreams using 16QAM increases, the spectral efficiency becomes closer to 4 bits per symbol.

The performance enhancement achieved by using error correcting coding is paid for by a decrease in spectral efficiency by a factor of R_c. Therefore, the spectral efficiency η_{coded} of the encoded system is calculated by:

$$\begin{aligned} \eta_{coded} &= \eta R_c \\ &= \left(1 + \frac{N_{QPSK} + 2N_{8PSK} + 3N_{16QAM}}{N_{total}}\right) R_c. \end{aligned} \tag{11}$$

TABLE I

THRESHOLD VALUES FOR DIFFERENT MODULATION SCHEMES.

Modulation	$\mathbf{BER_{target}}$=$\mathbf{10^{-2}}$	$\mathbf{BER_{target}}$=$\mathbf{10^{-3}}$, $\mathbf{R_c}$= 3/4
	SINR(dB)	SINR(dB)
QPSK	7.334	6.435
8PSK	12.064	11.033
16QAM	14.860	14.064

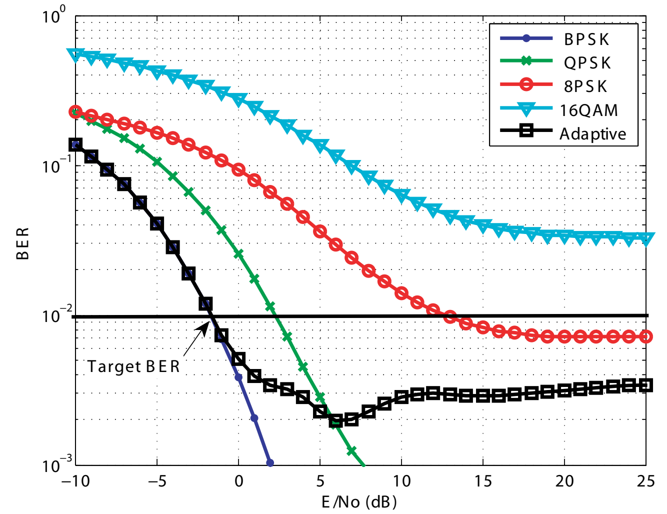
Equation (11) shows that using coded modulation, the spectral efficiency obtained will depend on the coding gain used.

V. Numerical Results

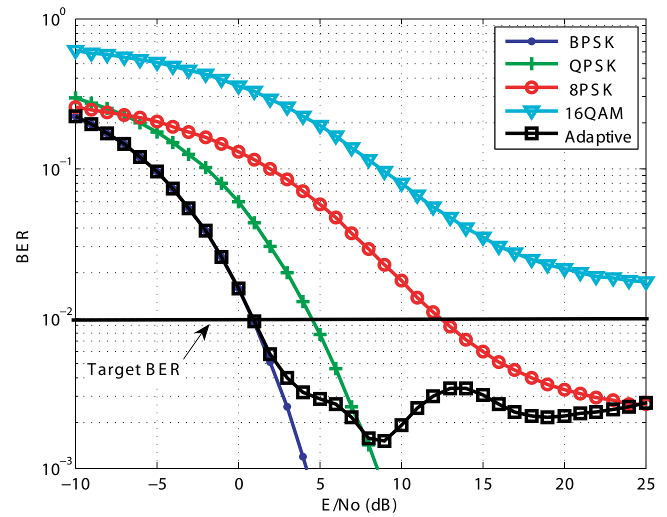
The parameters chosen for the OFCDM system correspond to those mentioned in [1] for 4G systems. The carrier frequency is f_c=5 GHz and the downlink channel bandwidth is 100 MHz. There are 256 subcarriers and we assume that each subcarrier experiences frequency non-selective fading and that fading is uncorrelated between subcarriers in the same group. The switching levels for using QPSK, 8PSK and 16QAM correspond to the SINR at which the desired BER is achieved; below these threshold levels, BPSK is used. Table 1 shows the threshold values for the different modulation schemes.

For the sake of comparison, we fix the total spreading factor (SF) to be 32. The total SF is equal to the SF in the time domain (SF_{time}) multiplied by the SF in the frequency domain (SF_{freq}). We investigate the BER performance for the uncoded case with a target BER of 10^{-2} and the encoded case with a target BER of 10^{-3} and coding rate $R_c = 3/4$ using BCH $(511, 385)$ code. We evaluate the performance with 16 users simultaneously assigned to the channel and N substreams transmitted simultaneously for each user to make the data rate identical in each analysis.

In Figs. 1(a) and 1(b), we plot the BER vs. E/N_o, where E denotes the symbol energy, for BPSK, QPSK, 8PSK, 16QAM and using the proposed adaptive modulation with SF of 16x2 and 2x16 respectively. SF of 16x2 represents a case with higher SF_{time} with respect to SF_{freq} while SF of 2x16 represents a case with higher SF_{freq}. The results show that at high E/N_o, we can obtain a BER of approximately 3.5x10^{-3} and 2.5x10^{-3} with SF of 16x2 and 2x16 respectively. This indicates that adaptive modulation improves the BER performance beyond what 8PSK and 16QAM can provide which allows the system to experience higher spectral efficiency than BPSK while achieving a better BER performance than 8PSK and 16QAM. An increase in the BER with adaptive modulation than the case of BPSK and QPSK at higher E/N_o is expected since no non-adaptive scheme provides better performance while simultaneously providing better spectral efficiency. From the results we also notice that, at high E/N_o more reduction in the BER is achieved when using a higher SF_{freq} because the frequency diversity gained by the system results in a better BER performance in the interference-limited region. It can also be seen from the figures that the adaptive system BER is better than the



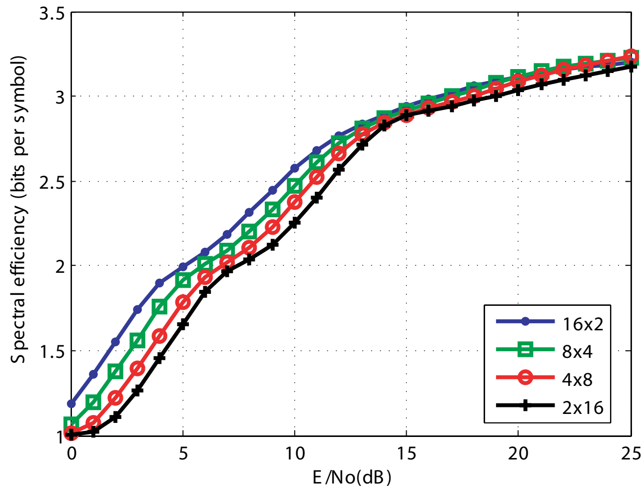
(a) 16x2 SF



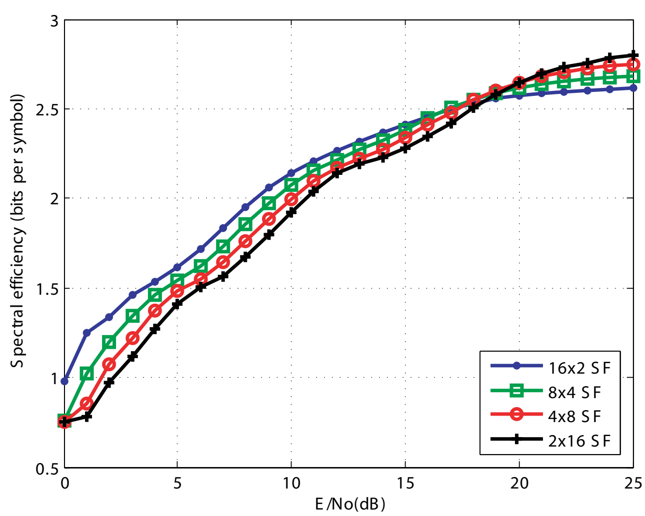
(b) 2x16 SF

Fig. 1. BER performance of different modulation schemes for BER_{target}=10^{-2} and different SF.

target BER, resulting in measured mean BER lower than the target except for very low values of E/N_o (noise-limited region). This can be explained by the group-adaptation regime which was based on the principle of using the lowest SINR in each group for modulation level estimation, leading to a pessimistic estimate for the entire group. This is necessary to guarantee that each user has a BER better than the target. Similar results are obtained when a coding rate R_c= 3/4 is used with a target BER of 10^{-3} for different spreading factors. The spectral efficiency can be determined by the number of substreams that uses each modulation scheme. In Figs. 2(a) and 2(b), we plot the spectral efficiency (bits per symbol) vs. E/N_o for the uncoded and encoded case with different spreading factors using the proposed adaptive modulation algorithm. We notice that as E/N_o increases, the spectral efficiency improves steadily as the system is able to choose more efficient modulation schemes. The results show that at high E/N_o, we can obtain a spectral efficiency up to 3.2 and 2.8 bits per symbol for the uncoded and encoded

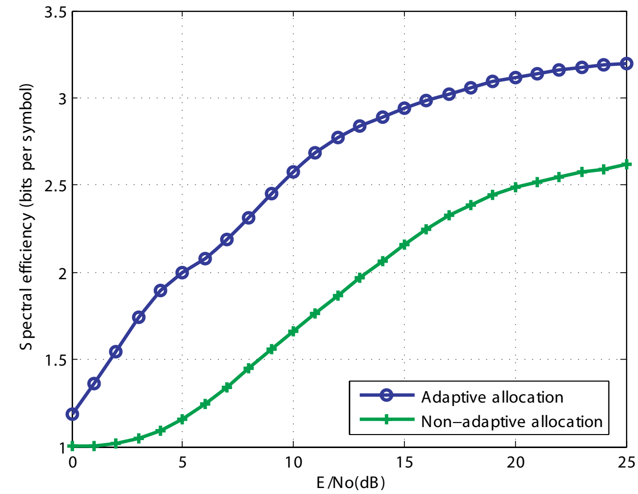


(a) uncoded with BER_{target}=10^{-2}

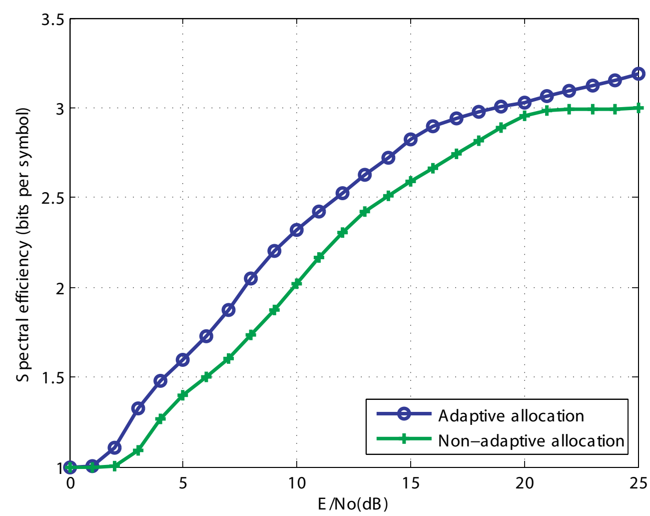


(b) BER_{target}=10^{-3} and R_c=3/4

Fig. 2. Spectral efficiency curves for adaptive modulation with different SF.



(a) 16x2 SF



(b) 2x16 SF

Fig. 3. Comparison curves of the spectral efficiency of uncoded adaptive modulation with adaptive (with non-adjacent subcarriers) and non-adaptive (with adjacent subcarriers) allocation for different SF.

case respectively. Also, at low E/N_o, the spectral efficiency increases with higher SF_{time} compared to SF_{freq} since more groups are available which provides a better chance of locating subcarriers with best fading characteristics which results in minimizing the MAI and maximizing the average SINR. As E/N_o increases, the OFCDM system makes a transition from being noise-limited to being interference-limited and thus the higher SF_{freq} gives better spectral efficiency due to the frequency diversity gained by transmitting over several subcarriers experiencing independent fading. This can be seen in Fig. 2(b) when E/N_o exceeds 18 dB.

In Figs. 3(a) and 3(b), we compare the spectral efficiency gained by using uncoded adaptive modulation in OFCDM systems when the adaptive subcarrier allocation technique [3] was used as opposed to using non-adaptive allocation (with adjacent subcarriers) [13] with 16x2 and 2x16 spreading factors respectively. As seen in Fig. 3(a), the spectral efficiency increased from 2.6 to 3.2 bits per symbol for E/N_o= 25 dB when using adaptive modulation in the OFCDM system combined with adaptive subcarrier allocation for SF of 16x2. From Fig. 3(b), we notice less increase in the spectral efficiency with SF of 2x16 compared to SF of 16x2. This is because for a fixed total SF, the subcarrier allocation technique is more effective with higher SF_{time} as more groups of subcarriers are available to maximize the average SINR. Also, for higher SF_{freq}, the frequency diversity gained by the system improves the signal power with respect to the noise while the subcarrier allocation technique increases the interference power proportional to the signal power which makes the subcarrier allocation technique less effective at higher SF_{freq} [3]. Similar results are obtained when a coding rate R_c= 3/4 is used with a target BER of 10^{-3} for different spreading factors.

Although using only higher order modulation can provide more increase in spectral efficiency than adaptive modulation, it degrades the BER performance by increasing the BER above the target values. This means that adaptive modulation provides a good tradeoff between spectral efficiency and BER.

VI. Conclusions

In this paper, we proposed an adaptive modulation algorithm for OFCDM combined with a subcarrier allocation technique. We used a fixed threshold adaptation algorithm to switch between modulation levels depending on the estimated SINR for each group. The performance of adaptive modulation for a target BER of 10^{-2} was investigated. Coding rate of 3/4 was used to accommodate a lower target BER of 10^{-3}. The proposed algorithm provides a spectral efficiency of up to 3.2 and 2.8 bits per symbol for a target BER of 10^{-2} and 10^{-3} respectively without increasing the total transmit power. The results showed that the adaptive system performs better than its target BER which suggests that we can further improve the spectral efficiency by using adaptive threshold for the switching between different modulation schemes. Also, a higher spectral efficiency is obtained when adaptive modulation is used with adaptive subcarrier allocation than with non-adaptive allocation for different spreading factors. Further enhancement in the performance is expected if adaptive modulation, coding and power adaptation are combined with the proposed adaptive system.

Acknowledgment

The authors would like to thank anonymous reviewers whose reviews and suggestions helped to improve the quality of this paper.

References

[1] T. Miki, T. Ohya, H. Yoshino, and N. Umeda, "The overview of the 4th generation mobile communication system," in *Proc. Fifth International Conference on Information, Communications and Signal Processing*, pp. 1551–1555, Dec. 2005.

[2] L. Xiao and Q. Liang, "A novel MC-2D-CDMA communication system and its detection methods," in *Proc. IEEE International Conference on Communications*, vol. 3, pp. 1223–1227, June 2000.

[3] R. Caldwell and A. Anpalagan, "Adaptive subcarrier allocaion in synchronous reverse links of a multicarrier CDMA system with time and frequency spreading," to appear in *IEEE Trans. Veh. Technol.*, May 2008.

[4] X. Qiu and K. Chawla, "On the performance of adaptive modulation in cellular systems," *IEEE Trans. Commun.*, vol. 47, pp. 884–895, June 1999.

[5] Y. J. Zhang and K. B. Letaief, "Multiuser adaptive subcarrier-and-bit allocation with adaptive cell selection for OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1566–1575, Sept. 2004.

[6] H. Long and Y. H. Chew, "An adaptive subcarrier allocation scheme for MC-DS-CDMA systems in the presence of multiple access interference," in *Proc. IEEE International Conference on Communications*, pp. 2894–2898, June 2004.

[7] W. T. Webb and R. Steele, "Variable rate QAM for mobile radio," *IEEE Trans. Commun.*, vol. 43, pp. 2223–2230, July 1995.

[8] K. Chang and Y. Han, "OFCDM based adaptive modulation with antenna array in fading channels," in *Proc. IEEE Vehicular Technology Conference*, pp. 122–126, Apr. 2003.

[9] K. Balachandran, S. R. Kadaba, and S. Nanda, "Channel quality estimation and rate adaptation for cellular mobile radio," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1244–1256, July 1999.

[10] K. Zheng, G. Zeng, and W. Wang, "Performance analysis for OFDM-CDMA with joint frequency-time spreading," *IEEE Trans. Broadcasting*, vol. 51, pp. 144–148, Mar. 2005.

[11] T. S. Rappaport, *Wireless Communications: Principles and Practice, Second Ed.* New York: Prentice Hall, Inc., 2002.

[12] X. Duan, X. Ma, Z. Niu, and J. Zheng, "Analysis of wireless transmission efficiency and its application: efficiency-based adaptive coding," in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, pp. 1516–1520, Sept. 2003.

[13] C. W. You and D. S. Hong, "Multicarrier CDMA systems using time-domain and frequency-domain spreading codes," *IEEE Trans. Commun.*, vol. 51, pp. 17–21, Jan. 2003.