Optimal Energy-Efficient Channel Exploration for Opportunistic Spectrum Usage

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***Abstract*—This letter studies the channel exploration problem for opportunistic spectrum usage systems, where exploring state information of each channel consumes time and energy. We formulate this problem as an optimal stopping problem and propose a myopic rule with low complexity, called one stage look ahead (1-SLA), to solve it. Moreover, the optimality of the 1-SLA rule for the energy-efficient channel exploration problem is proved, and simulation results are provided to show the effectiveness of the 1-SLA rule.**

***Index Terms*—Opportunistic spectrum usage, multichannel diversity, optimal stopping rule.**

I. INTRODUCTION

IN multichannel wireless systems, the user would ideally transmit on the channel with the strongest quality. This is referred to as *opportunistic spectrum usage*, which utilizes the diversity of channel characteristics to obtain increased throughput [1]. In such systems, the user has first to explore state information of the channels. Notably, channel exploration consumes resources (e.g., energy, time, and bandwidth). Also, as the number of explored channels increases, the multichannel diversity gain increases and so does the exploration overhead. Thus, there is a fundamental tradeoff between the multichannel diversity gain and the channel exploration overhead. The problem of channel exploration for opportunistic spectrum usage systems with exploration overhead has been well studied in the literature [1]–[6]. However, these work only consider time consumption for channel exploration; furthermore, most of them do not allow the user to use a previously explored channel, which leads to a relatively conservative design.

In this letter, the user is allowed to use previously explored channels. Moreover, both time and energy consumptions for channel exploration are explicitly considered. Such a consideration will be more appropriate than those which emphasize time consumption only, especially for systems that operate in low power regimes such as wireless sensor networks. There is a similar work on the energy-efficient channel exploration for cognitive radio networks [7]. The differences between our work and those reported there are: (i) the energy consumption in [7] is just for data transmission and that for channel exploration is not considered yet, and (ii) the user in this reference is not allowed to use any previously explored channel.

To characterize the impacts of time and energy consumptions for channel exploration, the optimization objective is to maximize the energy-efficient throughput. This is defined as the achievable throughput normalized by the aggregate energy consumption, including the consumed energy for both channel exploration and data transmission. The user sequentially explores the channels. After exploring each channel, the user decides to (i) stop channel exploration and begin to transmit on an explored channel with strongest condition, or (ii) proceed to explore the remaining channels. We formulate this sequential channel exploration problem as an optimal stopping problem [8] and propose a myopic rule with low complexity, called one stage look ahead (1-SLA), to solve it. Moreover, we prove that the 1-SLA rule is optimal for the energy-efficient channel exploration problem and investigate its properties.

II. SYSTEM MODEL

Suppose that there is a wireless system consisting of N orthogonal channels. All the channels are assumed to undergo block-fading, i.e., the channel gain of each channel is fixed in a slot and changes randomly in the next slot. Moreover, we assume that the channel fading characteristics are independent from channel to channel and from slot to slot. For simplicity of analysis, we consider homogeneous Rayleigh fading environment. Specifically, denote $\eta_n, n = \{1, \ldots, N\}$, as the power gain of channel n; then, all η_n are independent identically distributed (i.i.d.) exponential random variables with unit mean. As a result, the common probability density function of η_n is given by $f(x) = e^{-x}, x \geq 0$.

When the user decides to transmit on channel n, the instantaneous received signal-to-noise-ratio (SNR) is given by $\gamma_n = P\eta_n/\sigma^2$, where P represents the transmit power, σ^2 represents the variance of white Gaussian noise which is set to be one for simplicity, and η_n represents the instantaneous channel power gain. Let T denote the length of a slot, T_s denote the length of the required time for reliable exploration of one channel, and P_s denote the consumed power for one channel exploration. Without loss of generality, we assume that the above parameters satisfy $NT_s < T$ and $P_s < P$. For presentation, let $\tau = T_s/T$ and $\alpha = P_s/P$.

III. OPTIMAL ENERGY-EFFICIENT CHANNEL EXPLORATION RULE

A. Problem formulation

Since the channels are homogeneous in terms of fading characteristics, we can assume that the user explores the

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channels in the natural incremental order[1], i.e., the order is given by $O = \{1, 2, \ldots, N\}$. After exploring the first n channels, the user observes the sequence $\{\eta_n\}_{k=1}^{n}$. Then, if the user decides to stop channel exploration and chooses an explored channel with the strongest channel condition for transmission, it obtains the following normalized throughput:

$$R_n(\eta_1, ..., \eta_n) = R_n(\eta_n^{max}) = \frac{(1 - n\tau)\log(1 + P\eta_n^{max})}{n\tau\alpha P + (1 - n\tau)P}, \tag{1}$$

where $(1-n\tau)$ is the fraction of the effective transmission time in a slot, $\eta_n^{\max} = \max_{1\le k\le n}\{\eta_k\}$ is the maximum channel power gain among the explored channels and hence can be regarded the system state at channel n, and $(n\tau\alpha P + (1 - n\tau)P)$ represents the aggregate consumed energy for exploring the first n channels. In some earlier studies, e.g., [1], which did not allow the use of a previously explored channel, the achievable throughput of stopping at channel n is given by $R_n^{con} = \frac{(1-n\tau)\log(1+P\eta_n)}{n\tau\alpha P+(1-n\tau)P}$. Clearly, this leads to a relatively conservative design when compared with (1). It is seen from (1) that as n increases, $\eta_n^{\max}$ could increase but $(1 - n\tau)$ decreases. Thus, in order to maximize the normalized achievable throughput, the objective of the user is to choose the right time to stop exploration based on the current system state η_n^{max}.

The above objective belongs to the optimal stopping problem with finite horizon, as the user must stop at the last channel. In principle, such a problem can be solved by the method of backward induction [8]. Since the user must stop at channel N, we first find the optimal rule at channel $N-1$. Then, knowing the optimal rule at channel $N - 1$, we find the optimal rule at channel $N-2$ and so on back to the first channel (channel 1). Specifically, we define

$$V_N(\eta_1, \ldots, \eta_N) = R_N(\eta_N^{\max}), \tag{2}$$

and then inductively for $n = N - 1$, backward to $n = 1$,

$$\begin{aligned} &V_n(\eta_1, ..., \eta_n) \\ &= \max\left\{R_n(\eta_n^{\max}), \mathbf{E}_{\eta_{n+1}}[V_{n+1}(\eta_n^{\max}, \eta_{n+1})|\{\eta_k\}_{k=1}^{n}]\right\}, \end{aligned} \tag{3}$$

where $\mathbf{E}_{\eta_{n+1}}[\cdot]$ is the expectation operation over η_{n+1}. Inductively, $V_n(\eta_1, ..., \eta_n)$ represents the maximum normalized achievable throughput that the user can obtain starting from channel n after observing the effective channel gain sequence $\{\eta_k\}_{k=1}^{n}$. At channel n, we compare the achieved normalized throughput for stopping, i.e., $R_n(\eta_n^{\max})$, with the expected throughput by continuing the exploration and using the optimal rule for channel $n + 1$ through N, i.e., $\mathbf{E}_{\eta_{n+1}}[V_{n+1}(\eta_n^{\max}, \eta_{n+1})|\{\eta_k\}_{k=1}^{n}]\}$. Then, the optimal control is to stop channel exploration if the former is no less than the latter, and to continue otherwise [8].

Based on the above argument and analysis, the maximum normalized achievable throughput can be obtained if the user adheres to the optimal decision rule at each channel. Specifically, the optimal number of channels to explore before stopping is given by:

$$N_{opt} = \min\{1 \le n \le N : R_n \ge \mathbf{E}_{\eta_{n+1}}[V_{n+1}|\{\eta_k\}_{k=1}^{n}]\}, \tag{4}$$

where R_n is given by (1), and V_{n+1} is specified by (2)-(3).

It is seen from (4) that in order to make the optimal decision at channel n, the user has to calculate the sequence $\{\mathbf{E}_{\eta_{k+1}}[V_{k+1}|\{\eta_i\}_{i=1}^{k}]\}_{k=n}^{N}$ backward from $N-1$ to n. However, such a backward induction solution is a type of dynamic programming, which results in uncountable and infinite calculation space. Thus, the optimal stopping time specified by (4) is not feasible in practice and we need to propose an approach with low complexity to solve it.

B. One stage look ahead (1-SLA) rule

To reduce the complexity, we consider a truncated version of the optimal stopping problem given by (4). The simplest version of such truncation is the one-stage look ahead (1-SLA), with which the number of channels to explore before stopping is determined by:

$$N_s = \min\{1 \le n \le N : R_n \ge \mathbf{E}_{\eta_{n+1}}[U_{n+1}|\{\eta_k\}_{k=1}^{n}]\}, \tag{5}$$

where $U_{n+1} = \frac{(1-(n+1)\tau)\log(1+P\max\{\eta_{n+1},\eta_n^{max}\})}{(n+1)\tau\alpha P+(1-(n+1)\tau)P}$ represents the normalized throughput that the user can achieve by proceeding to explore channel $n + 1$ and then stop. For presentation, let $d(n) = \frac{1-n\tau}{n\tau\alpha P+(1-n\tau)P}$. Then, we have

$$\begin{aligned} &\mathbf{E}_{\eta_{n+1}}\left[U_{n+1}|\{\eta_k\}_{k=1}^{n}\right] \\ &= d(n+1)\mathbf{E}_{\eta_{n+1}}[\log(1 + P\max\{\eta_n^{\max}, \eta_{n+1}\})] \\ &= d(n+1)\Big\{\int_0^{\eta_n^{\max}} \log(1 + P\eta_n^{\max})e^{-x}dx \\ &\qquad + \int_{\eta_n^{\max}}^{\infty} \log(1 + Px)e^{-x}dx\Big\} \\ &= d(n+1)\left[\log(1 + P\eta_n^{\max}) + e^{\frac{1}{P}}\mathrm{Ei}(\eta_n^{\max} + \tfrac{1}{P})\right], \end{aligned} \tag{6}$$

where $\mathrm{Ei}(x)$ is the exponential integral (Cauchy principle value integral) defined as $\mathrm{Ei}(x) = \int_x^{\infty} \frac{e^{-t}}{t}dt$.

It is noted from (5) that 1-SLA calls for stopping at channel n if the achievable normalized throughput obtained at channel n is at least as large as the expected throughput of proceeding to explore channel $n + 1$ and then stopping. In practice, $\mathbf{E}_{\eta_{n+1}}[U_{n+1}|\{\eta_k\}_{k=1}^{n}]$ can be calculated by numerical methods with very low complexity than that of calculating (4). Specifically, the 1-SLA based channel exploration rule is described in Algorithm 1.

Algorithm 1: 1-SLA based channel exploration rule

Step 1: Initially, set $n = 1$.
Step 2: The user explores the nth channel; thus, it observes the channel gain sequence $\{\eta_k\}_{k=1}^{n}$. Then, it calculates the current obtained normalized throughput $R_n(\eta_1, ..., \eta_n)$ using (1) and the 1-SLA expected throughput $\mathbf{E}_{\eta_{n+1}}\left[U_{n+1}|\{\eta_k\}_{k=1}^{n}\right]$ using (6), respectively.
Step 3: If $R_n(\eta_1, ..., \eta_n) \ge \mathbf{E}_{\eta_{n+1}}\left[U_{n+1}|\{\eta_k\}_{k=1}^{n}\right]$, the user stops channel exploration and selects the explored channel with the strongest channel condition for transmission; else, set $n = n + 1$ and go to Step 2.

[1]By assuming that the channel fading characteristics are homogeneous, we mainly focus on designing the optimal channel exploration rule in this letter. Surely, when the channel characteristics are heterogeneous, the optimal channel exploration order is also a key concern [4], [5], [7]. But it is beyond the scope of this letter and could be studied in future.

C. Analysis of 1-SLA rule

Since the 1-SLA rule only considers the next unexplored channel, it is easy to implement and can be regarded as a myopic rule. In this subsection, we prove that 1-SLA is optimal for the channel exploration problem in opportunistic spectrum usage systems and investigate its properties.

Definition 1: Let A_n denote the events $\{R_n(\eta_1, ..., \eta_n) \geq \mathbf{E}_{\eta_{n+1}}[U_{n+1}|\{\eta_k\}_{k=1}^n]$, then we say that the stopping problem is monotone if

$$A_1 \subset A_2 \subset ... \subset A_N. \tag{7}$$

Namely, if the 1-SLA rule calls for stopping at stage n, then it will call for stopping at all future stages no matter what the future observations turn out to be [8].

Lemma 1. *In a finite horizon monotone stopping rule problem, the 1-SLA rule is optimal [8].*

Theorem 1. *In the considered opportunistic spectrum usage systems, the 1-SLA rule is optimal and hence maximizes the energy-efficient throughput of the user.*

Proof: For the time and energy consumption, the following inequality holds for all $n = \{1, \ldots, N-1\}$:

$$d(n) > d(n+1) \tag{8}$$

which can be derived from the assumptions of $0 < N\tau < 1$ and $\alpha < 1$. In addition, by some mathematical manipulations involving derivatives, it can be shown that the following inequality holds for $\forall n \in \{1, \ldots, N-2\}$:

$$d(n) - d(n+1) < d(n+1) - d(n+2) \tag{9}$$

Now, let us define the following function:

$$\begin{aligned} F_n(\eta_n^{\max}) &= R_n(\eta_1, ..., \eta_n) - \mathbf{E}_{\eta_{n+1}}[U_{n+1}|\{\eta_k\}_{k=1}^n] \\ &= G_n(\eta_n^{\max}) - H_n(\eta_n^{\max}), \end{aligned} \tag{10}$$

where

$$\begin{aligned} G_n(\eta_n^{\max}) &= [d(n) - d(n+1)]\log(1 + P\eta_n^{\max}) \\ H_n(\eta_n^{\max}) &= d(n+1)e^{\frac{1}{P}}\mathrm{Ei}(\eta_n^{\max} + \tfrac{1}{P}). \end{aligned} \tag{11}$$

It can easily be seen that $G_n(\eta_n^{\max})$ is a strictly monotone increasing function of $\eta_n^{\max}$ and $H_n(\eta_n^{\max})$ is a strictly monotone decreasing function of $\eta_n^{\max}$. Now, we compare $F_{n+1}(\eta_{n+1}^{\max})$ and $F_n(\eta_n^{\max})$ as follows:

$$\begin{aligned} &F_{n+1}(\eta_{n+1}^{\max}) - F_n(\eta_n^{\max}) \\ &= G_{n+1}(\eta_{n+1}^{\max}) - G_n(\eta_n^{\max}) + H_n(\eta_n^{\max}) - H_{n+1}(\eta_{n+1}^{\max}), \end{aligned} \tag{12}$$

where $\eta_{n+1}^{\max} = \max\{\eta_n^{\max}, \eta_{n+1}\} \geq \eta_n^{\max}$.

Now, combining the properties of $d(n)$ specified by (8) and (9), and the properties of $G_n(\eta_n^{\max})$ and $H_n(\eta_n^{\max})$, we have:

$$\begin{cases} G_{n+1}(\eta_{n+1}^{\max}) \geq G_n(\eta_n^{\max}) \\ H_n(\eta_n^{\max}) \geq H_n(\eta_{n+1}^{\max}) > H_{n+1}(\eta_{n+1}^{\max}) \end{cases} \tag{13}$$

Thus, the following inequality can be obtained immediately from (12) and (13):

$$F_{n+1}(\eta_{n+1}^{\max}) - F_n(\eta_n^{\max}) > 0 \tag{14}$$

Let us re-write the event A_n as follows:

$$A_n = \{\eta_n^{\max} : F_n(\eta_n^{\max}) \geq 0\} \tag{15}$$

By applying (14), we have

$$\eta_n^{\max} \in A_n \Rightarrow F_n(\eta_n^{\max}) > 0 \Rightarrow F_{n+1}(\eta_{n+1}^{\max}) > 0, \tag{16}$$

which is equivalent to $A_n \subset A_{n+1}$. Finally, the following can be inductively obtained:

$$A_1 \subset A_2 \subset ... \subset A_N. \tag{17}$$

That is, the channel exploration problem for opportunistic spectrum usage systems is monotone. Thus, according to Lemma 1, Theorem 1 follows. ■

Theorem 1 validates the optimality of 1-SLA. In the following, we investigate some properties of the 1-SLA rule for opportunistic spectrum usage systems.

Proposition 1. *Denote a_n as the solution of the following equation:*

$$F_n(\eta_n^{\max}) = 0, \tag{18}$$

where $F_n(\eta_n^{\max})$ is defined in (10), then a_n is unique and $a_n > a_{n+1}$.

Proof: It is known that the following always holds for $\forall n = \{1, \ldots, N\}$:

$$\begin{cases} F_n(0) = -d(n+1)e^{\frac{1}{P}}\mathrm{Ei}(\frac{1}{P}) < 0 \\ \lim\limits_{\eta_n^{\max} \to \infty} F_n(\eta_n^{\max}) = [d(n) - d(n+1)]\log(1 + P\eta_n^{\max}) > 0 \end{cases} \tag{19}$$

Moreover, combining the monotone properties of $G_n(\eta_n^{\max})$ and $H_n(\eta_n^{\max})$, it is known that $F_n(\eta_n^{\max})$ is a strict monotone increasing function of $\eta_n^{\max}$. Thus, according to (19), $F_n(\eta_n^{\max}) = 0$ has an unique root. Now, suppose $F_n(a_n) = 0$ and $F_{n+1}(a_{n+1}) = 0$, then the following inequality can be obtained by applying (8) and (9):

$$F_{n+1}(a_n) > F_n(a_n), \tag{20}$$

which yields the following inequality immediately:

$$F_{n+1}(a_n) > F_{n+1}(a_{n+1}). \tag{21}$$

We then have $a_n > a_{n+1}$, where we again use the property that $F_{n+1}(x)$ is a monotone increasing function over x. Therefore, Proposition 1 is proved. ■

Proposition 2. *The optimal stopping rule for the energy-efficient channel exploration problem can be described as:*

$$N_s = \min\{n \geq 1 : \eta_n^{\max} \geq a_n\}. \tag{22}$$

Proof: According to Algorithm 1, the optimal decision rule at each channel can be expressed as:

$$F_n(\eta_n^{\max}) \geq 0, \tag{23}$$

where $F_n(\eta_n^{\max})$ is specified by (10). It can easily be seen that $F_n(\eta_n^{\max})$ is a monotone increasing function of $\eta_n^{\max}$. Thus, according to Proposition 1, Proposition 2 follows. ■

It is seen from (22) that the optimal channel exploration policy is a threshold rule. Although there is no closed form expression for the thresholds, the result is important from an implementation point of view. The optimal thresholds can be computed numerically and stored on the device. Then the device only needs to compute $\eta_n^{\max}$ and compare it with the threshold a_n. This is much simpler than running Algorithm 1.

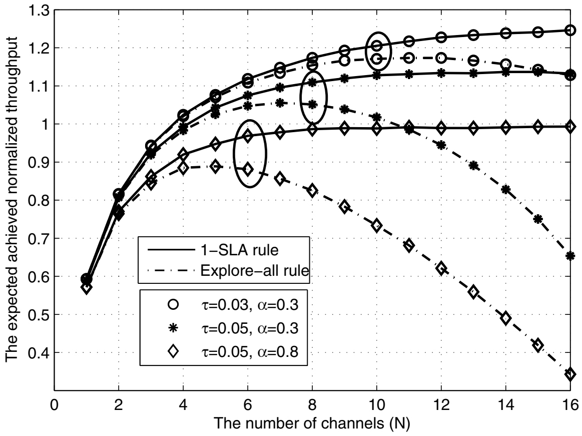

Fig. 1. Comparison with the explore-all rule.

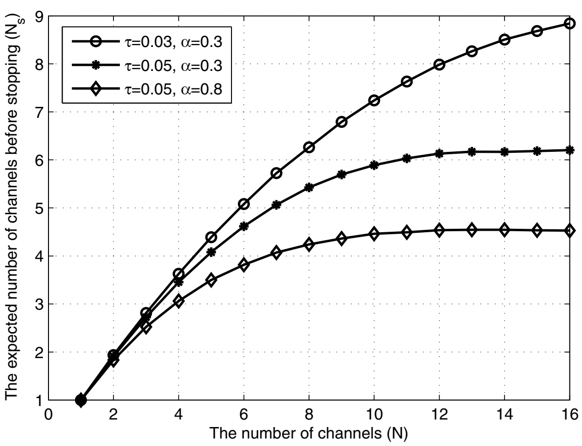

Fig. 2. The expected number of exploring channels before stopping.

Moreover, it is interesting to see $a_n > a_{n+1}$. This can be interpreted as follows. At the early stage, the probability of obtaining higher throughput in the future channels is relatively high since there is a large number of unexplored channels. Thus, the user is more willing to proceed to explore. However, as n increases, the number of unexplored channels decreases and the exploration overhead increases. As a result, the user is more likely to stop channel exploration. To summarize, in the energy-efficient channel exploration problem, the user behaves aggressively in the early stage while behaving more conservatively in the later stage.

IV. Simulation Results and Discussion

In the simulation study, the slot length is set to $T = 100\text{ms}$, and the transmit power is set to $P = 1\text{W}$; moreover, both τ and α take values between zero and one. Notably, τ and α jointly characterize the exploration overhead.

First, we compare the expected achievable normalized throughput by using the 1-SLA rule and the explore-all scheme in Fig. 1. In the explore-all scheme, the user explores all the channels and then selects the one with the highest normalized throughput for transmission. For a given exploration overhead, e.g., $\tau = 0.05$ and $\alpha = 0.03$, it is seen that there is a peak of the available normalized throughput for the explore-all rule, which shows the tradeoff between multichannel diversity and exploration overhead. It is noted from Fig. 1 that 1-SLA is always better because its decision is based on the observations so far. Furthermore, the achievable normalized throughput of 1-SLA moderates when the number of channels increases as that can be expected in any multichannel diversity system.

It is noted from Fig. 1 that when increasing the exploration overhead (i.e., increasing τ and α respectively, or increasing both concurrently), the achievable normalized throughput decreases as expected. Also, it is seen that the gap between the throughput obtained by the 1-SLA rule and that of the explore-all rule increases as the exploration overhead increases.

Secondly, we study the characteristics of the expected number of exploring channels, N_s, as shown in Fig. 2. It is noted from the figure that as the exploration overhead increases, the expected number of exploring channels decreases. The reason is that large exploration overhead leads to small achievable throughput for proceding to explore the remaining channels; thus, the user is more willing to stop. For a given exploration overhead, e.g., $\tau = 0.05$ and $\alpha = 0.03$, the expected number of exploring channels moderates when the number of channels increases. The reason is that the exploration overhead increases as the number of exploring channels increases; thus, even when there are sufficient channels, the user is more willing to explore only a part of the channels rather than to explore all the channels.

V. Conclusion

We formulated an optimal stopping problem to study the problem of channel exploration for opportunistic spectrum usage systems, where exploring state information of each channel consumes time and energy. We proposed a myopic 1-SLA rule and proved its optimality. In the 1-SLA rule, the user makes a decision based on the comparison result of the current achievable throughput and that of proceeding to explore the one channel and then stop. Thus, the 1-SLA rule is with low complexity and can be easily implemented. In future work, we intend to study the scenario with multiple users.

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