because the number of discarded packets was not high in the previous queue.

Tables I and II show that the FDR-dominant region expands in regard to transmission efficiency and packet loss rate as the queue size increases. A large-sized queue with AM can hold packets when the SINR worsens and then forward them when the SINR improves, thus compensating for performance degradation caused by interference. With respect to delay, a large-sized queue reduces the FDR-dominant region when the number of packets arriving from the upper layer is relatively high, compared with the supportable number of packets transmitted by AM. Otherwise, it expands the FDR-dominant region. In the former case, reliable packet transmission is a difficult proposition, as shown in Fig. 3. Moreover, note that the crossing SNR for the delay is not significantly changed by the size of the queue, as compared to the transmission efficiency and packet loss rate shown in Tables I and II. Therefore, in general, FDR proves more advantageous than HDR as the queue size increases.

V. Conclusion

This paper has proposed a framework for investigating the queuing aspects of FDR and HDR. We have first derived FSMC models with AM. Next, the transmission efficiency, packet loss rate, and delay have been evaluated and compared. Our results have shown that FDR is superior to HDR in the low-SNR region but have lost its advantage as the SNR increases. Unless the number of arriving packets from the upper layer is so high that it hinders reliable packet transmission, increased queue size generally benefits FDR more than HDR.

References

[1] IEEE Std. 802.16j, IEEE 802.16j-08/127r1: Proposal for STR Relaying on the Same Carrier, Jul. 2008.
[2] H. Ju, E. Oh, and D. Hong, "Improving efficiency of resource usage in two-hop full duplex relay systems based on resource sharing and interference cancellation," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3933–3938, Aug. 2009.
[3] D. W. Bliss, P. A. Parger, and A. R. Margetts, "Simultaneous transmission and reception for improved wireless network performances," in *Proc. IEEE Int. Workshop Stat. Signal Process.*, Aug. 2007, pp. 472–478.
[4] H. Ju, E. Oh, and D. Hong, "Catching resource-devouring worms in next-generation wireless relay systems: Two-way relay and full-duplex relay," *IEEE Commun. Mag.*, vol. 47, no. 9, pp. 58–65, Sep. 2009.
[5] D. Skraparlis, V. K. Sakarellos, A. D. Panagopoulos, and J. D. Kanellopoulos, "Outage performance analysis of cooperative diversity with MRC and SC in correlated lognormal channels," *EURASIP J. Wireless Commun. Netw.*, vol. 2009, pp. 1–7, 2009, Article ID 707839.
[6] T. Kwon, S. Lim, S. Choi, and D. Hong, "Optimal duplex mode for DF relay in terms of outage probability," *IEEE Trans. Veh. Technol.*, vol. 59, no. 7, pp. 3628–3634, Sep. 2010.
[7] P. Cheng, G. Yu, Z. Zhang, H. Jia, H. H. Chen, and P. Qiu, "Cross-layer performance analysis of two-hop wireless links with adaptive modulation," in *Proc. IEEE Int. Symp. Pers., Indoor, Mobile Radio Commun.*, Sep. 2006, pp. 1–5.
[8] K. Zheng, Y. Wang, L. Lei, and W. Wang, "Cross-layer queuing analysis on multihop relaying networks with adaptive modulation and coding," *IET Commun.*, vol. 4, no. 3, pp. 295–302, Feb. 2010.
[9] A. Müller and H. C. Yang, "Dual-hop adaptive packet transmission with regenerative relaying for wireless TDD systems," in *Proc. IEEE Int. Conf. Global Telecommun.*, Nov. 2009, pp. 552–557.
[10] X. Qiu and K. Chawla, "On the performance of adaptive modulation in cellular systems," *IEEE Trans. Commun.*, vol. 47, no. 6, pp. 884–895, Jun. 1999.
[11] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. New York: McGraw-Hill, 1991.
[12] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 2001.
[13] J. D. C. Little, "A proof of the queuing formula $L = \lambda W$," *Oper. Res.*, vol. 9, no. 3, pp. 383–387, May 1961.

Cooperative Sensing With Correlated Local Decisions in Cognitive Radio Networks

Lamiaa Khalid and Alagan Anpalagan

***Abstract*—In this paper, we analyze the impact of correlated secondary users' local decisions on the performance of cooperative spectrum-sensing schemes when the counting rule is employed at the fusion center. We employ a correlation model that is indexed by a single parameter ρ. We derive the system probabilities of detection and false alarm for the K-out-of-M counting rule when the secondary users' local decisions are correlated under both hypothesis. Our performance evaluations are based on two performance criteria, which are the Neyman–Pearson (NP) criterion and the minimization of the sensing errors. Our results show that, for each value of the correlation index, there exists an optimal value of K that satisfies each criterion. We use genetic algorithm to find the optimal setting that minimizes the total probability of sensing error since the optimization problem under the correlation model used in our analysis is a mixed integer nonlinear problem with nonlinear constraint.**

***Index Terms*—Cognitive radio, cooperative spectrum sensing, correlation index, decision fusion.**

I. Introduction

Cognitive radio (CR) is considered a promising approach for opportunistic access of underutilized licensed spectrum [1]. CRs employ spectrum sensing to determine vacant licensed frequency bands and restrict their secondary transmissions to empty bands to meet the regulatory requirements of limiting harmful interference to licensed systems [2]. Spectrum sensing is often considered as a detection problem, in which the main challenge is the detection of the weak signal from a primary transmitter through local observations of CR users. Several detection techniques can be used in spectrum sensing such as energy detection, matched filter detection, and cyclostationary feature detection [3]. In this paper, we focus on the energy detection approach since it has low computational and implementation complexities, and prior knowledge of the primary users' signal is not needed [4].

Cooperative spectrum sensing, in which information from multiple CR users are incorporated for the detection of the primary user, can improve the spectrum-sensing performance [4]. To combine the sensing information from multiple CR users, decision-based fusion [5] or data-based fusion [6] schemes can be used. For the decision fusion scheme, each CR user independently performs local spectrum sensing and then makes a binary decision and forwards this decision to a fusion center, which makes the final decision. For the data fusion scheme, each CR user directly sends its observation value to the fusion center for it to make the final decision. In [6], a linear cooperation strategy that is based on the optimal combination of the local statistics from spatially distributed CRs is developed. In [5], for the case of independent CR observations, it was shown that cooperating with all users in the network does not necessarily achieve optimum performance. They used a constant detection rate and a constant false alarm rate

Manuscript received November 26, 2010; revised May 30, 2011 and October 3, 2011; accepted November 2, 2011. Date of publication December 2, 2011; date of current version February 21, 2012. The review of this paper was coordinated by Prof. N. Arumugam.

The authors are with the Department of Electrical and Computer Engineering Ryerson University, Toronto, ON M5B 2K3, Canada.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2011.2177871

for optimally selecting the CR users with the highest primary user's signal-to-noise ratio (SNR) for cooperation using AND and OR fusion rules. In [7], it was shown that the optimal fusion rule when the sensor observations are conditionally independent is a *majority voting* (MV) rule in the case of binary local detectors. In [8], optimizing the K value of the K-out-of-M fusion rule was considered, and it was found that the optimal decision fusion rule to minimize the total error probability is the MV rule.

Although the independence assumption on the local detector observations simplifies the problem, this assumption is not practical in the case where the proximity of the local detector results in correlated observations. The observations at the local detectors will be dependent if one detects a random signal in noise or if the detector noise samples are correlated when detecting a deterministic signal in noise [9]. The performance of a distributed detection system with the given local decision rules and correlated local decisions was studied in [10], and the optimum decision fusion rule in the Neyman-Pearson (NP) sense was derived and analyzed. In [11], the authors studied the effect of correlated noise on the performance of a distributed detection system in terms of the probability of detection using the NP criterion when the fusion rule was fixed to one of the standard rules such as AND, OR, or the MV rule. They did not consider any specific correlation model but instead considered only symmetric multidimensional noise densities, which can be completely described by correlation coefficient. The optimal data fusion rule was developed for correlated local binary decisions, in terms of the conditional correlation coefficients conditioned on the hypothesis, for all orders in [12]. However, their model, in its general form, is computationally complex since it needs the estimation of $2(2^n - n - 1)$ correlation coefficients, where n is the number of detectors, to obtain the optimal log-likelihood ratio test. In [13], the authors studied the special case that the probability density functions under both hypotheses are multivariate Gaussian, with the aim of detecting the shift in the mean of a pair of dependent Gaussian random variables. The problem of fusing the decisions made at the local detectors when the CR users observe conditionally dependent data due to correlated shadowing was studied in [14], with the assumption that the noise observations are independent. A suboptimal temporal detector was proposed based on a linear quadratic (LQ) detector, which uses partial statistical knowledge to improve detection performance. Their results showed that the suboptimal LQ detector outperforms the counting rule only when the correlation between the secondary users is strong.

In this paper, we analyze the impact of correlated CR users' local decisions on the performance of cooperative spectrum-sensing schemes when the K-out-of-M counting rule is employed at the fusion center. The main contributions of this paper are given here.

1) We derive the system probability of detection and the probability of false alarm, when the CR local decisions are correlated under both hypotheses, for the K-out-of-M counting rule and the special cases of the AND, OR, and MV fusion rules.
2) We use the NP criterion to optimize the network probability of false alarm with constraint on the network probability of detection when the local decisions are correlated.
3) We show that there is an optimal value of K that satisfies the NP criterion for each correlation index.
4) Motivated by this finding, we formulate the problem of minimizing the total probability of sensing error under the correlation model used in our analysis as a mixed integer nonlinear programming (MINLP) problem.
5) To solve the problem, we employ genetic algorithm (GA) to find the optimal assignments for K and the local probability of false alarm for a certain correlation index.

The rest of this paper is organized as follows: The system model and correlation model adopted in this paper are described in Sections II and III, respectively. Based on the models, we derive the system probability of detection and false alarm for the different fusion rules in Section IV. In Section V, we formulate the problem of minimizing the probability of sensing error as a mixed integer nonlinear problem. Simulation results and comparisons are presented in Section VI. Section VII concludes this paper.

II. Cognitive Radio System Model

In this paper, we consider energy detection in an additive white Gaussian noise (AWGN) channel. We consider a CR network with M secondary CR users, which can opportunistically access the licensed spectrum allocated to primary users. The problem of detecting the presence of primary users is equivalent to distinguishing between the following two hypotheses:

$$x_i(n) = \begin{cases} v_i(n), & H_0 \\ h_i s(n) + v_i(n), & H_1 \end{cases} \tag{1}$$

where $x_i(n)$ is the received signal of the ith secondary user at the nth time instant; h_i is the ith user channel gain, which is assumed to be constant during the detection interval; $s(n)$ is the primary user's transmitted signal; and $v_i(n)$ is the AWGN. Without loss of generality, $s(n)$ and $v_i(n)$ are assumed to be independent. The goal of spectrum sensing is to decide between two hypotheses H_0 and H_1, which are the hypotheses that the primary user is absent and present, respectively.

The test statistics for the energy detector for the ith user Y_i is computed as the sum of the received signal energy over an interval of N samples and is given by [5]:

$$Y_i = \sum_{n=0}^{N-1} |x_i(n)|^2 . \tag{2}$$

Without loss of generality, in this paper, we assume that noise $v_i(n)$ is real Gaussian noise with zero mean and variance σ^2 and that the primary user signal $s(n)$ is a binary phase-shift keying modulated signal. For a large number of samples N ($N \geq 10$ [6]), using the central limit theorem [15], the distribution of the test statistics Y_i can be approximated by a Gaussian distribution such that $Y_i \sim \mathcal{N}(N\sigma^2, 2N\sigma^4)$ under H_0 and $Y_i \sim \mathcal{N}((N+\Gamma_i)\sigma^2, 2(N+2\Gamma_i)\sigma^4)$ under H_1 [6]. Assuming that the primary users have uniform transmitted power, i.e., $E[(s(n))^2] = 1$, the SNR of the primary signal at the ith secondary user will be $\Gamma_i = (N|h_i|^2)/(\sigma^2)$.

The decision on the occupancy of a certain subchannel can be obtained by comparing test statistic Y_i to threshold γ_i. The performance of the detection algorithm is characterized by two probabilities, i.e., the probability of detection P_d and the probability of false alarm P_f. Terms P_d and P_f are defined as the probabilities of detecting a primary user signal on the considered subchannel when the subchannel is occupied and vacant, respectively, and they are given by

$$P_d^i = P(Y_i > \gamma_i | H_1) = Q\left(\frac{\gamma_i - N(\sigma^2 + |h_i|^2)}{\sqrt{2N(\sigma^2 + 2|h_i|^2)\sigma^2}}\right) \tag{3}$$

$$P_f^i = P(Y_i > \gamma_i | H_0) = Q\left(\frac{\gamma_i - N\sigma^2}{\sqrt{2N\sigma^4}}\right). \tag{4}$$

III. Spectrum Sensing Under Correlated Cognitive Radio Observations

Cooperative sensing is the process of making a final decision for the network based on the sensing data that were collected from various secondary users. We consider the case in which each individual CR user i makes a one-bit hard decision d_i in the absence or presence of the primary user based on the sensing information, such that

$$d_i = \begin{cases} 1, & \text{if } Y_i \geq \gamma_i \\ 0, & \text{if } Y_i < \gamma_i. \end{cases} \tag{5}$$

Each CR user then sends this one-bit decision to the fusion center, which makes the final decision regarding the occupancy of each subchannel. We further assume that the local decisions are correlated and that the correlation coefficients are given by

$$\begin{aligned} E\left[\prod_{i\in I} d_i|H_j\right] &= E\left[d_{i_1}d_{i_2}\cdots d_{i_g}|H_j\right] \\ &= P\left(d_{i_1}=1, d_{i_2}=1,\ldots,d_{i_g}=1|H_j\right) \end{aligned} \tag{6}$$

where $E[x|H_j]$ and $P[x|H_j]$ are the conditional expectation and conditional probability, given H_j, where $j = 0$, 1, respectively; $I \subseteq \{1, 2, 3, \ldots, M\}$; $|I| = g$ is the cardinality number of set I; and $i_v \in I, v = 1, \ldots, g$.

For $|I_l| = 1$, we have

$$E[d_i|H_1] = P(d_i = 1|H_1) = P_d^i \tag{7}$$

$$E[d_i|H_0] = P(d_i = 1|H_0) = P_f^i \tag{8}$$

where P_d^i and P_f^i are the probability of detection and false alarm of the ith secondary user, respectively.

The correlation coefficient ρ_{xy}^j between two local decisions d_x and d_y under hypothesis H_j is given by

$$\rho_{xy}^j = \frac{E[d_xd_y|H_j] - E[d_x|H_j]E[d_y|H_j]}{\sqrt{\left(E[d_x^2|H_j] - (E[d_x|H_j])^2\right)\left(E[d_y^2|H_j] - (E[d_y|H_j])^2\right)}} \tag{9}$$

where $j \in \{0, 1\}$, x and $y \in \{1\cdots M\}$, and $x \neq y$. Since $d_x \in \{0, 1\}$, it follows that $d_x = (d_x)^2$. Therefore

$$\rho_{xy}^j = \frac{E[d_xd_y|H_j] - E[d_x|H_j]E[d_y|H_j]}{\sqrt{E[d_x|H_j]E[d_y|H_j]\,(1-E[d_x|H_j])\,(1-E[d_y|H_j])}}. \tag{10}$$

Assuming that the distance between any two CR users is small, compared with the distance between the CR user and the primary transmitter, the received signal at each CR will experience almost identical path loss. Therefore, in the case of an AWGN environment, we can assume equal SNRs for the different CR users [8]. Assuming that the same threshold for all CR users $\gamma_i = \gamma$, we have $P_f^i = P_f$. In the case of an AWGN channel, as previously assumed, we will also have $P_d^i = P_d$. Since $E[d_x|H_j] = E[d_y|H_j]$ from (7) and (8), ρ_{xy}^j is independent of x and y, and every pair of local detectors are equally correlated [16]. Therefore

$$\rho^1 = \frac{P[d_x = 1, d_y = 1|H_1] - P_d^2}{P_d(1 - P_d)} \tag{11}$$

$$\rho^0 = \frac{P[d_x = 1, d_y = 1|H_0] - P_f^2}{P_f(1 - P_f)} \tag{12}$$

where ρ^1 and ρ^0 are the correlation coefficients under H_1 and H_0, respectively. It is clear from (11) and (12) that ρ^j varies only with $P[d_x = 1, d_y = 1|H_j]$, which is a function of only P_d under H_1 and a function of only P_f under H_0 [17].

If we assume that the fusion center has input vector $D = [d_1, d_2, \ldots, d_M]$, where D can take 2^M possible realizations, the joint probability $P(D|H_j)$ at the fusion center will be given by [10]

$$P(D|H_j) = \sum_{i=0}^{r}(-1)^i\binom{r}{i}\epsilon_{M-r+i}^j. \tag{13}$$

If $A_\mu = \{i : d_i = \mu\}$, such that $1 < i < M$ and $\mu = 0, 1$, then $r = |A_0|$ and $M - r = |A_1|$ denote the number of secondary users that decide in favor of H_0 and H_1, respectively, and

$$\begin{aligned} \epsilon_q^j &= E[d_{i_1}d_{i_2}\cdots d_{i_q}|H_j] \\ &= \epsilon_1^j\prod_{s=0}^{q-2}\frac{\rho^j\left(s+1-\epsilon_1^j\right)+\epsilon_1^j}{1+s\rho^j} \quad \text{for} \quad q \geq 2 \end{aligned} \tag{14}$$

where q is the number of secondary detectors sending their decisions to the fusion center; ρ^j is the correlation index under H_j; $\epsilon_1^0 \triangleq E[d_x|H_0] = P_f$ and $\epsilon_1^1 \triangleq E[d_x|H_1] = P_d$; and for $q < 2$, $\epsilon_0^1 \triangleq 1$.

IV. Decision Fusion With Correlated Local Decisions

In cooperative spectrum sensing, each CR user independently performs local spectrum sensing and sends a one-bit binary decision in the absence or presence of the primary user to the secondary base station for fusion. In this paper, we consider the general K-out-of-M fusion rule as the decision fusion rule employed at the fusion center, where K is the number of users that claim that the primary user is present and M is the total number of cooperating users. For the sake of comparison, we give special attention to some special cases of the K-out-of-M fusion rule, which are the OR, AND, and MV fusion rules.

In the K-out-of-M rule, if K users or more decide in favor of H_1, i.e., if $M - r \geq K$, then the final decision declares that there is a primary user.

In the MV rule, the final decision is based on the majority of the individual decisions, i.e., $K = \lceil(M/2)\rceil$ in

$$P_D = \sum_{z=0}^{M-K}\sum_{i=0}^{M-K-z}(-1)^i\binom{M}{K+z}\binom{M-K-z}{i}\epsilon_{K+z+i}^1 \tag{15}$$

$$P_F = \sum_{z=0}^{M-K}\sum_{i=0}^{M-K-z}(-1)^i\binom{M}{K+z}\binom{M-K-z}{i}\epsilon_{K+z+i}^0 \tag{16}$$

where $\lceil x\rceil$ denotes the smallest integer not less than x.

In the OR fusion rule, the fusion center decides that the primary user is absent only if all secondary users decide the absence of the primary user signal [3], i.e., setting $K = 1$ in (15) and (16). This is equivalent to the following simplified form:

$$P_D = 1 - P(d_1 = 0, \ldots, d_M = 0|H_1) = 1 - \sum_{i=0}^{M}(-1)^i\binom{M}{i}\epsilon_i^1 \tag{17}$$

$$P_F = 1 - P(d_1 = 0, \ldots, d_M = 0|H_0) = 1 - \sum_{i=0}^{M}(-1)^i\binom{M}{i}\epsilon_i^0. \tag{18}$$

In the AND fusion rule, if all local detectors decide that there is a primary user, then the final decision at the fusion center declares that there is a primary user, i.e., setting $K = M$ in (15) and (16). This is equivalent to the following simplified form:

$$P_D = P(d_1 = 1, d_2 = 1, \ldots, d_M = 1|H_{1,}) = \epsilon_M^1 \tag{19}$$

$$P_F = P(d_1 = 1, d_2 = 1, \ldots, d_M = 1|H_0) = \epsilon_M^0. \tag{20}$$

Next, we will show that the number of users K that optimizes the system probability of false alarm (detection) is dependent on the threshold γ and the correlation index ρ. It is clear that both P_D and P_F are dependent on ϵ_q^j in (14). Since P_d and P_f are both dependent on γ from (3) and (4) and assuming $\rho^1 = \rho^0 = \rho$, we can rewrite ϵ_q^j as follows:

$$\epsilon_q^j = f(\gamma) \prod_{s=0}^{q-2} \frac{\rho\,(s+1-f(\gamma)) + f(\gamma)}{1+s\rho} \tag{21}$$

where $f(\gamma)$ represents a function of the local sensing threshold. Taking the logarithm of both sides, we get

$$\log \epsilon_q^j = \log f(\gamma) + \sum_{s=0}^{q-2} \log\left(\rho\,(s+1-f(\gamma)) + f(\gamma)\right) - \log(1+s\rho). \tag{22}$$

Taking the derivative with respect to s and rearranging the terms, we get

$$\frac{\partial \epsilon_q^j}{\partial s} = \rho \epsilon_q^j \sum_{s=0}^{q-2} \left(\frac{1}{\rho\,(s+1-f(\gamma)) + f(\gamma)} - \frac{1}{1+s\rho} \right). \tag{23}$$

To find the optimal value of q that optimizes ϵ_q^j, we set $\partial \epsilon_q^j / \partial s = 0$. Assuming that $\rho \neq 0$ and $\epsilon_q^j \neq 0$, we get

$$\sum_{s=0}^{q-2} \frac{1}{\rho\,(s+1-f(\gamma)) + f(\gamma)} = \sum_{s=0}^{q-2} \frac{1}{1+s\rho}. \tag{24}$$

Therefore, the optimal value of K, which is directly related to q as seen from (15) and (16), is a function of both ρ and γ, as shown in (24).

Next, we plot the receiver operating characteristic (ROC) curves for the different fusion rules when the local decisions of the CR users are correlated. The degree of correlation between the CR local decisions is described by parameter ρ, which is assumed equal under the two hypotheses H_0 and H_1 [16], [18]. In [11], it was shown that the correlation coefficient between two sensor decisions cannot exceed that between the corresponding sensor observations. Based on this, our results provide a lower bound on the degree of correlation between CR local observations. We consider a network with M CR users, with all the CR nodes participating in the decision fusion, and fix the fusion rule to one of three rules. The number of detection symbols N is set to 100, and we assume that all the cooperating users have equal noise variance, such that $\sigma_i^2 = 1\ \forall i$ and equal received SNR.

In Fig. 1, we plot the ROC curves for the MV rule with $\rho = 0, 0.05, 0.1, 0.2,$ and 0.6, respectively, when $M = 5$ and the SNR is set to -10 dB. From the figure, we observe that the highest P_D is obtained when $\rho = 0$, which represents the case when CR observations are independent. The value of P_D then decreases with the increase in the correlation between the observations of the CR users until, eventually, the system is reduced to the case of a single CR user (no cooperation) as ρ becomes closer to 1. Similar results are obtained when the OR and AND fusion rules are considered. Our results show that, for low values of P_F ($P_F < 0.1$), depending on the fusion rule considered, we can have up to a 7%–13% increase in the system probability of detection when ρ decreases from 0.6 to 0.1.

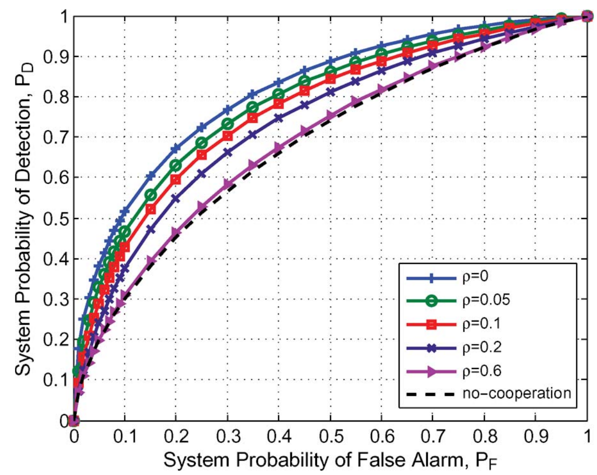

Fig. 1. ROC curves (P_D versus P_F) for different correlation indexes for the MV fusion rule with $M = 5$ and SNR $= -10$ dB.

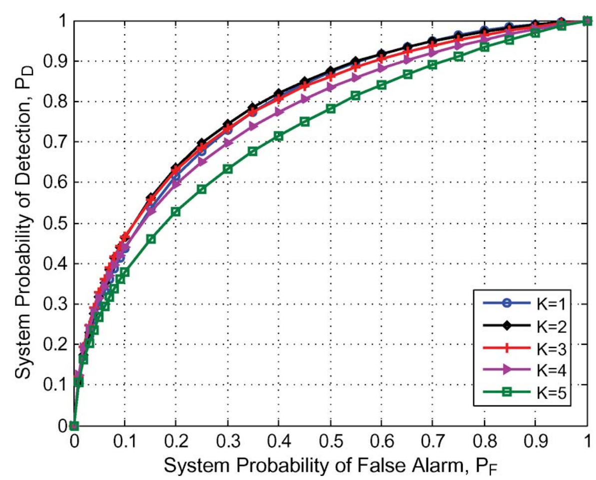

Fig. 2. Comparison of ROC curves (P_D versus P_F) for the OR, AND, and MV rules when $\rho = 0.05$.

Figs. 2 and 3 show the ROC curves for different numbers of cooperating users K with $\rho = 0.05$ and 0.2, respectively, when $M = 5$ and the SNR is set to -10 dB. From Fig. 2, we notice that, when $\rho = 0.05$, although the MV rule $(K = 3)$ is not the optimal fusion rule, the MV fusion rule still outperforms the OR $(K = 1)$ and AND $(K = 5)$ fusion rules, in terms of the probability of detection, for low values of the system probability of false alarm ($P_F < 0.1$ for the considered case). As P_F increases, the OR and AND fusion rules get progressively closer to the MV rule. From Fig. 3, we observe that, as the correlation index (ρ) increases $(\rho > 0.1)$, the OR fusion rule outperforms all the considered fusion rules.

Next, we evaluate the performance of the cooperative spectrum-sensing algorithm under the NP criterion. We aim to minimize the probability of false alarm P_F for a given probability of detection P_D for the K-out-of-M rule when the local decisions are correlated. In Fig. 4, we plot the network probability of false alarm versus the number of users K that are in favor of H_1 out of the M cooperating users for $M = 30$, an SNR of -10 dB, and $P_D = 0.9$. We show that there is an optimal number of users K that gives the minimum probability of false alarm for the network for a given probability of detection. This value of K changes with correlation index ρ, as shown in the figure.

In the remainder of this paper, we will consider the K-out-of-M rule at the fusion center, together with the correlation model described

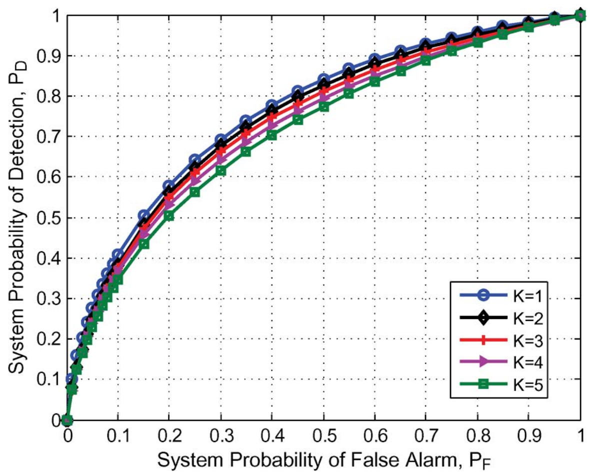

Fig. 3. Comparison of ROC curves (P_D versus P_F) for the OR, AND, and MV rules when $\rho = 0.2$.

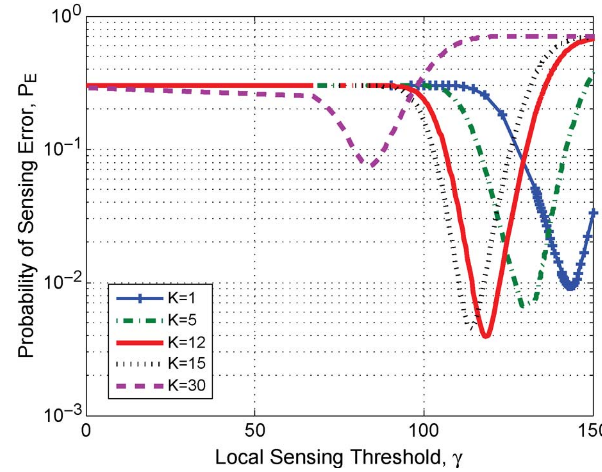

Fig. 5. Probability of sensing error versus the local sensing threshold for different numbers of users K when $\rho = 0.05$.

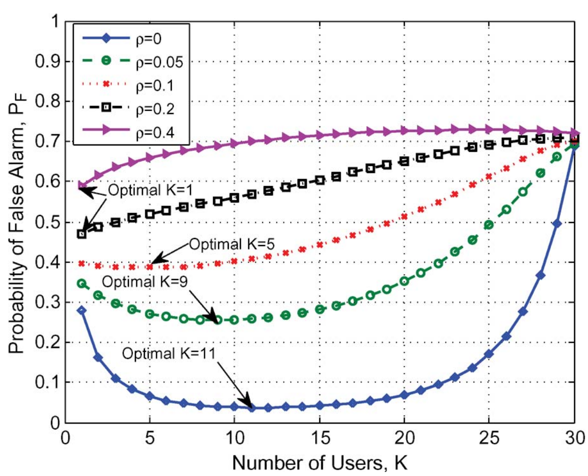

Fig. 4. System probability of false alarm versus the number of users K, with $M = 30$, SNR $= -10$ dB, and $P_D = 0.9$.

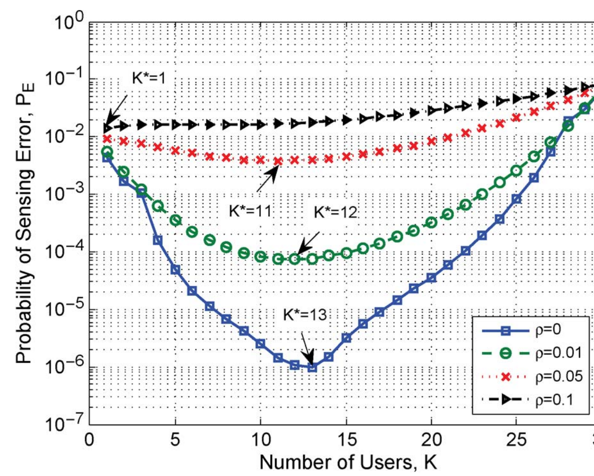

Fig. 6. Probability of sensing error versus the number of users K when $\beta = 0.9$.

in Section III. We will formulate the problem of finding the optimal settings that minimize the total probability of sensing error in the next section.

V. Problem Formulation and GA-Based Solution

Motivated by the aforementioned reported results, we aim to find the optimal number of users (K^*) and the local sensing threshold (γ) that jointly minimize the probability of sensing error at the fusion center subject to a limit on the probability of detection for a given correlation index. Using this sensing objective, we are jointly considering the sensing errors that occur when the channel is busy and idle, i.e., P_F and $1 - P_D$. Therefore, the problem can be formulated as

$$\arg\min_{K,\gamma} \; P_E(K,\gamma) = (1 - P_D(K,\gamma))\, P_B + P_F(K,\gamma)P_I \quad (25)$$

$$\text{s.t.} \qquad P_D(K,\gamma) \geq \beta \quad \text{and} \quad K \in \{1,\ldots,M\} \quad (26)$$

where P_E is the probability of sensing error. The terms P_B and P_I are the prior probabilities that the primary user is present and absent on the channel, respectively. Those two terms will depend on the channel occupancy model [19]. The total probabilities of detection P_D and false alarm P_F for the K-out-of-M rule are given in (15) and (16), respectively. The limit on the total P_D, i.e., β, is used to guarantee a satisfactory level of protection for the primary users.

The aforementioned optimization problem is an MINLP problem that constitutes simultaneously minimizing two conflicting nonlinear objective functions subject to a lower limit on a nonlinear function. Due to the complexity and nonlinearity of the problem, we propose to use GA, which is a general-purpose optimization algorithm developed by Holland [20], to minimize the probability of a sensing error.

For a certain correlation index ρ, the optimal values of γ and K^* are obtained using GA as follows.

Step 1 Generate a population of S chromosomes by randomly assigning values for γ such that the constraint in (26) is satisfied. The value of γ will depend on the desired local probability of false alarm in (4).

Step 2 Encode the value of γ by representing them in binary form.

Step 3 Evaluate the fitness function for each of the chromosomes by converting P_E in (25) to a function to be maximized for the encoded version of γ.

Step 4 Choose a number of chromosomes S_{best}, such that $0 < S_{\text{best}} < S$, with the best fitness value (elite chromosomes), and directly place them in the next generation.

Step 5 Select $S - S_{\text{best}}$ parents from the entire population according to their fitness value by using the roulette wheel selection method.

TABLE I
MEAN AND STANDARD DEVIATION OF THE MINIMUM PROBABILITY OF ERROR OVER 20 RUNS

		K=1	K=12	K=15	K=30
ρ=0.01	Mean	0.00538	7.1715×10^{-5}	9.1572×10^{-5}	0.06894
	Standard deviation	5.1012×10^{-7}	3.3325×10^{-7}	2.4072×10^{-7}	1.2228×10^{-7}
ρ=0.05	Mean	0.00926	0.00385	0.00450	0.07515
	Standard deviation	1.3108×10^{-5}	3.3474×10^{-6}	2.2261×10^{-6}	6.2564×10^{-7}
ρ=0.1	Mean	0.01429	0.01711	0.01957	0.08116
	Standard deviation	7.8609×10^{-6}	3.0690×10^{-6}	2.1175×10^{-6}	2.7098×10^{-7}

Step 6 Perform crossover and mutation on the selected chromosomes from Step 5 with probability p_c and p_m, respectively, and obtain a new population of S chromosomes.

Step 7 Compute the fitness value for the new population. Terminate the algorithm if the budget of the fitness function evaluations is exhausted; otherwise, return to Step 1.

Step 8 Repeat the aforementioned steps for $1 \leq K \leq M$.

Step 9 Choose the value of K that gives the maximum fitness value (minimum P_E), K^*.

The performance of GA greatly depends on the selection of parameters such as the population size, selection method, probabilities of crossover and mutation, and the termination criteria. Based on a number of tests, we choose the GA parameters that are well suited for our optimization problem. We use the roulette selection method with population size $S = 60$ and two elite chromosomes. We use one-point crossover with a crossover probability of 0.75. The probability of mutation is set to 0.002, and the termination criterion is 2000 evaluations.

VI. RESULTS AND DISCUSSION

In this section, we evaluate the performance in terms of the total probability of sensing error when using the K-out-of-M fusion rule with correlated local decisions. Since the K-out-of-M fusion rule with the optimal K outperforms the OR and AND fusion rules for weak correlation coefficients, as shown before, we consider values of ρ less than 0.1. In our evaluation, we use the following parameters: number of detection symbols $N = 100$, SNR $= -5$ dB, the number of secondary users $M = 30$, the probability that the primary user is absent $P_I = 0.3$, and the probability that the primary user is present $P_B = 0.7$. We then use the genetic algorithm to find the optimal pair (K^*, γ) that minimizes the probability of sensing error.

First, we numerically evaluate the total probability of sensing error P_E, with different local sensing thresholds γ. In Fig. 5, we plot P_E versus γ for different numbers of cooperating secondary users K when $\rho = 0.05$. The figure shows that, for each value of K, as P_E varies with γ, there exists only one minimum value for P_E. This minimum value of P_E will be different for different values of ρ. This relation can also be deduced from (24). The figure also shows that, for a certain value of ρ, using the optimal pair (K^*, γ), we can obtain a much lower P_E when compared with using nonoptimal settings. Next, we present the simulation results obtained by using GA to find the optimal P_E for a given value of correlation index ρ. Fig. 6 shows P_E versus the number of users K, with $M = 30$, for different values of ρ. The limit on P_D, i.e., β in (26), is set to 0.9. It is clear from the figure that, for $\rho < 0.1$, the fusion rule that minimizes P_E is the K-out-of-M rule with the optimal value of K^*, depending on correlation index ρ. However, as ρ gets closer to 0.1, optimizing K becomes less critical since a similar performance can be obtained using the OR fusion rule. This observation agrees with the previous results presented in Section IV. We also notice that the minimum value of P_E decreases with the increase in the degree of correlation between secondary users. Using GA, we were able to obtain the optimal pair (K^*, γ) that gives the minimum P_E for different values of ρ. The mean and standard deviation of the minimum probability of error over 20 runs for different numbers of users K and different values of ρ when β is set to 0.9 are shown in Table I.

VII. CONCLUSION

In this paper, we have studied the problem of cooperative spectrum sensing when the local decisions of the CR users are correlated and the counting rule is employed at the fusion center. Based on a correlation model that is indexed with a single parameter ρ, we have derived the system probabilities of detection and false alarm for the K-out-of-M fusion rule and the special cases of the AND, OR, and MV fusion rules. We have shown that the detection performance of the cooperative spectrum-sensing scheme degrades with the increase in the correlation between CR local decisions for all the considered fusion rules. We have also shown that, for different values of the correlation index, the number of cooperating users that optimizes the performance in terms of minimizing the total probabilities of false alarm and sensing error also differs. For $\rho < 0.1$, the MV rule outperforms the AND and OR fusion rules. However, we have shown that better performance than that achieved by the MV rule can be obtained by optimizing the number of cooperating CR users, K. For, $\rho > 0.1$, the optimal fusion rule is the OR rule $(K = 1)$. Based on our observations, we have used GA to find the optimal setting that minimizes the probability of sensing error in CR networks when the correlation index is known at the fusion center.

REFERENCES

[1] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[2] H. Arslan, *Cognitive Radio, Software Defined Radio, and Adaptive Wireless Systems*. Dordrecht, The Netherlands: Springer-Verlag, 2007.

[3] E. Hossain and V. Bhargava, *Cognitive Wireless Communication Networks*. New York: Springer-Verlag, 2007.

[4] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys Tuts.*, vol. 11, no. 1, pp. 116–130, Mar. 2009.

[5] E. Peh and Y. Liang, "Optimization for cooperative sensing in cognitive radio networks," in *Proc. IEEE Wireless Commun. Netw. Conf.*, Mar. 2007, pp. 27–32.

[6] Z. Quan, S. Cui, and A. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 28–40, Feb. 2008.

[7] P. K. Varshney, *Distributed Detection and Data Fusion*. New York: Springer-Verlag, 1996.

[8] W. Zhang, R. K. Mallik, and K. B. Letaief, "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 5761–5766, Dec. 2009.

[9] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors—Part I: Fundamentals," *Proc. IEEE*, vol. 85, no. 1, pp. 54–63, Jan. 1997.

[10] E. Drakopoulos and C. C. Lee, "Optimum multisensor fusion of correlated local decisions," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 27, no. 4, pp. 593–606, Jul. 1991.

[11] V. Aalo and R. Viswanathan, "On distributed detection with correlated sensors: Two examples," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 25, no. 3, pp. 414–421, May 1989.

[12] M. Kam, Q. Zhu, and W. Gray, "Optimal data fusion of correlated local decisions in multiple sensor detection systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 3, pp. 916–920, Jul. 1992.

[13] P. Willett, P. F. Swaszek, and R. S. Blum, "The good, bad, and ugly: Distributed detection of a known signal in dependent Gaussian noise," *IEEE Trans. Signal Process.*, vol. 48, no. 12, pp. 3266–3279, Dec. 2000.

[14] J. Unnikrishnan and V. V. Veeravalli, "Cooperative sensing for primary detection in cognitive radio," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 18–27, Feb. 2008.

[15] B. V. Gendenko and A. N. Kolmogorov, *Limit Distributions for Sums of Independent Random Variables*. Reading, MA: Addison-Wesley, 1954.

[16] M. W. Baidas, A. S. Ibrahim, K. G. Seddik, and K. J. R. Liu, "On the impact of correlation on distributed detection in wireless sensor networks with relays deployment," in *Proc. IEEE Conf. Commun.*, Jun. 2009, pp. 1–6.

[17] Y. Liao, "Distributed decision fusion in signal detection: A robust approach," Ph.D. dissertation, Duke Univ., Durham, NC, 2005.

[18] J. G. Chen and N. Ansari, "Adaptive fusion of correlated local decisions," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 28, no. 2, pp. 276–281, May 1998.

[19] B. Vujicic, N. Cackov, S. Vujicic, and L. Trajkovi, "Modeling and characterization of traffic in public safety wireless networks," in *Proc. Int. Symp. Perform. Eval. Comput. Telecommun. Syst.*, Jul. 2005, pp. 212–233.

[20] J. H. Holland, *Adaptation in Natural and Artificial Systems*. Ann Arbor, MI: Univ. Michigan Press, 1975.

Performance Analysis of MRC Diversity for Cognitive Radio Systems

Dong Li, *Member, IEEE*

***Abstract*—In this paper, we analyze the effect of maximum ratio combining (MRC) diversity on the performance of cognitive radio systems, in which the cognitive user (CU) shares the same spectrum with a primary user (PU) and the transmit power of the CU satisfies both the transmit and interference power constraints. Utilizing the derived cumulative distribution function (cdf) of the received signal-to-noise ratio (SNR), we investigate the ergodic capacity and the average symbol error rate (SER) of the considered system and derive new expressions for their asymptotic performance. Both analytical and simulation results show that the MRC diversity can provide full diversity order and a capacity-scaling law as a logarithmic function of the number of cognitive receive antennas when the transmit power of the CU is dominated by the transmit power constraint.**

***Index Terms*—Average symbol error rate (SER), capacity-scaling law, cognitive radio, diversity order, ergodic capacity, maximum ratio combining (MRC).**

I. Introduction

The electromagnetic spectrum is a precious resource for wireless communication systems. However, under the current command-and-control spectrum management policy, spectrum resources are not sufficiently utilized as reported by the Federal Communications Commission [1] and become crowded due to the increasing number of various bandwidth-consuming wireless applications. Recently, cognitive radio has been proposed as an effective solution to deal with these problems by allowing cognitive users (CUs) to share the same spectrum with a primary user (PU) [2]. One challenge in such spectrum-sharing systems is that the CUs should satisfy two conflicting goals, i.e., maximizing the system performance of CUs while causing no harmful interference to the PU.

Related works on spectrum sharing can be found in [3]–[5]. In [3], it was shown that significant capacity gains can be achieved in fading channels, compared with that in additive white Gaussian noise (AWGN) channels. In [4], the authors studied cognitive system performance with imperfect channel knowledge in terms of ergodic capacity and average bit error rate (BER). The author in [5] concluded that the capacity performance of the CU could benefit more from the average interference power constraint than from the peak power interference. However, they only consider a simple point-to-point system model.

It is well known that maximum ratio combining (MRC) diversity can significantly improve system performance by equipping the receiver with multiple antennas. However, applying MRC to cognitive radio systems has not been well studied. It was shown that the capacity performance of spectrum-sharing systems could be considerably improved through MRC diversity by mitigating severe fading [6] and channel estimation error [7] in the interference channel between the cognitive transmitter and the primary receiver. A major limitation on [6] and [7] is that they ignored and failed to exploit the effect of the transmit power constraint. As such, the transmit power of the CU will approach infinity when the interference channel experiences weak channel conditions. Furthermore, the symbol error rate (SER) performance for cognitive MRC systems is still unknown, which has not been presently available in the technical literature as far as we know.

To tackle these problems, in this paper, we thoroughly evaluate the performance of spectrum-sharing systems with MRC diversity in Rayleigh fading channels by utilizing the derived cumulative distribution function (cdf) of the received signal-to-noise ratio (SNR). Both the transmit and interference power constraints on the transmit power of the CU are considered. Specifically, the contributions of this paper can be summarized here.

1) We investigate the ergodic capacity of the considered system and analyze its asymptotic performance when the number of cognitive receive antennas is large. Analytical and simulation results show that the ergodic capacity scales as a logarithmic function of cognitive receive antennas when the transmit power of the CU is dominated by the transmit power constraint.
2) We also investigate the average SER of the considered system and analyze its asymptotic performance for low-noise-power region. Analytical and simulation results show that full diversity order, which is equal to the number of cognitive receive antennas, can be achieved when the transmit power of the CU is dominated by the transmit power constraint.

The rest of this paper is organized as follows: Section II describes the system model. Asymptotic analyses for the ergodic capacity and average SER are given in Sections III and IV, respectively. Section V gives the simulation results, and Section VI concludes this paper.

II. System Model

The system under consideration is composed of a primary transmitter–receiver pair and a cognitive transmitter–receiver pair. Both the PU and the CU are equipped with a single antenna, except

Manuscript received July 3, 2011; revised September 7, 2011, October 15, 2011 and November 17, 2011; accepted December 30, 2011. Date of publication January 4, 2012; date of current version February 21, 2012. The review of this paper was coordinated by Dr. D. W. Matolak.

The author is with the Faculty of Information Technology, Macau University of Science and Technology, Taipa, Macau (e-mail: dli@must.edu.mo).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2012.2182787