

# Cooperative Subcarrier and Power Allocation for a Two-Hop Decode-and-Forward OFCDM Based Relay Network

S. Senthuran, A. Anpalagan, and O. Das

**Abstract**—In this article, subcarrier and power allocation schemes are proposed and analyzed for different scenarios for a two-hop decode-and-forward OFCDM based relay network. In subcarrier allocation, the effect of considering the channel state information (CSI) of source-base station and source-relay link are evaluated in a cooperative diversity system. Results show that allocation of subcarriers based on source-relay node CSI provides better BER performance at higher  $E_b/N_o$ , and at lower  $E_b/N_o$  both the source-relay and source-base station links need to be considered. From our numerical simulation, we also noticed that the cross-over  $E_b/N_o$  point (around which frequency spreading gives better performance than time spreading) moves towards the lower  $E_b/N_o$  when the subcarrier allocation is done giving more weight to source-base station link rather than the source-relay link which provides additional flexibility in operating environment for OFCDM systems. In power allocation, a cooperative power allocation ratio  $\lambda$  (=source node power/total power) is defined and BER performance is evaluated for different values of  $\lambda$  in the relay network. It is found that there exists an optimal power allocation ratio for different operating environment such as source-to-relay channel gains and time-frequency spreading factors. It is reported that: (a) When all three channels (source-to-relay, source-to-destination and relay-to-destination) have equal gains, power ratio is found to be  $\lambda \approx 0.8$  (i.e., 80% and 20% of the total power is distributed among source and relay node respectively). The performance degrades at much faster rate when  $\lambda$  increases above the optimal value at higher  $E_b/N_o$ . On the other hand, the performance remains almost the same when the decrement in  $\lambda$  is less than the optimal value. (b) For a network with stronger source-to-relay link, the optimal  $\lambda$  remains almost the same as the case with equal channel gains at higher  $E_b/N_o$ ; however, the optimal power ratio moves toward lower value of  $\lambda$  of 0.65 at lower  $E_b/N_o$ . (c) The optimal  $\lambda$  remains almost the same with different time-frequency spreading factors.

**Index Terms**—Relay, two-hop, decode-and-forward, OFCDM, subcarrier allocation, power allocation.

## I. INTRODUCTION

FOR wideband applications, code division multiple access (CDMA) based systems deployed on multicarriers could provide effective multiple access capabilities. First was the multicarrier direct-sequence CDMA (MC-DS-CDMA) [1]

system where multicarrier modulation was used with time spreading. The other was the multicarrier CDMA [2] where multicarrier modulation and frequency domain spreading were used. Even though MC-CDMA provides frequency diversity, it is vulnerable to multiple access interference (MAI) in frequency selective subcarriers. In order to realize the benefits from both systems, a multicarrier CDMA system was proposed in [3] where both frequency and time spreading codes were used according to the channel conditions. This proposed system was shown to provide frequency diversity while reducing the MAI.

Multicarriers can be selected orthogonally, and orthogonal frequency and code division multiplexing (OFCDM) system was proposed in [4]. OFCDM uses data spreading in both time and frequency domain, where each data stream is segmented into multiple substreams and spread over multiple subcarriers and several OFCDM symbols, exploiting additional frequency and time diversity. OFDM technology is incorporated in standard IEEE 802.11 for wireless local area networks and in IEEE 802.16 for metropolitan area networks [5]. In [6], OFDM based two dimensional spreading was proposed for future 4G wireless networks and field test was done by NTT DoCoMo to support the needs of OFCDM technology in future wideband communications [7]. There has been quite an amount of research work on subcarrier grouping and allocation to further improve the performance in OFCDM systems. For example, subcarriers are grouped and adaptively allocated in [8] to users by improving the signal to interference and noise ratio (SINR) while minimizing the MAI.

On the other hand, the transmission range of the next generation wireless networks is limited due to the higher operating frequencies [9]. Cooperative relay systems can improve the throughput, coverage and the reliability [10], [11] in wideband wireless communications. In order to get better performance, relaying was incorporated in the standard IEEE 802.16j (WiMAX) amendment and in 4G networks [12].

In future generations networks, OFCDM based relay networks would provide better performance. CDMA technique was introduced in relay systems in [13] [14] to take advantage of spread spectrum techniques. In [15], MC-CDMA decode-and-forward relay system performance was analyzed. The power allocation was analyzed in [16] for a CDMA based decode-and-forward relay system. However, there have been not much work done for OFCDM based relay networks. In this article, we investigate the BER performance of an

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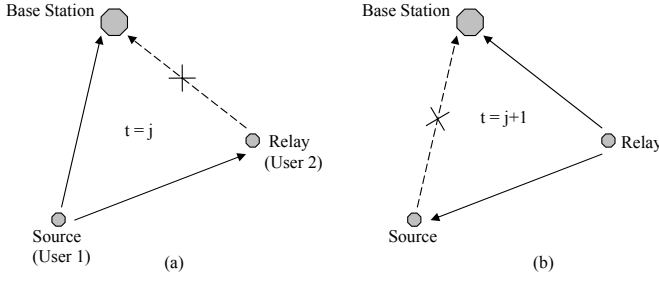


Fig. 1. System model (a) at time  $t = j$ , (b) at time  $t = j + 1$ .

OFCDM based decode-and-forward cooperative relay network for different power and subcarrier allocation schemes.

The rest of the article is organized as follows: In Section II, the system model for a two-hop relay network is presented. In Section III, relevant literature for one-hop and BER derivation for a two-hop OFCDM based decode-and-forward relay network are presented. In Section IV, a modified adaptive subcarrier allocation algorithm is proposed and the power allocation scheme between source and relay nodes is presented. Required BER performance analysis is shown in Section V. Section VI presents the performance comparison for different subcarrier allocation and power allocation schemes for different time-frequency spreading codes. Finally, this article concludes in Section VII with future work.

## II. TWO-HOP COOPERATIVE RELAY SYSTEM MODEL

We consider an uncoded two-hop wireless system where OFCDM is employed as multiple access technology. Users are denoted by  $U_k$  and total number of users is  $K$  (assumed to be even number). Odd numbered users ( $U_1, U_3, \dots, U_{K-1}$ ) are classified into one group and even numbered users ( $U_2, U_4, \dots, U_K$ ) into another group as done in [15]. Further, it is assumed that there is a one-to-one mapping between two groups of users and they are paired (as partners) for cooperative relaying. Source nodes and relaying nodes are allocated separate channels (time slots) during the transmission. Each node transmits its own data as well as the estimated data of its partner which was received in the previous time slot.

All the users have their own spreading codes. Each partner node is aware of its own spreading code as well as its partner's spreading code. Every user's information is retransmitted using that user's spreading code. That is, source transmits its own information using its own spreading code. Relay node decodes and forwards the information of its partner using its partner's spreading code and, at the same time transmit its own information using its own code [17].

Fig. 1 shows a couple of transmission cycles to understand the pair-wise communication ( $t = j, j + 1$ ) and Table I shows 4 consecutive transmission time instances. The User 1 is considered as source and the User 2 as relay. Further, the binary data stream of user  $k$  is denoted by  $\mathbf{b}_j^k = \pm 1$  and its estimate at the relay node by  $\hat{\mathbf{b}}_j^k$  where  $j \in \mathbb{N}$  indicates the time instance.

In the analysis, the channel is assumed to be slowly varying with respect to the OFCDM symbol duration. It is also assumed that subcarrier spacing is larger than the coherence

TABLE I  
TWO USER'S COOPERATIVE COMMUNICATION DURING 4 TIME SLOTS

Transmitting node	Time instance			
	$t=j$	$t=j+1$	$t=j+2$	$t=j+3$
User 1	$b_j^1$	-	$b_{j+1}^2, b_{j+2}^1$	-
User 2	-	$\hat{b}_j^1, b_{j+1}^2$	-	$\hat{b}_{j+2}^1, b_{j+3}^2$

bandwidth of the channel in a group and hence, frequency non-selective fading occurs on each subcarrier within a subcarrier group [3]. This allows the frequency domain channel to be modeled as:

$$H_{j,m}^k = \alpha_{j,m}^k e^{i\phi_{j,m}^k},$$

where  $i = \sqrt{-1}$ . The term  $\alpha_{j,m}^k$  is the Rayleigh fading gain for the  $m^{\text{th}}$  subcarrier of the  $k^{\text{th}}$  user during the  $j^{\text{th}}$  transmitted bit. The phase is a uniformly distributed random variable over the interval  $(0, 2\pi]$ , which is assumed to be independent for each bit, user, and subcarrier.

## III. BER DERIVATION FOR OFCDM BASED SYSTEM

In this section, BER performance for an one-hop OFCDM system with adaptive subcarrier allocation is first reviewed and for the two-hop decode-and-forward OFCDM based relay system is derived later.

### A. Single Link (One-hop) Transmission

For a direct one-hop transmission, the adaptive subcarrier allocation for an OFCDM system was proposed in [8]. In order to recover the data from User 1 at the receiver, the received signal is copied to the subcarrier branches that are assigned to User 1 and then signal on each subcarrier is restored to the baseband considering the phase response of the channel. Then each branch is multiplied by the synchronized time domain spreading sequence and the frequency domain pseudo-random (PN) chip. The despread signal is then multiplied by the fading gain,  $\alpha_{j,m}^1$ , according to the maximal ratio combining algorithm. Finally, the subcarriers are summed, integrated over the bit period, and sampled to yield the decision variable for User 1 as:

$$Z_j^1 = D_j^1 + I_j^1 + \eta_j^1, \quad (1)$$

where  $D_j^1$  is the desired signal,  $I_j^1$  is the interference from the other users on group  $y$ ,  $y = \{1, 2, \dots, Y\}$ , where  $Y$  is total number of subcarrier groups and  $\eta_j^1$  is the noise term which is considered as the AWGN noise signal with a double-sided power spectral density of  $N_o/2$  during  $j^{\text{th}}$  bit duration. Further,  $G_y$ ,  $M_y$  and  $K_y$  denote the set of subcarriers, number of subcarriers and the number of users respectively occupying a group  $y$  simultaneously. Since we are not concerned with the absolute performance of the receiver/detection schemes, we could use any receiver. MRC receiver has been widely used in the literature [3], [8], [18]–[20] for similar work. A multi-user detection (MUD) scheme such as MMSE can also be used; however, it will not affect our conclusions, though one would expect relative BER performance degradation in MRC receiver with an increase in the number of active users compared to MUD schemes.

The desired signal can be written as [8] for a single link direct transmission,

$$D_j^1 = N\sqrt{\varepsilon_c}b_j^1 \sum_{m \in G_y} (\alpha_{j,m}^1)^2 \quad (2)$$

where  $\varepsilon_c$  is the chip energy and  $N$  is the length of the PN sequence.

The power of the desired signal for user 1 is determined by computing the variance  $D_j^1$ . Since the bit stream,  $b_j^1$  has zero mean, the power of the desired signal can be written as:

$$P_{D_j^1}^1 = \text{Var}[D_j^1] = N^2\varepsilon_c \left[ \sum_{m \in G_y} (\alpha_{j,m}^1)^2 \right]^2, \quad (3)$$

where  $\text{Var}[\cdot]$  is the variance.

The interference term is calculated by considering the correlation between the received signals from all  $K_y$  users occupying subcarrier group  $y$  simultaneously, with the time and frequency domain spreading sequences for User 1.

The interference power can be approximated as a Gaussian random variable when the number of users is moderate to large, and the input data stream is random [3] [21]. The resulting interference power is,

$$P_{I_j}^1 = N\varepsilon_c(K_y - 1)E[(\alpha_y)^2] \sum_{m \in G_y} (\alpha_{j,m}^1)^2, \quad (4)$$

where  $E[(\alpha_y)^2]$  is the average fading gain for the  $K_y$  users and  $M_y$  subcarriers in group  $y$ . It can be calculated as follows:

$$E[(\alpha_y)^2] = \frac{1}{K_y M_y} \sum_{k \in G_y} \sum_{m \in G_y} (\alpha_{j,m}^k)^2$$

The noise power at output of the correlator can be written as:

$$P_{\eta_j}^1 = \text{Var}[\eta_j^1] = NN_o \sum_{m \in G_y} (\alpha_{j,m}^1)^2 \quad (5)$$

Based on the equations (3)-(5), the SINR for User 1 during the  $j^{\text{th}}$  bit assigned to subcarrier group  $y$ ,  $\gamma_{j,y}^1$  can be written as:

$$\gamma_{j,y}^1 = \frac{N\varepsilon_c \sum_{m \in G_y} (\alpha_{j,m}^1)^2}{(K_y - 1)\varepsilon_c E[(\alpha_y)^2] + N_o} \quad (6)$$

Further, the probability of bit error can be determined for the  $j^{\text{th}}$  bit as follows assuming BPSK modulation,

$$\text{Pr}_{b_j}^1 = Q\left(\sqrt{2\gamma_{j,y}^1}\right) \quad (7)$$

### B. Two-hop Cooperative Transmission

Based on the results in the previous section for one-hop system, the BER performance of the two-hop relay system is derived in this section. Few additional notations  $s \rightarrow r$ ,  $s \rightarrow bs$  and  $r \rightarrow bs$  are incorporated in the channel gain to differentiate the links, denoting source-relay link, source-base station link, and relay-base station link respectively. As shown in Fig. 1, the same source information is received at the base station ( $bs$ ) during two consecutive time slots. In the  $j^{\text{th}}$  time slot, the information is received from the source ( $s$ ) and, during the  $(j+1)^{\text{th}}$  time slot the information is decoded and forwarded by the corresponding relaying ( $r$ ) node. The

combined decision variables with equal confidence on both the links of User 1 after two consecutive time slots ( $j, j+1$ ) at the base station receiver can be written as below. The source node and the relay node chip energies are denoted by  $\varepsilon_{c_s}$  and  $\varepsilon_{c_r}$  respectively. The desired signal from (2) is:

$$\begin{aligned} D_{j+1,bs}^1 &= D_{j,bs}^{1,s \rightarrow bs} + D_{j+1,bs}^{1,r \rightarrow bs} \\ &= N\left(\sqrt{\varepsilon_{c_s}}b_j^1 \sum_{m \in G_{y,s}} (\alpha_{j,m}^{1,s \rightarrow bs})^2 \right. \\ &\quad \left. + \sqrt{\varepsilon_{c_r}}\hat{b}_{j+1}^1 \sum_{m \in G_{y,r}} (\alpha_{j+1,m}^{2,r \rightarrow bs})^2\right), \end{aligned} \quad (8)$$

where  $m \in G_{y,s}$  is the set of subcarriers in group  $y$  while source transmitting and,  $\alpha_{j,m}^{1,s \rightarrow bs}$ ,  $\alpha_{j+1,m}^{2,r \rightarrow bs}$  are the Rayleigh fading gain for the  $m^{\text{th}}$  subcarrier of the 1<sup>st</sup> user during the  $j^{\text{th}}$  bit transmission in the link  $s \rightarrow bs$  and, of the 2<sup>nd</sup> user during the  $(j+1)^{\text{th}}$  bit transmission in the link  $r \rightarrow bs$  respectively.

We can rewrite the desired decision variable at the base station on the condition of correct ( $D_{j+1,bs}^{1,c}$ ) and incorrect ( $D_{j+1,bs}^{1,f}$ ) decoding at the relay node as:

$$\begin{aligned} D_{j+1,bs}^{1,c} &= [D_{j+1,bs}^1 | b_j^1 = 1, \hat{b}_{j+1}^1 = 1] \\ &= N(\sqrt{\varepsilon_{c_s}} \sum_{m \in G_{y,s}} (\alpha_{j,m}^{1,s \rightarrow bs})^2 \\ &\quad + \sqrt{\varepsilon_{c_r}} \sum_{m \in G_{y,r}} (\alpha_{j+1,m}^{2,r \rightarrow bs})^2) \end{aligned} \quad (9)$$

$$\begin{aligned} D_{j+1,bs}^{1,f} &= [D_{j+1,bs}^1 | b_j^1 = 1, \hat{b}_{j+1}^1 = -1] \\ &= N(\sqrt{\varepsilon_{c_s}} \sum_{m \in G_{y,s}} (\alpha_{j,m}^{1,s \rightarrow bs})^2 \\ &\quad - \sqrt{\varepsilon_{c_r}} \sum_{m \in G_{y,r}} (\alpha_{j+1,m}^{2,r \rightarrow bs})^2) \end{aligned} \quad (10)$$

and the interference and noise power terms can be derived from (4) and (5) as shown in (11).

Based on the above result, the probability of bit error can be determined for the  $j^{\text{th}}$  bit for User 1 at the base station as:

$$\begin{aligned} \text{Pr}_{j,bs}^1 &= (1 - \text{Pr}_{j,r}^1) Q\left(\frac{\sqrt{2}(D_{j+1,bs}^{1,c})}{\sqrt{\Omega_{j+1,bs}^1}}\right) \\ &\quad + (\text{Pr}_{j,r}^1) Q\left(\frac{\sqrt{2}(D_{j+1,bs}^{1,f})}{\sqrt{\Omega_{j+1,bs}^1}}\right), \end{aligned} \quad (12)$$

where  $\Omega_{j+1,bs}^1$  is defined in (11) and  $\text{Pr}_{j,r}^1$  is defined as the probability of bit error of User 1 at the relay node while decoding before retransmitting to the base station. It can be calculated using (7) with

$$\gamma_{j,y}^1 = \frac{N\varepsilon_{c_s} \sum_{m \in G_{y,s}} (\alpha_{j,m}^{1,s \rightarrow r})^2}{(K_{y,s} - 1)\varepsilon_{c_s} E[(\alpha_y^{s \rightarrow r})^2] + N_o} \quad (13)$$

### IV. SUBCARRIER AND POWER ALLOCATION IN A RELAY NETWORK

In this section, a modified adaptive subcarrier allocation algorithm for an OFCDM based relay network is first proposed and then the power allocation parameters and the scenarios are discussed for the subsequent analysis.

$$\begin{aligned}\Omega_{j+1,bs}^1 &= N \sum_{m \in G_{y,s}} (\alpha_{j,m}^{1,s \rightarrow bs})^2 ((K_{y,s} - 1)\epsilon_{c_s} E[(\alpha_{j,y}^{s \rightarrow bs})^2] + N_o^{s \rightarrow bs}) \\ &+ N \sum_{m \in G_{y,r}} (\alpha_{j+1,m}^{2,r \rightarrow bs})^2 ((K_{y,r} - 1)\epsilon_{c_r} E[(\alpha_{j+1,y}^{r \rightarrow bs})^2] + N_o^{r \rightarrow bs}).\end{aligned}\quad (11)$$

### A. Subcarrier Allocation

In [8], an adaptive subcarrier allocation algorithm was proposed for a single link transmission to improve the overall BER performance of a OFCDM system. The subcarriers are assigned to users that are having higher SINR at the same time reducing the interference caused by this assignment to other users in the same subcarrier group. This algorithm is modified later in this section for a decode-and-forward OFCDM based two-hop relay system.

In a two-hop relay system, there are 3 links, namely ( $s \rightarrow r$ ,  $s \rightarrow bs$  and  $r \rightarrow bs$ ) that contribute to the overall BER performance of the system. As described earlier, the two consecutive time slot transmission from the source and the relay are assumed to fade independently; hence, the subcarrier allocation is also done separately.

During the  $j^{\text{th}}$  time slot, the source will transmit and, the base station and the cooperating relay node will receive on a same set of subcarrier group ( $G_{y,s}$ ). Therefore, the subcarrier allocation algorithm is carried out based on the availability of the channel state information (CSI) of the links  $s \rightarrow r$ , and  $s \rightarrow bs$ .

We introduce a parameter  $\nu$  ( $0 \leq \nu \leq 1$ ), called *cooperative CSI (C-CSI)* which weighs in end-to-end SINR for a two-hop transmission and hence provides different impact on subcarrier allocation.  $\nu=0.5$  means that the subcarrier allocation algorithm tries to optimize both links ( $s \rightarrow bs$  and  $s \rightarrow r$ ) simultaneously giving equal weight. In this case, maximizing the SINR while minimizing the MAI would be performed on both links together with equal weight. When  $\nu=1$ , the subcarrier allocation is done solely based on the CSI of the link  $s \rightarrow bs$  whereas when  $\nu=0$ , it is solely on the CSI of the link  $s \rightarrow r$ .  $\nu$  is therefore called *cooperative CSI* that adjusts the weight of different cooperative links.

Hereafter, the time instance  $j$  is dropped in the notations for simplicity. Decision variable (i.e., SINR) in subcarrier allocation for a two-hop relay system for user  $k$  is defined giving different weights to cooperative links as:

$$\gamma_y^k = \nu \gamma_y^{k,bs} + (1 - \nu) \gamma_y^{k,r}, \quad (14)$$

where  $\gamma_y^{k,bs}$  and  $\gamma_y^{k,r}$  are defined as SINR of user  $k$  belonging to subcarrier group  $y$  at the base station and the relay node respectively and they are calculated as in (13).

The subcarrier allocation algorithm is described briefly as follows:

(a) Subcarrier group that has the largest SINR  $\gamma_y^k$  as defined in (14) for all  $k$  users is found as follows:

$$\gamma_{max}^k = \max\{\gamma_1^k, \gamma_2^k, \dots, \gamma_Y^k\}$$

The index denoted  $y_{max}^k$  is recorded.

(b) From the result in (a), the subcarrier group with the smallest SINR is found as follows:

$$\gamma_{min} = \min\{\gamma_{max}^1, \gamma_{max}^2, \dots, \gamma_{max}^K\}$$

The index of the user with the lowest SINR value, denoted  $k_{min}$  is recorded.

(c) User  $k_{min}$  is assigned to the subcarrier group  $y_{max}^{k_{min}}$  and the SINR is re-calculated for that assigned group using (13). The algorithm is repeated until all the data streams are assigned.

The subcarrier group with the highest SINR for each user will produce the best BER performance for each user (in step (a)). When the user with the smallest  $\gamma_{max}^k$  (in step (b)) is provisioned to the subcarrier group first (in step (c)), it reduces the average fading gain in the group, and consequently reduces the amount of interference to other users.

### B. Power Allocation

The source node and the relay node chip energies are denoted by  $\epsilon_{c_s}$  and  $\epsilon_{c_r}$  respectively. The total chip energy ( $\epsilon_c$ ) is kept as constant and can be written as

$\epsilon_c = \epsilon_{c_s} + \epsilon_{c_r}$ . The cooperative power ratio  $\lambda$  ( $0 \leq \lambda \leq 1$ ) is defined as:

$$\lambda = \frac{\text{source node power}}{\text{total power of source and relay nodes}} = \frac{\epsilon_{c_s}}{\epsilon_c} \quad (15)$$

and  $\epsilon_{c_r}$  can be defined as  $(1 - \lambda)\epsilon_c$ . In the numerical analysis, the various power allocation strategies among the source and the relay node are discussed with different subcarrier allocation schemes. Further, the BER performance analysis is carried out for two different scenarios.

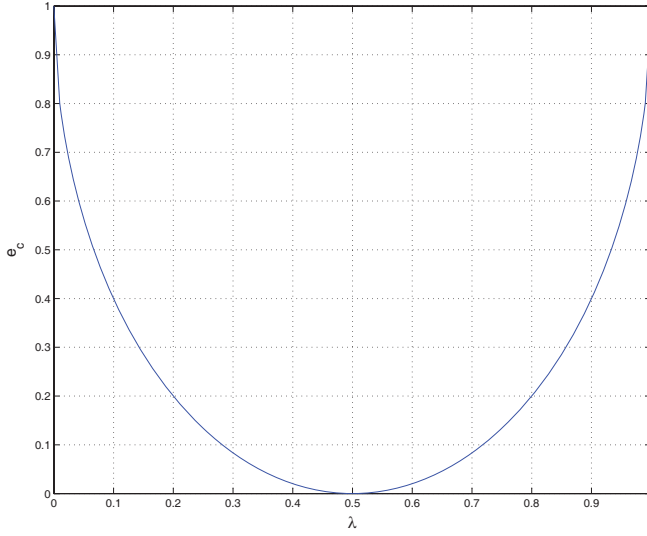
- Case 1: Equal channel gains for all three links ( $s \rightarrow bs$ ,  $s \rightarrow r$  and  $r \rightarrow bs$ )
- Case 2: Strong source to relay link ( $\xi$  dB better than other two links). This case is more practical as one would normally resort to relay communication under this condition.

The power allocation strategies are discussed in the Section VI.

## V. BER PERFORMANCE ANALYSIS

The analysis for the overall BER performance is done in this section for the scheme where the subcarrier allocation is done giving equal weight to the links  $s \rightarrow bs$  and  $s \rightarrow r$  ( $\nu=0.5$ ). This analysis can be extended to other schemes as well. For the analytical purpose, equation (12) is simply denoted as:

$$P_e = (1 - e_r)e_c + e_r e_f, \quad (16)$$


 Fig. 2. Normalized curve for  $e_c$  vs  $\lambda$ .

where each term represents the corresponding term in (12). First, we consider the term  $e_c = Q\left(\frac{\sqrt{2}(D_{j+1,b}^{1,c})}{\sqrt{\Omega_{j+1,b}^1}}\right)$ . It can be assumed that both the links  $s \rightarrow bs$  and  $r \rightarrow bs$  have similar fading characteristics when the relay node is used for the diversity advantage. Hence, the number of subcarriers and the total channel gains within a subcarrier group can be assumed approximately the same in (8)-(10). Further, in an interference-limited system, the noise power can also be neglected compared to the interference power. Based on these assumptions,  $e_c$  can be approximated as:

$$e_c \approx Q\left(\frac{K(\sqrt{\epsilon_{cs}} + \sqrt{\epsilon_{cr}})}{\sqrt{\epsilon_{cs} + \epsilon_{cr}}}\right), \quad (17)$$

where  $K$  is constant for a fading block. It can be further simplified by substituting  $\epsilon_{cs} + \epsilon_{cr} = \epsilon_c$  as:

$$e_c \approx Q\left(K(\sqrt{\lambda} + \sqrt{1-\lambda})\right). \quad (18)$$

Equation (18) is a convex function and it has the minimum when the power is equally shared between the source and the relay node (i.e.,  $\lambda=0.5$ ) as shown in the Fig. 2.

Similarly, it can be shown that the term  $e_f$  has the worst performance when  $\lambda=0.5$ . On the other hand, the term  $e_r$  monotonically decreases with  $\lambda$  as the  $Q(\cdot)$  function does. Therefore, the term  $(1 - e_r)$  is an increasing function. These observations are used to interpret the results later.

#### A. Higher $E_b/N_o$

The  $s \rightarrow r$  link error probability ( $e_r$ ) is comparatively minimal at higher  $E_b/N_o$  range even though the power allocation strategies vary at the source. In this case, the second term in (16),  $e_r.e_f$ , would be negligibly smaller compared to the other term  $(1 - e_r)e_c$ . As we discussed in previous section, when  $\lambda$  is closer to one, the the performance of  $e_c$  and  $(1 - e_r)$  are getting worse; hence, the system performance is worse at higher  $E_b/N_o$  for higher  $\lambda$ . At lower  $\lambda$ , even though  $e_c$  is

getting worse,  $(1 - e_r)$  is improving and hence, slightly better performance can be noticed compared to higher  $\lambda$  values (this can be seen later in Fig. 5 at 20 dB).

#### B. Lower $E_b/N_o$

At lower  $E_b/N_o$  values, the  $s \rightarrow r$  link performance will vary widely based on the received SINR at the relay node. If the transmit power is higher at the source node or  $s \rightarrow r$  link is having better channel gains, the BER at the relay node would be better. At lower  $E_b/N_o$  values, the lower power allocation (lower  $\lambda$ ) to the source node leads to higher BER ( $e_r$ ) at the relay node. Further, the higher power allocation to the relay node makes the relay node transmit the incorrectly decoded bits with high power. This affects the term  $e_f$  further, but the performance of the term  $e_c$  remains almost the same irrespective to the power allocation. Hence, the second term  $e_r.e_f$  is highly determining the overall BER of the system at lower  $E_b/N_o$  (as shown later in the Fig. 5 at 0 dB for Case 1). The performance should be analyzed using both the terms in (16) when the link  $s \rightarrow r$  has different channel gain compared to the other two links.

### VI. NUMERICAL SIMULATION AND DISCUSSION

In this section, the simulation scenarios and the BER performance for different schemes are discussed. The BER performance results are numerically evaluated using Monte-Carlo simulation. The subcarrier allocation is done based on the availability of the CSI between the links  $s \rightarrow r$  and  $s \rightarrow bs$ . Subcarriers are grouped so that within a group, all the subcarriers undergo independent fading. Further, the uplink bandwidth is assumed to be 20MHz with 128 subcarriers. There are 64 users grouped into two and paired. In one time slot, 32 users are assigned to the channel by the subcarrier allocation algorithm. Each of the users is assumed to be traveling at a velocity of 5km/h, which corresponds to a Doppler frequency of 23.14Hz and to a coherence time of approximately 18μs. It is assumed that delay spread is 6.4μs which is common in urban areas [21].

The numerical simulation is carried out for the decode-and-forward OFCDM based two-hop relay system with different power allocation schemes. When a time domain spreading factor of 4 is utilized with a frequency domain processing gain of 8, it is denoted by  $4 \times 8$ .

#### A. Subcarrier Allocation with $\lambda=0.5$

In this section, different subcarrier allocation schemes are evaluated for a system with equal power sharing between the source and the relay nodes ( $\lambda=0.5$ ).

Fig. 3(a) shows the different BER performance of a  $4 \times 8$  (time x frequency) spread two-hop decode-and-forward relay system for different C-CSI ( $\nu$ ) values. From this figure, we can notice that the subcarrier allocation of a decode-and-forward two-hop relay depends on the channel state information of the link  $s \rightarrow r$  as further explained next.

In a typical relay system, the CSI of the link  $s \rightarrow bs$  is usually available. When the CSI of both links  $s \rightarrow bs$  and  $s \rightarrow r$  are available and equal weight is given (i.e.,  $\nu=0.5$ ) in the

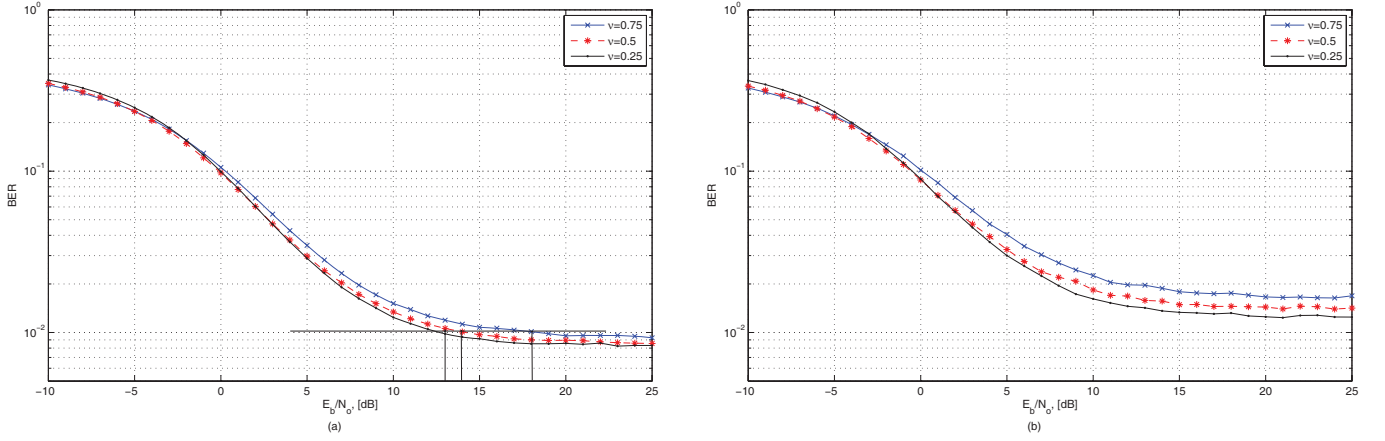


Fig. 3. BER performance comparison for different spreading systems with different C-CSI ( $\nu$ ): (a)  $4 \times 8$  (b)  $8 \times 4$ .

subcarrier allocation, it gives 4 dB gain over the subcarrier allocation with  $\nu=0.75$ , where subcarrier allocation is done mainly based on the link  $s \rightarrow bs$  at BER of  $10^{-2}$ . Interestingly, when the subcarrier allocation is done mainly based on the link  $s \rightarrow r$  ( $\nu=0.25$ ), it outperforms the allocation which is done mainly based on the link  $s \rightarrow bs$  ( $\nu=0.75$ ) by 5 dB at BER of  $10^{-2}$ . The decision variable at the receiver ( $bs$ ) is considered during the decoding process giving equal weight on both the received signal from the source and the forwarded signal from its partner. Therefore, the incorrectly decoded bits at the relay node will cause higher error probability at the base station. At higher  $E_b/N_o$ , it is better to do the subcarrier allocation considering mainly the inter-partner (source-relay) CSI, provided it is accurate.

Similar characteristic is observed in Fig. 3(b) for a system with spreading factor of  $8 \times 4$ . We can notice that the error floor improves when the value of  $\nu$  decreases, i.e., the subcarrier allocation is done giving higher weight to the link  $s \rightarrow r$ . The error floor is 0.0169 when  $\nu=0.75$  whereas it is 0.0125 when  $\nu=0.25$ . The performance is highly interference-limited at higher  $E_b/N_o$  for higher time spreading schemes as shown in Fig. 3. Further, the error floors for the schemes  $16 \times 2$  and  $2 \times 16$  are shown in Table II and explained in detail in Section VI-B2.

In the case of a specific spreading factor (for example  $8 \times 4$ ), different  $E_b/N_o$  values influence in selecting the index  $\nu$  when both link CSIs are perfectly available. For example, even though  $\nu=0.25$  is performing better at higher  $E_b/N_o$ , for  $E_b/N_o < 0$  dB,  $\nu=0.5$  is preferred over  $\nu=0.25$  for an  $8 \times 4$  system as shown in Fig. 3. We also observed the significance of this selection in higher time spreading systems such as  $16 \times 2$  and  $32 \times 1$  (for which results are not included in this article). Even though the selection of  $\nu$  in subcarrier allocation is less significant for the schemes with higher frequency spreading, it influences the schemes with higher time spreading.

In Fig. 4, BER performance is shown for different spreading factors, from higher (lower) time (frequency) spreading to lower (higher), by varying the parameter C-CSI ( $\nu$ ). At lower  $E_b/N_o$ , it is better to use higher spreading in time, and lower spreading in frequency as also reported in [8]. The *cross-over point* is defined as an  $E_b/N_o$  value around which the

TABLE II  
ERROR FLOOR FOR DIFFERENT  
SPREADING AND  $\nu$ .

C-CSI ( $\nu$ )	BER	
	$16 \times 2$	$2 \times 16$
0.25	0.0208	0.0057
0.5	0.0253	0.0058
0.75	0.0294	0.0059

TABLE III  
CROSS-OVER POINT FROM  
FREQUENCY TO TIME SPREADING.

C-CSI ( $\nu$ )	$E_b/N_o$ (dB)
0	5
0.25	4
0.5	3
0.75	1
1	-1

performance of the frequency spreading becomes better than the time spreading. In Fig. 4, when  $\nu$  increases from 0.25 to 0.75, the cross-over point ( $E_b/N_o$ ) value moves from 4 dB to 1 dB. It is also tabulated for different values of  $\nu$  the Table III and it shows a relationship between the cross-over point ( $E_b/N_o$ ) and the C-CSI ( $\nu$ ) which provide additional flexibility in operating environment for OFCDM systems between time and frequency spreading.

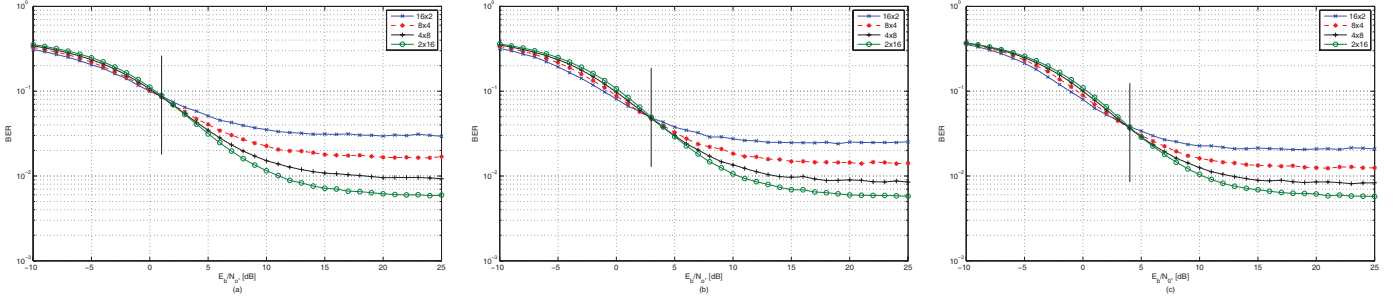
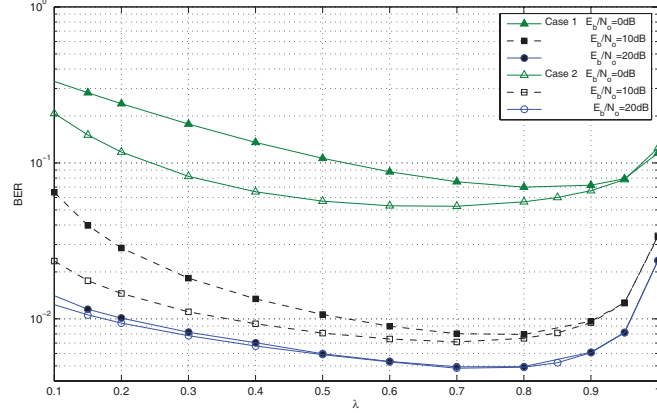
### B. Power Allocation with $\nu=0.5$

In this section, different power allocation schemes are evaluated for a system where the subcarrier allocation is done giving equal weight to the links  $s \rightarrow bs$  and  $s \rightarrow r$  ( $\nu=0.5$ ).

1) *Different Source to Relay Channel Gains*: Fig. 5 shows the BER performance for different cooperative power ratio ( $\lambda$ ) at different  $E_b/N_o$  values for a  $2 \times 16$  scheme for two different cases described in Section IV-B.

For the equal link gain (Case 1, at higher  $E_b/N_o$  (20 dB)), the BER performance is optimum when  $\lambda \approx 0.8$ . When  $\lambda$  increases further, that is source node power is increased, the performance deteriorates at a faster rate than when  $\lambda$  decreases. Therefore, when  $\lambda$  takes the value less than the optimum, it is less sensitive to the BER performance and hence, it is safe to operate in the region  $0.6 \leq \lambda \leq 0.85$  with less than 10% performance degradation. On the other hand, at lower  $E_b/N_o$  values, the BER performance is also sensitive when  $\lambda$  decreases. Hence, the operating region shrinks to  $0.75 \leq \lambda \leq 0.85$ .




 Fig. 4. BER performance comparison with various spreading factor: (a)  $\nu=0.25$  (b)  $\nu=0.5$  (c)  $\nu=0.75$ .

 Fig. 5. BER performance comparison for  $2 \times 16$  scheme with different power allocation at  $E_b/N_o=0$  dB, 10 dB and 20 dB ( $\nu=0.5$ ).

In the case of stronger  $s \rightarrow r$  link (Case 2) considering here we consider  $\xi=5$  dB, we can notice a similar trend at higher  $E_b/N_o$  values but at lower  $E_b/N_o$  values, Case 2 performance is better when lower power is allocated to the source node compared to Case 1. That is, the optimal cooperative power ratio moves towards  $\lambda \approx 0.65$  for lower  $E_b/N_o$  values when the  $s \rightarrow r$  has 5 dB better channel gain compared to the other two links as shown in Fig. 5.

Fig. 6(a) and Fig. 6(b) show the BER performance for different cooperative power ratio for both the cases when the subcarrier allocation is done giving equal weight ( $\nu=0.5$ ) on both the links  $s \rightarrow bs$  and  $s \rightarrow r$ . As we discussed earlier, at higher  $E_b/N_o$ , BER performance reaches the interference-limited error floor according to the power allocation scheme in both cases. Fig. 6(b) shows that when the channel gain on the link  $s \rightarrow r$  is better, it gives better performance on low-to-moderate  $E_b/N_o$  region; but at higher  $E_b/N_o$ , it reaches the same error floors. Further, it shows 5 dB, 1.5 dB and 1 dB gains at  $\lambda=0.25$ ,  $\lambda=0.5$  and  $\lambda=0.75$  respectively at BER of  $10^{-2}$  for the system with better channel gain of 5 dB between the  $s \rightarrow r$  link than the one having equal gain. Table IV shows the above results.

We can notice at higher  $E_b/N_o$  when the link  $s \rightarrow r$  is stronger, the interference-limited error remains the same for both the cases and there is no significant difference in optimal power allocation strategy ( $\lambda$ ). On the other hand, at lower  $E_b/N_o$  values, different power allocation strategies have different impact.

 TABLE IV  
PERFORMANCE COMPARISON AT BER OF  $10^{-2}$  FOR DIFFERENT  $\lambda$  ( $\nu=0.5$ ).

Power Allocation Ratio ( $\lambda$ )	Required $E_b/N_o$ at BER of $10^{-2}$	
	Equal Gain	Strong $s \rightarrow r$ Link
0.25	8.25	7.25
0.50	10.5	8.0
0.75	17.0	13.0

2) *Different Time-Frequency Spreading Schemes*: Fig. 7 shows the BER performance of different spreading schemes ( $4 \times 8$ ,  $8 \times 4$ ) for different  $E_b/N_o$  values. We notice the similar phenomena as discussed earlier for  $2 \times 16$  scheme. We also notice that at low  $E_b/N_o$  (0 dB) the performance is better for an  $8 \times 4$  scheme than the  $4 \times 8$  scheme whereas, at higher  $E_b/N_o$  (10 dB, 20 dB),  $4 \times 8$  scheme outperforms the other.

At lower  $E_b/N_o$ , the performance is limited by thermal noise. Higher number of groups with few subcarriers lead to allocate subcarriers with best channel conditions. Therefore, lower frequency spreading with higher time spreading offers better performance. Also the MAI is minimal since a relatively lower number of users is sharing each group of subcarriers. On the other hand, at higher  $E_b/N_o$ , the system performance is limited by MAI. Therefore, selecting a better channel does not necessarily offer better performance. Further, larger group size offers better frequency diversity and at higher  $E_b/N_o$  it is better to have higher frequency spreading with lower time spreading as also reported in [8].

The optimal power allocation strategy is not dependent on different spreading factors, but the power allocation affects differently at the extreme power allocation conditions ( $\nu \approx 0/1$ ) for different spreading schemes.

## VII. CONCLUSIONS

We first proposed a cooperative CSI-based subcarrier allocation algorithm for a OFCDM-based two-hop decode-and-forward relay system. Its BER performance was evaluated via Monte Carlo simulation. In the case of equal confidence given at the base station receiver for the source and its partner's forwarded data in a decode-and-forward OFCDM two-hop relay diversity system, CSI of the source-relay link (with lower *cooperative CSI*) influences the subcarrier allocation at moderate to high  $E_b/N_o$  values. Further, when subcarrier allocation is done giving higher weight on the CSI of the link

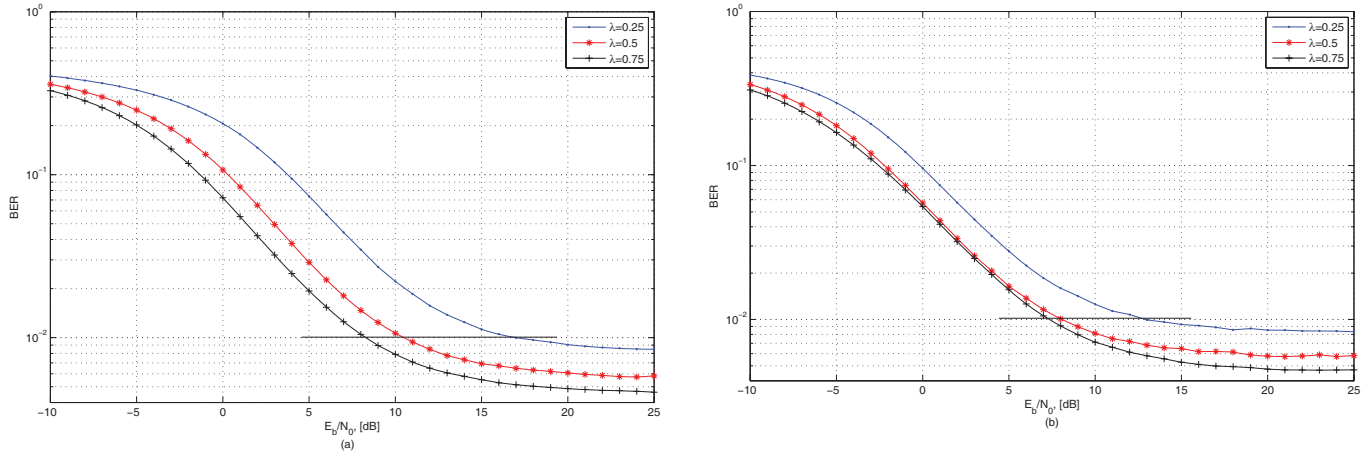


Fig. 6. BER performance comparison for  $2 \times 16$  scheme with C-CSI,  $\nu=0.5$ , for various power allocation with: (a) Equal Channel Gain (Case 1) (b) 5 dB higher  $s \rightarrow r$  link (Case 2).

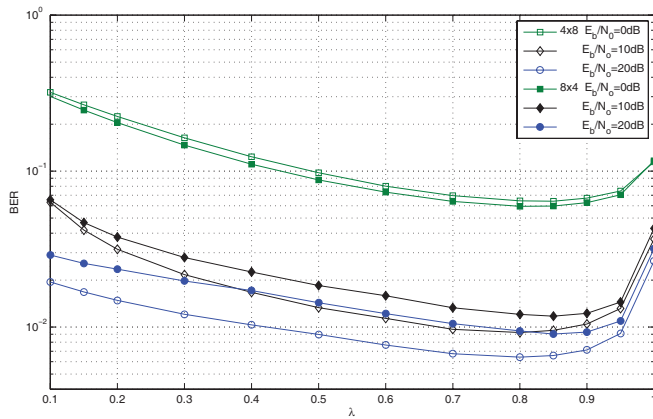


Fig. 7. BER performance comparison for  $4 \times 8$  and  $8 \times 4$  schemes with different power allocation at  $E_b/N_0 = 0$  dB, 10 dB and 20 dB ( $\nu=0.5$ ).

$s \rightarrow bs$  (as  $\nu$  is increased) the time-frequency spreading cross-over point moves towards lower  $E_b/N_0$  values.

Secondly we evaluated the power allocation schemes. There is an optimal power allocation between the source node and the relay node and that varies for different operating  $E_b/N_0$  values. When all three links have equal gains, the optimal power allocation ratio is  $\lambda \approx 0.8$ . Further, if a system has better  $s \rightarrow r$  link, it will improve the performance at lower  $E_b/N_0$  values but not the interference-limited error floor. For a such system, at lower  $E_b/N_0$  values, the optimal power allocation ratio is  $\lambda \approx 0.65$ . Further, the optimal power allocation strategy is not highly dependent on different time-frequency spreading factors.

We can conclude from the above results that optimal power allocation between the source-relay nodes combined with proper subcarrier allocation significantly improves the system performance of a OFCDM based decode-and-forward relay network.

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