Throughput Analysis of Opportunistic Access Strategies in Hybrid Underlay–Overlay Cognitive Radio Networks

S. Senthuran, A. Anpalagan, and O. Das

***Abstract*—In cognitive radio networks, it is important to effectively use the under-utilized spectrum resources without affecting the primary users. In an underlay system, secondary users are allowed to share the channel simultaneously with primary users (with the restriction on interference level) but not in an overlay system. In this article, we consider a system where a secondary user can switch between overlay and underlay modes of operation in order to improve its throughput with limited sensing capability (i.e. sensing only one channel at a time). The results based on Markov chain analysis are satisfactorily verified using Monte-Carlo simulation. It is found that proper selection of transmission mode can provide greater improvement in throughput for a secondary user. The mode selection depends on the transition characteristics of primary users and the throughput ratio between the two modes of operation.**

***Index Terms*—POMDP, opportunistic access, cognitive radio, underlay, overlay, interweave.**

I. INTRODUCTION

THE under-utilization of the scarce spectrum triggered the need for opportunistic spectrum sharing among mobile radio users recently [1], [2]. The users who own the spectrum usually get higher access privilege while the cognitive users (also known as secondary users) usually look for opportunistic access [3]. In an overlay cognitive system, the unoccupied spectrum holes should be shared by secondary users with minimal collision with primary users whereas in an underlay system, concurrent transmission and the interference threshold to the primary users are the main concerns [4]–[8]. In this article, we consider a cognitive radio communication system that can operate either in underlay or overlay mode depending on the primary user characteristics. Generally, when the primary user is transmitting, the secondary users can not transmit on the same channel as the generated interference to the primary receiver will likely exceed the tolerable interference level. When the generated interference is below the interference threshold of the primary system, the secondary users can operate with low power in the underlay mode of operation.

In cognitive radio networks, reliable and faster sensing is very important [9], [10]. We assume that a secondary user can sense only one channel at a time and the primary user channel can change the state at the end of each time slot [11]. Our sensing focus is based on the primary user traffic prediction. Even though, hidden Markov model was used for primary user prediction in [12] and a multivariate time series approach was used in [13], we use statistical analysis of the past sensed data of the primary users [10], [14] in the selection of the sensing channel. Therefore, the objective of the access scheme is to maximize the long-term throughput of a secondary user with minimal interference to the primary users and the limited sensing information. Based on this scenario, every primary user channel state is modeled as an independent but identical three state discrete Markov chain. The states represent the overlay (*State B* and *State I*) and underlay (*State U*) transmissions and the detailed explanations are given later.

This work mainly focuses on the study of the transmission mode (underlay/overlay) selection to maximize secondary user's throughput based on the traffic characteristics and interference thresholds of the primary users. From our analysis, we can determine which mode (underlay/overlay) of operation provides better throughput benefit to the secondary user. That is, whether the secondary user should stay in underlay transmission or look for overlay transmission across the primary channels to get better long term throughput. We verified the throughput performance of both modes analytically under different conditions using a Markov chain model as well as using Monte-Carlo simulation. Our contributions are summarized as follows: We

- propose a new analytical study on transmission mode selection with channel switching in hybrid underlay/overlay systems based on (i) achievable secondary user throughput ratio between underlay and overlay transmission modes and (ii) the primary user traffic statistics,
- develop a three-state Markov framework to analyze such a hybrid mode of transmission for a positively correlated primary user traffic and analytically derive the optimal thresholds for transmission mode selection considering throughput performance of secondary users, and
- show both analytically and via simulation that proper transmission mode selection can provide throughput advantage in hybrid cognitive radio networks.

In the analysis, we assume that a secondary user can sense only one channel at a time with no sensing error. The sensing

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The authors are with the Department of Electrical and Computer Engineering, Ryerson University, Toronto, Canada (e-mail: ssenthur@ryerson.ca, {alagan, odas}@ee.ryerson.ca).

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error will cause interference to the primary users [15] as well as under-utilization of resources [16]. Therefore, our results can only be considered as to provide an upper bound with respect to sensing errors. Also cooperative wideband sensing techniques have been proposed in the literature to sense multiple channels simultaneously [17]; however, they introduce complexity in terms of communication overhead, coordination and processing. Since interference thresholds are used in deciding on the transmission mode selection, sensing errors will negatively impact the decision while multiple channel sensing will improve the channel selection and hence positively impact the decision.

The rest of the article is organized as follows: In Section II, the channel modeling and the notations are explained. In Section III, the two modes of operation (strategies) are explained and the respective throughput analysis are done using a Markov chain model. Section IV presents the analytical results that are verified through simulation. Finally, this article concludes with summary and future work in Section V.

II. PRIMARY USER SYSTEM MODEL

In a cognitive radio network, the secondary users use the primary channels opportunistically. We classify the state of a primary user channel into three states in the view of a secondary user. They are:

1) The channel is not being used by the primary user and hence the channel is said to be idle. The secondary user can opportunistically use that channel and its state is denoted by *State I* (idle).
2) The primary user occupies the channel and the secondary user occupancy will cause interference to the primary user. Since the primary users get higher priority, the secondary users are not allowed to use that channel. This state of the channel is denoted by *State B* (busy).
3) In few instances (where the interference caused by the secondary users to the primary user is below a certain threshold), secondary users are allowed to share the channel with primary users. In that case, both primary and secondary users share the channel but secondary user transmits with low power. Hence, the data rate would be low. This channel state is denoted by *State U* (similar to underlay).

Therefore, we use a three state Markov chain to model each primary user channel as shown in Fig. 1. The states are defined for a single primary user channel as summarized below:

- *State I*: The channel is idle and it can be occupied by the secondary user.
- *State B*: The channel is occupied by the primary user and no secondary user can share this channel.
- *State U*: The channel is occupied by the primary user but it can be used by the secondary user with low transmit power.

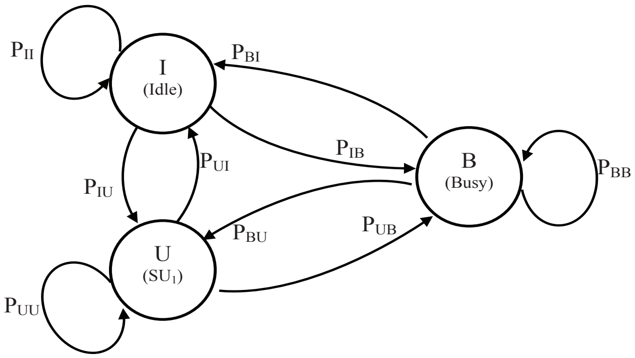

Fig. 1. Markov channel model for a primary user channel.

We assume that, based on the sensing result and predefined interference threshold, secondary user decides whether to share the channel with primary user or not. Let Γ_S be the sensed power level and Γ_O and Γ_U ($> \Gamma_O$) be the sensed power thresholds for overlay and underlay (sharable) modes. All these parameters are defined at the cognitive transmitter. Primary receiver interference threshold is denoted by γ_S. The detail operational descriptions are stated in the subsection III-D.

- If $\Gamma_S > \Gamma_U$, then no transmission is allowed.
- If $\Gamma_S < \Gamma_O$, then overlay mode of transmission. The overlay threshold is tighter to protect primary user transmission during time slots (with the acceptable tolerance of mis-detection and false alarm). During that slot, secondary user will operate with its full power to gain maximum throughput.
- If $\Gamma_O < \Gamma_S < \Gamma_U$, then underlay mode of transmission. Underlay threshold is little compromised to allow for limited secondary user transmission. When the secondary user finds that the sensed power level is above the overlay threshold but safely below the underlay threshold, then it will transmit with lower power to gain some throughput.

Hence, based on the sensing result of the primary user's signal and the predefined thresholds, the secondary user decides whether the primary user channel is in state U or B [18][19]. The transition probability from *State B* to *State I* is denoted by P_{BI}. Similarly, other transition probabilities are denoted.

The corresponding transition matrix is given by,

$$T = \begin{pmatrix} P_{BB} & P_{BI} & P_{BU} \\ P_{IB} & P_{II} & P_{IU} \\ P_{UB} & P_{UI} & P_{UU} \end{pmatrix}. \tag{1}$$

The channel state prediction is done based on the primary user state transition probabilities and the state of a specific channel during the latest sensing [11]. With these information, the state of that channel during the next time slot can be found using the Baye's rule.

The state of a channel k at time slot t is denoted by $S_k(t)$ and the channel sensed during the time slot t is denoted by $a(t)$. The predicted state probability of a channel k, during the time slot t, being in *State B, State I* and *State U* are denoted by $\omega_{k_B}(t)$, $\omega_{k_I}(t)$ and $\omega_{k_U}(t)$ respectively. They can be calculated as in (2), where $X \in \{B, I, U\}$, P_{BU}=1-P_{BB}-P_{BI}, P_{IU}=1-P_{IB}-P_{II}, P_{UU}=1-P_{UB}-P_{UI} and $\omega_{k_B}(t)$+$\omega_{k_I}(t)$+$\omega_{k_U}(t)$=1.

III. PROPOSED STRATEGIES AND THROUGHPUT ANALYSIS

The objective is to maximize the throughput of a secondary user without affecting the primary users. We assume that the

$$\omega_{k_X}(t+1) = \begin{cases} P_{BX}, & a(t){=}k,\ S_{a(t)}(t){=}B; \\ P_{IX}, & a(t){=}k,\ S_{a(t)}(t){=}I; \\ P_{UX}, & a(t){=}k,\ S_{a(t)}(t){=}U; \\ \omega_{k_B}(t)P_{BX} + \omega_{k_I}(t)P_{IX} + \omega_{k_U}(t)P_{UX}, & a(t){\neq}k. \end{cases} \tag{2}$$

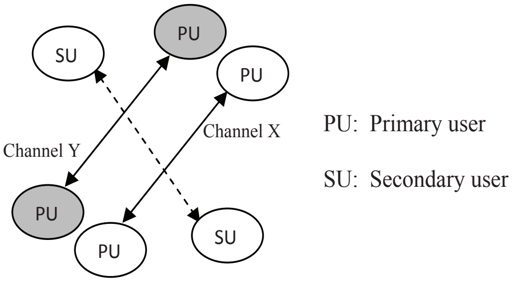

Fig. 2. System model (two primary user channels and one cognitive user pair).

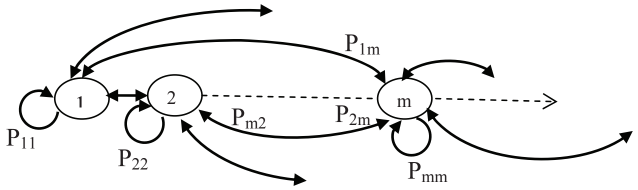

Fig. 3. Markov chain model for the secondary user: states represent the length (L) of the stay of a secondary user in a channel continuously.

secondary user has enough data to transmit continuously and it looks for a better selection of a primary user channel to transmit its data. Also we assume that the secondary user can sense only one channel at once. Therefore, secondary user does not know the states of the other channels when secondary user senses one channel, but it can predict the states of the other channels based on the previously sensed data [11].

In this work, the initial analysis is done for a system that has two identical but independent primary channels and one secondary user that tries to opportunistically occupy a primary user channel as shown in Fig. 2. The critical decision for a secondary user is whether to stay in a channel and transmit at a lower rate or, leave that channel and sense the other channels when the currently sensed channel is in *State U*. The other channel can be in one of the three states. If it is in *State I*, then the switching would be beneficial. If it is in *State B*, then it would reduce the throughput.

In this article, we evaluate the following two strategies for different conditions such as traffic characteristics of the primary users and different sensitivity levels of throughput reduction when sharing a channel.

1) *Strategy A*: Secondary user occupies the primary user channel when the channel is either in *State I* (idle) or in *State U* (share with the primary user with lower data rate).
2) *Strategy B*: Secondary user occupies the channel only when it is idle (*State I*) (no coexistence with the primary user).

A. Markov chain model for secondary user's channel occupancy

The analysis is done for a two primary user channel system with one secondary user as mentioned earlier. The secondary user stays in a channel or switches the channel based on the above strategy. The secondary user's behaviour is analyzed using a Markov chain where each state represents the length of the continuous stay of a secondary user in terms of time slots ($L = 1, 2, 3, \ldots$) before switching to another channel. It is shown in Fig. 3, where P_{mn} denotes the transition probability of secondary user for staying in a channel for $L = m$ slots and switching to the other channel for $L = n$ slots. The probability of staying in a channel for a consecutive L slots is written as P_L. Note that each primary user channel is modeled as in Fig. 1 and each secondary user's stay in a channel is modeled as in Fig. 3.

In the literature, for a two state case (B or I), the analysis was done for positively and negatively correlated primary user traffic . The positively correlated primary user traffic implies that $P_{BB} > P_{IB}$. A strategy was proposed in [11] as follows: in positively correlated traffic, the secondary user switches the channel when it senses the primary channel as busy, and in negatively correlated traffic, the secondary user stays in that channel when it senses the channel as busy. Secondary user does the opposite when it senses as idle.

In our case with three states, we considered a scenario in which the primary user traffic is positively correlated with time. That is, when a primary user occupies a channel, it stays in that channel for few slots ($P_{BB} > \{P_{BI}$ or $P_{BU}\}$). Hence, if the currently sensed channel is busy then the predicted state of that channel during the next slot would be busy with higher probability. In that case, the secondary user prefers to switch the channel expecting to transmit in the other channel. In a negatively correlated traffic, it would be other way. That is, the secondary user may find it beneficial to stay in that channel without any transmission when the sensed slot is busy and it will wait for the next slot hoping that the primary user may vacate that channel. We believe that positively correlated primary user traffic represents a more common case of a stable system with enough data at primary user for transmission. Therefore, we assume that secondary user switches the channel when it senses the primary channel as busy in the analysis.

B. Strategy A: Cognitive access with simultaneous channel sharing

In this strategy, the secondary user occupies a channel when it is in *State I* (idle) or *State U* (coexistence with primary user). When the channel state becomes *State B*, secondary user leaves that channel without transmitting as we assumed that primary user traffic is positively correlated. In order to

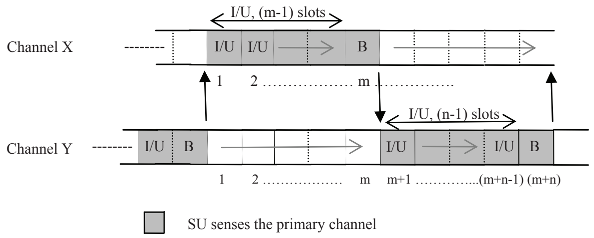

Fig. 4. Sensing: One cognitive user accesses two primary channels.

compute the secondary user's throughput in this strategy, we need to find both the transition and the steady state probabilities of the Markov chain given in Fig. 3.

The calculation of a transition probability in a two primary user channel system can be illustrated as follows: A secondary user had stayed in the first channel (say channel X) for $L = m$ slots, then moved to the second channel (say channel Y) when the channel X became busy (*State B*). Then it stayed in the channel Y for $L = n$ slots before moving back to the channel X in our two primary user channel system as shown in Fig. 4.

m and n can be any positive integers but we consider the case for $n \geq 2$ first. Always a secondary user switches the channel if that channel becomes busy to allow for primary user's privileged access. If the first slot of the channel Y is in *State I* (slot $m + 1$), then the transition probability should be $P_{BI}^{(m+1)}$ where $P_{BI}^{(m+1)}$ denotes the transition probability of a channel becoming *State I* from *State B* after $m + 1$ slots. Similarly, if the slot $m + 1$ is in *State U*, then the transition probability should be $P_{BU}^{(m+1)}$. For staying the rest of $n - 1$ slots in the channel Y, the channel should be either in *State I* or *U* for the next $n - 2$ slots (i.e., from $(m + 2)^{th}$ slot to $(m + n - 1)^{th}$ slot) and of course, the last slot ($(m+n)^{th}$ slot) should be in *State B*. We consider an extracted transition matrix $\bar{T}$ as defined in (5) from the original transition matrix T to characterize the channel transition. The transition probability for staying in a channel for $L = n, n \geq 2$ slots provided secondary user had stayed in the other channel for $L = m$ slots before switching to this channel can be written as in (3) for four different scenarios. Hence,

$$\begin{aligned} P_{mn} &= P_{BI}^{(m+1)}(\bar{P}_{II}^{(n-2)} P_{IB} + \bar{P}_{IU}^{(n-2)} P_{UB}) \\ &+ P_{BU}^{(m+1)}(\bar{P}_{UI}^{(n-2)} P_{IB} + \bar{P}_{UU}^{(n-2)} P_{UB}), \end{aligned} \tag{4}$$

where $\bar{P}_{II}^{(n-2)}$ is defined as transition probability of a channel from *State I* to become *State I* after $n-2$ slots (there should not be any *State B* within $n-2$ slots) and other notations can also be interpreted similarly. The transition probabilities are calculated from the transition matrix $\bar{T}$ which is defined as,

$$\bar{T} = \begin{pmatrix} P_{II} & P_{IU} \\ P_{UI} & P_{UU} \end{pmatrix}. \tag{5}$$

Finally, for the case when $n = 1$, the secondary user stays only one time slot because the primary channel is in *State B*. Hence, the transition probability for that case can be written as $P_{m1} = P_{BB}^{(m+1)}$.

In the steady state, we can write the steady state equation for the first two states ($L = 1, 2$) as,

$$P_1 = \sum_{i=1}^{\infty} P_i P_{BB}^{(i+1)}, \tag{6}$$

$$P_2 = \sum_{i=1}^{\infty} P_i (P_{BI}^{(i+1)} P_{IB} + P_{BU}^{(i+1)} P_{UB}), \tag{7}$$

with

$$\sum_{i=1}^{\infty} P_i = 1. \tag{8}$$

Further, we can write the steady state equation for the states $L > 2$ as in (9).

In order to solve equations (6)-(8), we need to simplify the infinite series. For that we need to find the relationship between the states. The states are sub-divided as shown below to find the relationships among them. From (7), we can write:

$$P_2 = P_{2_1} + P_{2_2} \tag{10}$$

where $P_{2_1} = \sum_{i=1}^{\infty} P_i P_{BI}^{(i+1)} P_{IB}$ and $P_{2_2} = \sum_{i=1}^{\infty} P_i P_{BU}^{(i+1)} P_{UB}$. Also, from (9) for a specific state $L, L > 2$, we can write

$$P_L = P_{L_1} + P_{L_2} + P_{L_3} + P_{L_4},$$

where

$$\begin{aligned} P_{L_1} &= \sum_{i=1}^{\infty} P_i P_{BI}^{(i+1)} \bar{P}_{II}^{(L-2)} P_{IB}, \\ P_{L_2} &= \sum_{i=1}^{\infty} P_i P_{BI}^{(i+1)} \bar{P}_{IU}^{(L-2)} P_{UB}, \\ P_{L_3} &= \sum_{i=1}^{\infty} P_i P_{BU}^{(i+1)} \bar{P}_{UI}^{(L-2)} P_{IB}, \\ P_{L_4} &= \sum_{i=1}^{\infty} P_i P_{BU}^{(i+1)} \bar{P}_{UU}^{(L-2)} P_{UB}. \end{aligned}$$

From the above we can find the relationship as,

$$P_{L_1} = P_{2_1} \bar{P}_{II}^{(L-2)}, \tag{11a}$$

$$P_{L_2} = P_{2_1} \frac{P_{UB}}{P_{IB}} \bar{P}_{IU}^{(L-2)}, \tag{11b}$$

$$P_{L_3} = P_{2_2} \frac{P_{IB}}{P_{UB}} \bar{P}_{UI}^{(L-2)}, \tag{11c}$$

$$P_{L_4} = P_{2_2} \bar{P}_{UU}^{(L-2)}, \tag{11d}$$

and from (6), (10) and (11),

$$P_1 = P_1 P_{BB}^{(2)} + P_2 P_{BB}^{(3)} + \sum_{i=3}^{\infty} (P_{L_1} + P_{L_2} + P_{L_3} + P_{L_4}) P_{BB}^{(i+1)}$$

$$P_{mn} = \begin{cases} P_{BI}^{(m+1)} \bar{P}_{II}^{(n-2)} P_{IB}, & \text{Slot } (m+1) \text{ and } (m+n-1) \text{ are in } \textit{State I}; \\ P_{BI}^{(m+1)} \bar{P}_{IU}^{(n-2)} P_{UB}, & \text{Slot } (m+1) \text{ and } (m+n-1) \text{ are in } \textit{State I} \text{ and } \textit{State U} \text{ respectively}; \\ P_{BU}^{(m+1)} \bar{P}_{UI}^{(n-2)} P_{IB}, & \text{Slot } (m+1) \text{ and } (m+n-1) \text{ are in } \textit{State U} \text{ and } \textit{State I} \text{ respectively}; \\ P_{BU}^{(m+1)} \bar{P}_{UU}^{(n-2)} P_{UB}, & \text{Slot } (m+1) \text{ and } (m+n-1) \text{ are in } \textit{State U}. \end{cases} \quad (3)$$

$$P_L = \sum_{i=1}^{\infty} P_i \left(P_{BI}^{(i+1)} (\bar{P}_{II}^{(L-2)} P_{IB} + \bar{P}_{IU}^{(L-2)} P_{UB}) + P_{BU}^{(i+1)} (\bar{P}_{UI}^{(L-2)} P_{IB} + \bar{P}_{UU}^{(L-2)} P_{UB}) \right). \quad (9)$$

$$\begin{aligned} P_1 &= P_1 P_{BB}^{(2)} + P_2 P_{BB}^{(3)} \\ &+ P_{2_1} \sum_{i=3}^{\infty} \left(\bar{P}_{II}^{(i-2)} + \frac{P_{UB}}{P_{IB}} \bar{P}_{IU}^{(i-2)} \right) P_{BB}^{(i+1)} \\ &+ P_{2_2} \sum_{i=3}^{\infty} \left(\frac{P_{IB}}{P_{UB}} \bar{P}_{UI}^{(i-2)} + \bar{P}_{UU}^{(i-2)} \right) P_{BB}^{(i+1)} \quad (12) \end{aligned}$$

The series sum can be calculated using eigen value decomposition as shown in Appendix A. Solving the liner equations (6)-(8) after simplifying them as shown above, we can find the steady state probabilities of each state analytically.

The steady state probability P_L gives the probability that a secondary user continuously stays in a channel for L number of slots. The last slot should be in *State B* and previous $L-1$ slots can be either in *State I* or *State U*. As mentioned earlier, if it is in *State U*, the secondary user shares the channel with primary user and the throughput would be reduced. Therefore, we need to identify the probability distribution of *State I* and *State U* for each state P_L in the throughput calculation. That is, for each state $L \geq 2$, the probability of staying in *State I* (P_{L_I}) and *State U* (P_{L_U}) needs to be calculated. This can be done using a tree diagram [20] concept. For that calculation, we need to know the probability of the first slot being in *State I* (P_{I_f}) and *State U* (P_{U_f}) for the states $L \geq 2$ and P_{I_f} can be found as,

$$P_{I_f} = \sum_{i=1}^{\infty} P_i P_{BI}^{(i+1)},$$

and similar equation can be written for the probability of the first slot being in *State U* (P_{U_f}). We assume that the secondary user's data rate is R_O when it is in *State I* and R_U when the channel is in *State U*. Then the transmitted data can be defined as,

$$C_I = \sum_{i=2}^{\infty} (i-1) P_i (P_{i_I} R_O + P_{i_U} R_U). \quad (13)$$

The average length of a continuous stay of a secondary user in a channel (in number of slots) can be found as,

$$\bar{L}_I = \sum_{i=1}^{\infty} i P_i. \quad (14)$$

Using (13) and (14), we can write the throughput of the secondary user as,

$$\bar{C}_I = \frac{C_I}{\bar{L}_I}$$

C. Strategy B: Cognitive access without simultaneous channel sharing

In this section, the Strategy B is discussed with throughput analysis. In the analysis, as defined earlier in Fig. 3, an infinite state Markov chain having states $L = 1, 2, 3 \ldots$ is used. As secondary user senses only one channel at a time, the state of the other channel (*B/U/I*) is predicted from the last visit. If a channel was visited n slots before and it was in *State B*, then the probability of being in *State I* in the next slot would be $P_{BI}^{(n+1)}$. Similarly, we can predict the other transition probabilities. In this prediction, we are using the state of the channel when it was sensed during the last visit.

In Strategy B, a secondary user occupies a channel only when it is in *State I* (idle) and leaves a channel if the sensed channel was either in *State B* or *State U*. That is, if a secondary user senses the channel as either in *State B* or *State U*, then it switches the sensing to the next channel in the next slot. During the sensing slot, if that channel is in *State B*, the secondary user will not transmit during that slot; on the other hand, if the sensed channel is in *State U*, then secondary user will transmit with lower power during that slot and switch the sensing to the other channel. Hence, it is noted that secondary user stays in only one slot ($L = 1$) even though it does not transmit when the channel is in *State B*, and it transmits only in one slot at lower rate when the sensed channel is in *State U* before switching the sensing to the other channel. It is considered in the throughput calculation of state $L = 1$. For $L \geq 2$ states, the first $L-1$ slots are in *State I* and the last slot is *State B* or *State U*. In the analysis, we denote it with W, that is, channel state of the last slot can be either *State B* or *State U*. The probability of last slot being in *State U* before the secondary user leaves a channel can be written as $\delta = P_{IU}/(P_{IU} + P_{IB})$ for $L \geq 2$. Similarly, we can write that probability of last slot being in *State B* as $1 - \delta$.

As mentioned earlier, the $L = 1$ state is either due to *State B* or *State U*. For the analytical purpose, the *State* $L = 1$ is divided into six states ($1_{B_B}, 1_{B_U}, 1_{U_U}, 1_{U_B}, 1_{B_W}$ and 1_{U_W}) and for $L \geq 2$, each state is divided into three states (L_B, L_U and L_W) based on the state of the previous visit. That is, *State* 1_{B_U} denotes that secondary user stays in only one slot ($L = 1$) (during the sensing slot) and then, leaves it as that channel is in *State B*; and the secondary user had left that channel on its

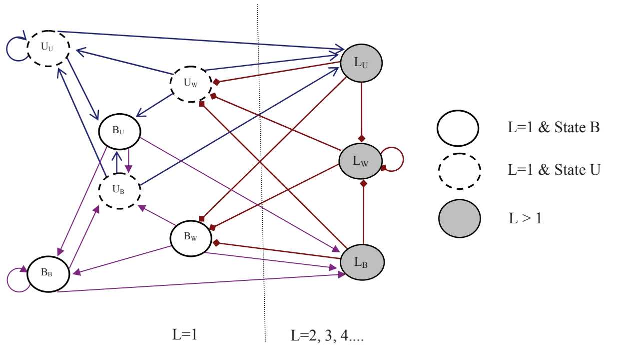

Fig. 5. Modified Markov chain model for the secondary user: states represent length of the stay of a secondary user (L) in a channel continuously.

previous visit as it was in *State U* (denoted by the subscript letter U). For simplicity in notation, we use B_B instead of 1_{B_B} (state notation without "1"). The corresponding steady state probabilities are denoted by $P_{B_B}, P_{B_U}, P_{U_U}, P_{U_B}, P_{B_W}$ and P_{U_W} for the state $L = 1$ and P_{L_B}, P_{L_U} and P_{L_W} for the states $L \geq 2$, where $P_1 = P_{B_B} + P_{B_U} + P_{U_U} + P_{U_B} + P_{B_W} + P_{U_W}$, and $P_L = P_{L_B} + P_{L_W} + P_{L_U}$ for $L \geq 2$.

The modified representation of Fig. 3 is shown in Fig. 5. This modification is done to facilitate the analysis for Strategy B.

The subscript of the state notation denotes the state of channel during the previous visit. That is, L_B denotes that the secondary user stays L slots continuously in a channel and the state of the channel during secondary user's previous visit was *State B*. L_U and L_B states are only one step transition states. That is, whenever a secondary user moves from a $L = 1$ state to L (≥ 2) states, it will go to one of the L_U states ($L \geq 2$) if its previous state was $L = 1$ & *State U* or it will go to one of the L_B states ($L \geq 2$) if its previous state was $L = 1$ & *State B*. Since L_B and L_U are transition states, there shall not be any transitions within L_B or L_U states or between L_B and L_U states. After staying L (≥ 2) slots either in L_B or L_U state, secondary user can move to either *State* B_W ($L = 1$ & *State B*) or *State* U_W ($L = 1$ & *State U*) or *State* L_W ($L \geq 2$, the first $L - 1$ slots are in *State I* and last slot is either in *State B* or *State U*). That is, the transition depends on the state of the newly switched channel. The state transitions are possible within *State* L_W. That is, we can find the transition probability, if the secondary user moves from any *State* L_W ($L \geq 2$) to any *State* L_W ($L \geq 2$).

The state descriptions are summarized below. The state of the channel during the last visit is used in the transition probability calculations.

- B_B - The secondary user left the channel on its previous visit as the channel was in *State B*, it stays in only one slot ($L = 1$) during the current sensing slot, and then leaves that channel as the channel is in *State B*.
- B_U - The secondary user left the channel on its previous visit as the channel was in *State U*, it stays in only one slot ($L = 1$) during the current sensing slot, and then leaves that channel as the channel is in *State B*.
- U_B - The secondary user left the channel on its previous visit as the channel was in *State B*, it stays in only one slot ($L = 1$) during the current sensing slot, and then leaves that channel as the channel is in *State U*.
- U_U - The secondary user left the channel on its previous visit as the channel was in *State U*, it stays in only one slot ($L = 1$) during the current sensing slot, and then leaves that channel as the channel is in *State U*.
- B_W - The secondary user left the channel on its previous visit as the channel was in either *State B* or *State U*, it stays in only one slot ($L = 1$) during the current sensing slot, and then leaves that channel as the channel is in *State B*.
- U_W - The secondary user left the channel on its previous visit as the channel was in *State B* or *State U*, it stays in only one slot ($L = 1$) during the sensing slot, and then leaves that channel as the channel is in *State U*.
- L_B - The secondary user left the channel on its previous visit as the channel was in *State B*, it stays in L slots during the current visit.
- L_U - The secondary user left the channel on its previous visit as the channel was in *State U*, it stays in L slots during the current visit.
- L_W - The secondary user left the channel on its previous visit as the channel was in either *State B* or *State U*, it stays in L slots during the current visit.

At steady state, we can write the steady state equation for *State* 2_B ($L = 2$) as follows:

$$\begin{aligned} P_{2_B} &= P_{B_B} P_{BI}^{(2)} (P_{IB} + P_{IU}) + P_{B_U} P_{UI}^{(2)} (P_{IB} + P_{IU}) \\ &+ P_{B_W} (\delta P_{UI}^{(2)} + (1 - \delta) P_{BI}^{(2)}) (P_{IB} + P_{IU}), \end{aligned}$$

where $\delta = P_{IU}/(P_{IU} + P_{IB})$. For $L > 2$,

$$\begin{aligned} P_{L_B} &= P_{B_B} P_{BI}^{(2)} P_{II}^{L-2} (P_{IB} + P_{IU}) \\ &+ P_{B_U} P_{UI}^{(2)} P_{II}^{L-2} (P_{IB} + P_{IU}) \\ &= P_{B_W} (\delta P_{UI}^{(2)} + (1 - \delta) P_{BI}^{(2)}) P_{II}^{L-2} (P_{IB} + P_{IU}), \end{aligned}$$

and hence we can write,

$$P_{L_B} = P_{2_B} P_{II}^{L-2}. \tag{15a}$$

Similarly, the following relationships can be found.

$$P_{L_W} = P_{2_W} P_{II}^{L-2}, \tag{15b}$$

$$P_{L_U} = P_{2_U} P_{II}^{L-2}. \tag{15c}$$

Using (15),

$$\begin{aligned} 1 &= \sum_{i=1}^{\infty} P_i \qquad (16) \\ &= (P_{B_B} + P_{B_U} + P_{B_W} + P_{U_B} + P_{U_U} + P_{U_W}) \\ &+ \frac{P_{2_B} + P_{2_W} + P_{2_U}}{1 - P_{II}} \\ 1 &= P_1 + \frac{P_2}{1 - P_{II}} \end{aligned}$$

The steady state equations can be written as shown in Appendix B. Solving these linear equations (21) with (16), we can find the steady state probabilities of each state. The average length of a continuous stay of a secondary user in a channel can be found (in number of slots) as,

$$\bar{L}_{II} = \sum_{i=1}^{\infty} i P_i = P_1 + \frac{P_2(2-P_{II})}{(1-P_{II})^2}. \tag{17}$$

When $L = 1$, the secondary user switches the channel after staying in only one slot, and that slot should be either *State U* or *State B*. If it is in *State U*, then secondary user transmits at lower rate (R_U) before the switch. On the other hand, if the channel is in *State B*, secondary user leaves that channel without transmitting as we assumed that primary user traffic is positively correlated. Then the transmitted data can be calculated based on the probability of *State U* when $L = 1$ as,

$$C_a = (P_{U_U} + P_{U_B} + P_{U_W}) R_U. \tag{18}$$

For $L \geq 2$, in the last slot, the secondary user leaves the channel due to the channel becoming either *State U* or *State B*. If the reason for leaving the channel was due to *State U*, then the secondary user would have transmitted in the last slot with a lower rate before leaving. Hence, the transmitted data in the last slot can be calculated as,

$$C_b = \sum_{i=2}^{\infty} P_i \delta R_U = \frac{P_2}{1-P_{II}} \delta R_U, \tag{19}$$

where as defined earlier, $\delta = \frac{P_{IU}}{P_{IU}+P_{IB}}$. When $L \geq 2$, the data transmitted in other than the last slot is definitely due to the *State I* of the channel; hence, it can be calculated as,

$$C_c = \sum_{i=2}^{\infty} (i-1) P_i R_O = \frac{P_2}{(1-P_{II})^2} R_O. \tag{20}$$

The throughput can be calculated as,

$$\bar{C}_{II} = \frac{C_a + C_b + C_c}{\bar{L}_{II}},$$

where the average length of a continuous stay of a secondary user in a channel in terms of slots $\bar{L}_{II}$ is given in (17).

D. Operational Details

For the overlay transmission, the cognitive user should accurately detect the presence of the primary user. When the primary user is not active or not within the range of the cognitive user transmission, the cognitive user can transmit. There are many sensing algorithms in the literature [9], [10] that can be used to detect the primary user transmission. On the other hand, in an underlay transmission mode, a cognitive user can transmit while a primary user is active as long as the interference to the primary system is within the threshold.

We propose to use two different (fixed) power levels $\tilde{P}_O$ and $\tilde{P}_U (< \tilde{P}_O)$ during the overlay and underlay modes of transmission of the cognitive users; however, more than two levels can also be considered. As these power levels are known to the cognitive transmitter, the throughput ratio (R_U/R_O) of the cognitive user between the overlay and underlay transmission modes can be roughly calculated before the transmission. In our implementation, noting that achievable data rate is proportional to transmit power, we initially set $R_U/R_O = \tilde{P}_U/\tilde{P}_O$ where $\tilde{P}_U$ and $\tilde{P}_O$ are underlay and overlay mode transmission power respectively. During the transmission, throughput can be reported back to the cognitive transmitter by the cognitive receiver and hence fine-tuned.

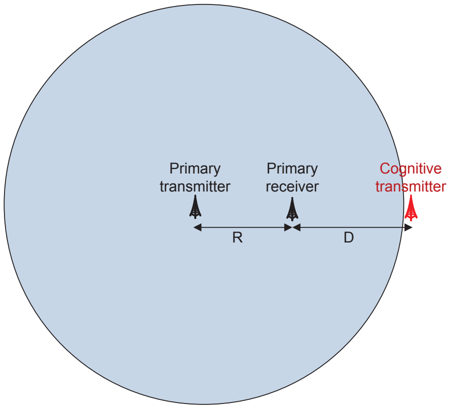

Fig. 6. System model: cognitive user in the presence of primary transceiver.

We use the system model commonly used in [21],[22][23] as shown in Fig. 6.

In this model, it is assumed that a cognitive user knows the pilot power of the primary transmitter ($\tilde{P}_p$), interference threshold at the primary receiver (γ_s) and communication range of the primary transmitter (R). With the pilot power, the distance between the primary and cognitive transmitters can be calculated ($R + D$) where D is the interference range of the cognitive transmitter. If the cognitive user's underlay mode transmission with power $\tilde{P}_U$ is not causing interference to the primary receiver, then the underlay transmission is possible. This interference threshold constraint can be written as $\frac{\tilde{P}_p L(R)}{\tilde{P}_U L(D) + \tilde{P}_n} > \gamma_s$, where $L(d)$ and $\tilde{P}_n$ are denoted by total path loss at distance d from the transmitter, and the noise power level at the primary receiver respectively [21]. After estimating the distance of the primary user with the detected primary signal level during the sensing at the cognitive transmitter, the above interference constraint can be checked to see if it is met or not; and if it is, underlay transmission with power $\tilde{P}_U$ is decided.

We define following notations:

γ_s	The interference threshold at the primary receiver. If the interference power is above this threshold, primary receiver cannot decode the signal properly.
Γ_S	The sensed power level at the cognitive transmitter.
Γ_O	Power threshold at cognitive transmitter that determines the presence of the primary transmission. If the sensed power level at the cognitive transmitter is below this threshold (i.e., $\Gamma_S < \Gamma_O$), then it can be assumed safely that primary user is not active on that sensed channel. This threshold can be calculated based on the probability of false alarm and detection requirements [24] and [25].
Γ_U	Power threshold at cognitive transmitter for underlay transmission. If the sensed power level (Γ_S) is such that $\Gamma_O < \Gamma_S < \Gamma_U$, then it can be assumed that cognitive user can safely transmit in underlay

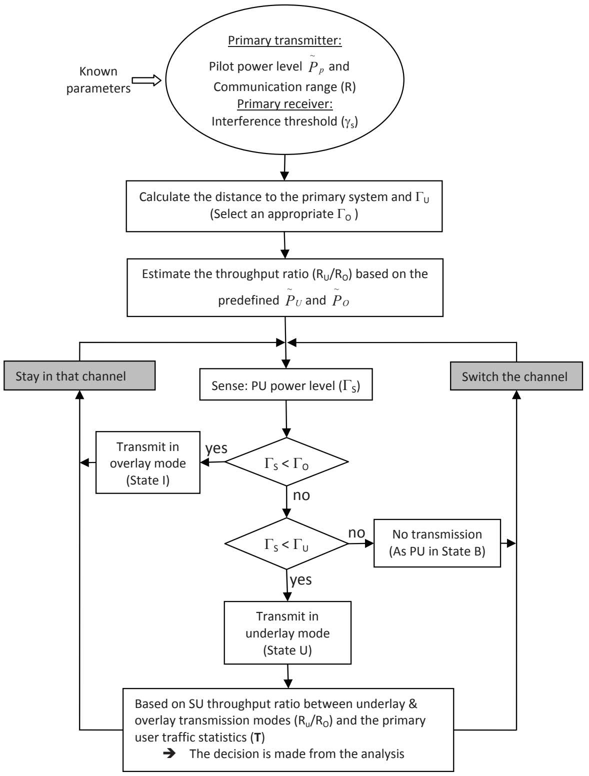

Fig. 7. Flowchart: The decisions made in the shaded boxes are by assuming that the primary user occupancy is positively correlated with time.

mode without exceeding the interference threshold of the primary receiver. If the sensed power level at the cognitive transmitter is above this threshold (i.e., $\Gamma_S > \Gamma_U$), then it can be assumed that the primary user is active and underlay transmission of the cognitive user may affect the primary system. This threshold can be calculated using the sensed signal level $\tilde{P}_p L(R + D)$. Following, the location of the primary transmitter can be estimated $(R + D)$ and then the interference range D can be calculated as the communication range of the primary transmitter (R) is assumed to be known. Hence, with the received signal level Γ_S, the distance and the corresponding interference level can be calculated for a specific transmission power level $\tilde{P}_U$. Note that Γ_U is a specific sensed power level, at which the underlay transmission with power $\tilde{P}_U$ causes interference to exceed the interference threshold (γ_s) to the primary system.

The above discussed scenarios are summarized as:

- State B : $\Gamma_S > \Gamma_U$, No transmission
- State I : $\Gamma_S < \Gamma_O$, Overlay transmission
- State U : $\Gamma_U > \Gamma_S > \Gamma_O$, Underlay transmission

The primary user traffic characteristic can be positively or negatively correlated with time. In this paper, we consider a case where the primary user traffic is positively correlated with time. When the primary user is in State I/B/U, it will stay in that state for a while. That is, if the channel in State B, it will stay in that channel for few slots (in State B) (as $(P_{BB} > \{P_{BI}$ or $P_{BU}\})$). Hence, if the currently sensed channel is busy, then the predicted state of that channel during the next slot would be busy with higher probability. In that case, the secondary user prefers to switch the channel expecting to transmit in the other channel. In a negatively correlated traffic, it would be other way. That is, the secondary user may find it beneficial to stay in that channel without any transmission when the sensed slot is busy and it will wait for the next slot hoping that the primary user may vacate that channel. We believe that positively correlated primary user traffic represents a more common case of a stable system with enough data at primary user for transmission. Therefore, we assumed in the analysis that secondary user switches the channel when it senses the primary channel as busy. That is, the channel prediction simply leads the cognitive user to switch the channel when the channel is busy and stay in that channel when it is idle.

When the channel is in State U, the cognitive user can stay in that channel with underlay transmission until that channel becomes State B or, cognitive user can switch that channel and look for overlay transmission (State I). When switching, cognitive user may end up with a channel with State B and it may reduce the throughput. Following our framework and analysis, a cognitive user can decide, based on the primary user network statistics, which mode of transmission (underlay/overly) gives better long term throughput as evident from the figures from Section IV.

Further, we assume that the primary user's transition matrix (T) as a long term statistic and not instantaneous quantity. The secondary user can monitor the network over a period of time and, based on the sensed power level of the primary user Γ_S with the thresholds Γ_O and Γ_U, it can build and model the primary user statistics (state transition matrix, T) [26]. Also, the secondary users can use any learning algorithms to model the primary user statistics [27][28]. Another approach is to use databases where these statistics can be stored and provided to secondary users upon request.

It is important to note that secondary users do sense in every slot and make the (instantaneous) decision based on the sensing result to transmit or not. On the other hand, the channel switching strategy depends on the primary user (long term) transition matrix as described in the flowchart in 7. If a secondary user finds the sensed channel to be in State U (underlay transmission), then that secondary user can stay in that channel and transmits at lower power for few slots or else, it can switch the channel sensing in the next slot expecting that channel to be in State I (overlay transmission). For a given transition matrix and underlay-overlay throughput ratio, the channel switching decision can be made and then, the sensing is performed before transmission. Hence, the interference constraint is not compromised. The use of long term transition matrix may lead to lower throughput as the channel switching decisions are made based on it; but it can be updated based on the sensing results in each slot. The frequency of the updates depends on the dynamic nature of the primary system and the complexity limitation of the secondary system.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we present and discuss the performance difference between the two schemes. The transition probabil-

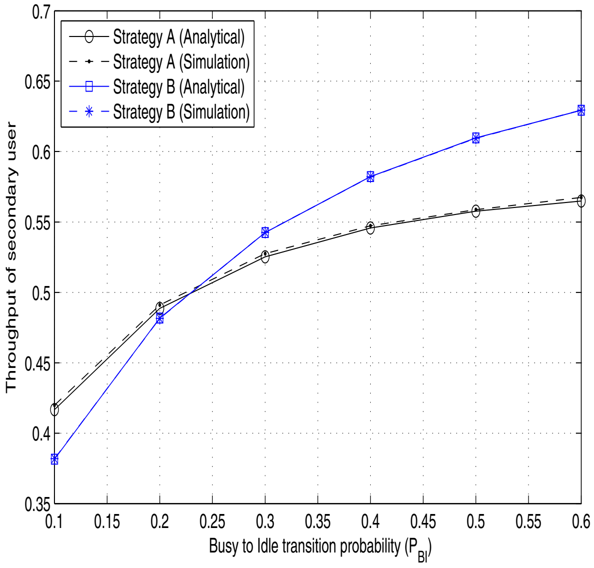

Fig. 8. Analytical and simulation throughput performance comparison for different P_{BI}. $\frac{R_U}{R_O} = 0.6, P_{BU} = 0.1, P_{UB} = 0.3, P_{UI} = 0.1, P_{IU} = 0.3, P_{IB} = 0.2$.

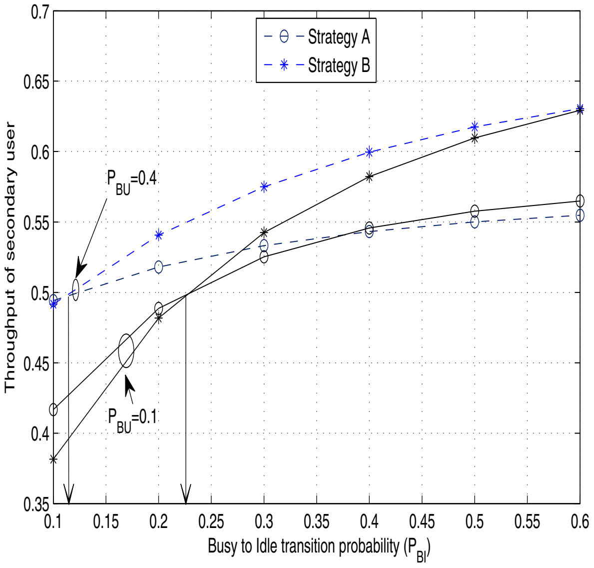

Fig. 9. Throughput performance for different P_{BI} and P_{BU} for $\frac{R_U}{R_O} = 0.6$. $P_{UB} = 0.3, P_{UI} = 0.1, P_{IU} = 0.3, P_{IB} = 0.2$.

ities are randomly picked to explain the analysis. The Monte-Carlo simulation results are compared with analytical results in Fig. 8 for different transition probabilities of P_{BI} (busy to idle) when the data ratio $\frac{R_U}{R_O}$ is 0.6. As noted in the figure, analytical and simulation results are in close agreement. We verified the analysis with simulation results for other transition probabilities as well.

As we mentioned earlier, we assumed that the primary user traffic is positively correlated with time. That is, when a primary user occupies the channel, it stays in that channel for few slots $(P_{BB} > \{P_{BI} \text{ or } P_{BU}\})$. Based on this assumption, we evaluated the underlay-overlay mode selection strategy for the secondary user. Our analysis and simulation show the best strategy selection under this scenario (that is, secondary user switches the channel when it senses the primary channel as busy). The strategy selection may change for other scenarios such as with $(P_{BB} < \{P_{BI} \text{ or } P_{BU}\})$ and, those cases are not shown in the plots. We believe the analysis for other scenarios can be done similarly by following the same steps and principles for the considered case.

A. Different primary user characteristics in channel sharing

When the primary user occupies the channel, it can stay in that channel for longer/shorter duration or it can share the channel with the secondary user. Longer the primary user occupies the channel alone, lower the throughput of the secondary user will be. In Fig. 9, the throughput performance for different transition probabilities P_{BU} are considered for the above scenario. As $P_{BB} + P_{BI} + P_{BU}$=1, for a specific P_{BI}, if P_{BU} is higher, then P_{BB} would be lower. That is, if a primary user uses the channel (busy) for a short duration then the secondary user's throughput would be higher. We can see from Fig. 9 that depending on the length of stay of a primary user, different strategies can be chosen by the secondary user. When the primary users stays longer in a channel (i.e., P_{BB} is higher), the Strategy A is useful. That is, secondary user should look for an idle/sharing channel. The exact strategy switching thresholds can be found from the analysis for different primary user characteristics. In Fig. 9, it is found that the Strategy B performs better when $P_{BI} > 0.1$ for $P_{BU} = 0.4$ and when $P_{BI} > 0.22$ for $P_{BU} = 0.1$ respectively. Similarly, we can select the best strategy provided the statistical data of primary users are given.

B. Different data rates in channel sharing $\left(\frac{R_U}{R_O}\right)$

The strategy selection not only depends on the primary user characteristics but also on the data rate of the secondary user when sharing the channel. As discussed earlier, there exists a strategy switching threshold that directs the best strategy to follow. This threshold changes with different data rates. We consider the ratio between the data rate of the secondary user when sharing the channel with primary user (R_U) and the data rate when using the channel alone (R_O). We do not assume any specific data rates, rather consider the ratio to make the analysis more generic.

The throughput performance comparison is shown in Fig. 10 and Fig. 11 for different data rates while sharing the channel with primary user for the ratio of 0.3 and 0.6 respectively. As expected, Strategy A performs better when the $\frac{R_U}{R_O}$ ratio increases. Hence, we can also see that for $P_{IU} = 0.1$, the strategy switching point shifts from $P_{IB} = 0.3$ to $P_{IB} = 0.2$ when $\frac{R_U}{R_O}$ changes from 0.3 to 0.6 while other transition probabilities are kept fixed. That is, when the shared channel data rate (R_U) of a secondary user increases, the Strategy A performs better. These analytical figures give better understanding of the throughput performance of the secondary users in these different strategies. In Fig. 10, we can see that proper use of strategy gives up to 8% throughput advantage when

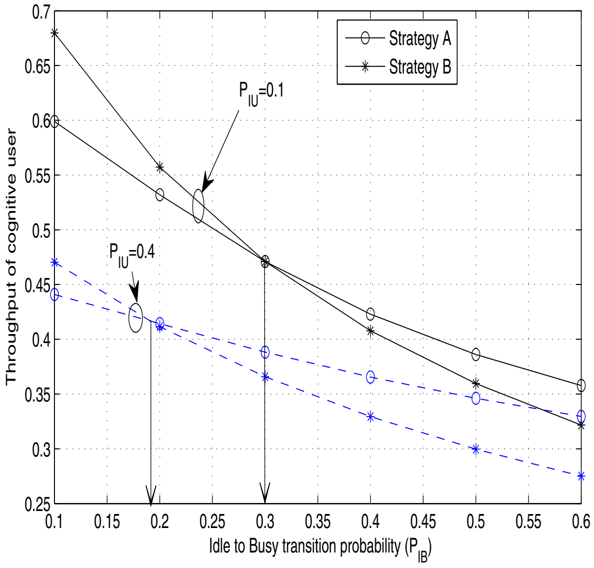

Fig. 10. Throughput performance for different P_{IB} and P_{IU} for $\frac{R_U}{R_O} = 0.3$. $P_{UB} = 0.1, P_{UI} = 0.2, P_{BU} = 0.3, P_{BI} = 0.1$.

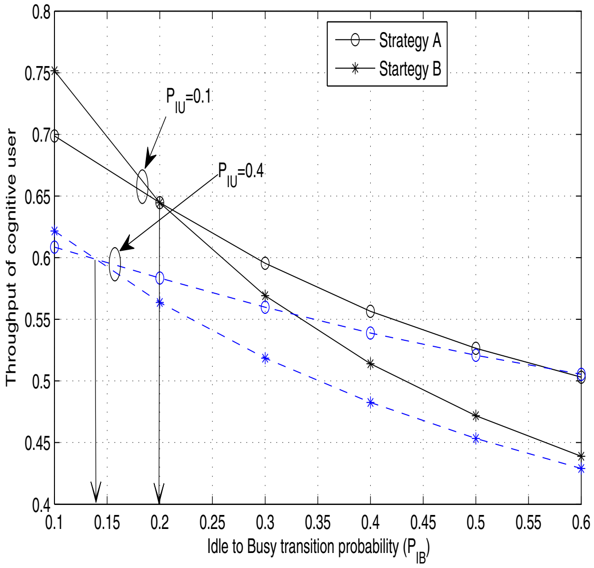

Fig. 11. Throughput performance for different P_{IB} and P_{IU} for $\frac{R_U}{R_O} = 0.6$. $P_{UB} = 0.1, P_{UI} = 0.2, P_{BU} = 0.3, P_{BI} = 0.1$.

$P_{IB} = 0.1$. Similarly, we can find different thruput advantage for different conditions.

In Fig. 12, the strategy selection criteria for different throughput ratio (R_U/R_O) are shown for different set of primary user statistics. For example, when $R_U/R_O = 0.6$ and $P_{BI} > 0.23$, it is better to select Strategy B. Also, we can see from Fig. 10 that for higher R_U/R_O, the strategy selection is towards Strategy B (underlay) when the probability P_{UU} is higher. These data can be calculated analytically form our framework for different primary user statistics and can be stored in a lookup table, if needed, to speed up the decision process by the secondary users to select the optimal transmission mode.

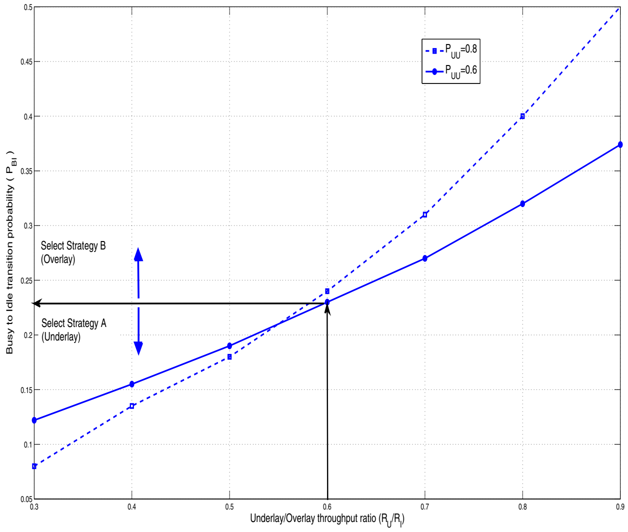

Fig. 12. Strategy selection for different throughput ratio ($\frac{R_U}{R_O}$) ($P_{BU} = 0.1, P_{UB} = 0.3, P_{UI} = 0.1, P_{IU} = 0.3, P_{IB} = 0.2$)

V. CONCLUSION

In this article, we analyzed two different opportunistic access strategies with different primary user channel conditions where the secondary user can use an idle channel alone or can share with primary user under certain circumstances. When the primary user channel characteristics are given, we can find the best access strategy for the secondary user from our analysis. When the data rate ratio (R_U/R_O) increases, the Strategy A performs better (access with sharing the channel) for a positively correlated primary user traffic. That is, it is beneficial for a secondary user to look for an idle/shared channel. If the ratio decreases, it is better to look for an idle channel only (Strategy B, access without sharing the channel). If a primary user occupies the channel (busy) for a long period, then it is better to look for Strategy A. The exact strategy switching thresholds (mode selection) can be found from our analysis and it depends on the transition probabilities of the primary user and the throughput ratio between the two modes of operation. The proper strategy selection can provide throughput gain over the other based on the scenario. It is shown analytically and via simulation that proper transmission mode selection can provide up to 8% throughput advantage. The performance loss due to imperfect sensing and the gain due to sensing of multiple channels can be evaluated when using the proposed transmission mode selection.

APPENDIX A

The square matrix T defined in (1) can be written as,

$$T = VEV^{-1},$$

where E and V denote a diagonal matrix of eigenvalues and a full matrix whose columns are the corresponding eigenvectors of matrix T respectively. The diagonal elements of the matrix

E can be denoted by λ_1, λ_2 and λ_3. Similarly the matrix $\bar{T}$, defined in (5) can be written as $\bar{T} = \bar{V}\bar{E}\bar{V}^{-1}$ where the diagonal elements are denoted by $\bar{E}_{(1,1)} = \mu_1$ and $\bar{E}_{(2,2)} = \mu_2$ where $E_{(m,n)}$ denotes the m^{th} row and n^{th} column element of the matrix $\bar{E}$ for any positive integer m, n. Using the above properties, $P_{BU}^{(i)}$, for any positive integer i, can be calculated as $P_{BU}^{(i)} = (VE^iV^{-1})_{(1,3)}$. The infinite series sum can be found as shown below,

$$\begin{aligned}\sum_{i=3}^{\infty} \bar{P}_{II}^{(i-2)} P_{BI}^{(i+1)} &= (\bar{V}_{(1,1)} \bar{V}_{(1,1)}^{-1} W_{\mu_1})_{(1,2)} \\ &+ (\bar{V}_{(1,2)} \bar{V}_{(2,1)}^{-1} W_{\mu_2})_{(1,2)},\end{aligned}$$

where

$$W_{\mu_1} = V \begin{pmatrix} \frac{\mu_1(\lambda_1)^4}{1-\lambda_1\mu_1} & 0 & 0 \\ 0 & \frac{\mu_1(\lambda_2)^4}{1-\lambda_1\mu_1} & 0 \\ 0 & 0 & \frac{\mu_1(\lambda_3)^4}{1-\lambda_1\mu_1} \end{pmatrix} V^{-1},$$

$$W_{\mu_2} = V \begin{pmatrix} \frac{\mu_2(\lambda_1)^4}{1-\lambda_1\mu_2} & 0 & 0 \\ 0 & \frac{\mu_2(\lambda_2)^4}{1-\lambda_1\mu_2} & 0 \\ 0 & 0 & \frac{\mu_2(\lambda_3)^4}{1-\lambda_1\mu_2} \end{pmatrix} V^{-1}.$$

Similarly, we can find the other infinite series sum.

APPENDIX B

In Strategy B, the secondary user stays in only when the state of the channel is *State I*. As mentioned earlier, we consider a two primary channel system (say, channel X and Y). The *State* U_B denotes that state of the current channel (say channel X) is *State U* and stays in only one slot ($L = 1$). The state of the previous channel (channel Y) was *State B* denoted by the subscript. Since the secondary user stays in only one slot in channel X (as it is in *State* U_B) and switches back to the previous channel (channel Y), we can find the transition probability of a channel (in this example, channel Y) from *State B* to *State B* after 2 slots as $P_{BB}^{(2)}$. Note that, $P_{BB}^{(2)}$ denotes the transition probability of a channel becoming *State B* from *State B* after 2 slots. Hence, we can write the transition probability from *State* U_B to *State* B_U as $P_{BB}^{(2)}$. Similarly, we can write the transition probability from *State* U_U to *State* B_U as $P_{UB}^{(2)}$. Next we will find the transition probability from *State* U_W to *State* B_U. The *State* U_W denotes that the current state of the channel is *State U* & $L = 1$, and the state of the previous channel's last slot was either *State B* or *State U* (denoted by subscript W). That is, the secondary user stayed in the previous channel for L (≥ 2) slots and left that channel since the last slot of that channel was either *State B* or *State U*. Since the secondary user was in that channel for more than one slot, the one before the last slot should be *State I*. Hence, the probability of last slot being in *State U* can be written as $\delta = P_{IU}/(P_{IU} + P_{IB})$ and last slot being in *State B* can be written as $1-\delta$. Hence, we can write the transition probability from *State* U_W to *State* B_U as $\delta P_{UB}^{(2)} + (1-\delta)P_{BB}^{(2)}$. As we already know the transition probabilities from *State* U_U to *State* B_U, *State* U_B to *State* B_U and *State* U_W to *State* B_U, we can write the steady state equation for the State B_U as,

$$\begin{aligned}P_{B_U} &= P_{UB}^{(2)} P_{U_U} + P_{BB}^{(2)} P_{U_B} \\ &+ (\delta P_{UB}^{(2)} + (1-\delta) P_{BB}^{(2)}) P_{U_W}\end{aligned} \tag{21a}$$

Similarly, we can write the steady state equation for the other states as,

$$\begin{aligned}P_{U_U} &= P_{UU}^{(2)} P_{U_U} + P_{BU}^{(2)} P_{U_B} && \text{(21b)} \\ &+ (\delta P_{UU}^{(2)} + (1-\delta) P_{BU}^{(2)}) P_{U_W} \\ P_{U_B} &= P_{BU}^{(2)} P_{B_B} + (\delta P_{UU}^{(2)} && \text{(21c)} \\ &+ (1-\delta) P_{BU}^{(2)}) P_{B_W} + P_{UU}^{(2)} P_{B_U} \\ P_{2_B} &= P_{BI}^{(2)} (P_{IB} + P_{IU}) P_{B_B} && \text{(21d)} \\ &+ P_{UI}^{(2)} (P_{IB} + P_{IU}) P_{B_U} \\ &+ (\delta P_{UI}^{(2)} (1-\delta) P_{BI}^{(2)}) (P_{IB} + P_{IU}) P_{B_W} \\ P_{B_B} &= P_{BB}^{(2)} P_{B_B} + (\delta P_{UB}^{(2)} + (1-\delta) P_{BB}^{(2)}) P_{B_W} && \text{(21e)} \\ &+ P_{UB}^{(2)} P_{B_U} \\ P_{2_U} &= P_{UI}^{(2)} (P_{IB} + P_{IU}) P_{U_U} + P_{BI}^{(2)} (P_{IB} + P_{IU}) P_{U_B} \\ &+ (\delta P_{UI}^{(2)} + (1-\delta) P_{BI}^{(2)}) (P_{IB} + P_{IU}) P_{U_W} && \text{(21f)} \\ P_{B_W} &= P_{BB}^{(3)} P_{2_B} + (\delta P_{UB}^{(3)} + (1-\delta) P_{BB}^{(3)}) P_2 + P_{UB}^{(3)} P_{2_U} \\ &+ P_{BB}^{(4)} P_{3_B} + (\delta P_{UB}^{(4)} + (1-\delta) P_{BB}^{(4)}) P_3 + P_{UB}^{(4)} P_{3_U} \\ &+ \ldots\ldots \\ &= \sum_{i=2}^{\infty} P_{BB}^{(i+1)} P_{i_B} \\ &+ (\delta P_{UB}^{(i+1)} + (1-\delta) P_{BB}^{(i+1)}) P_i + P_{UB}^{(i+1)} P_{i_U}\end{aligned}$$

Using (15),

$$\begin{aligned}P_{B_W} &= P_{2_B} \sum_{i=2}^{\infty} P_{BB}^{(i+1)} P_{II}^{i-2} \\ &+ P_2 \sum_{i=2}^{\infty} (\delta P_{UB}^{(i+1)} + (1-\delta) P_{BB}^{(i+1)}) P_{II}^{i-2} \\ &+ P_{2_U} \sum_{i=2}^{\infty} P_{UB}^{(i+1)} P_{II}^{i-2} \\ &= W_{(1,1)} P_{2_B} + W_{(3,1)} P_{2_U} \\ &+ (\delta W_{(3,1)} + (1-\delta) W_{(1,1)}) P_2\end{aligned} \tag{21g}$$

where

$$W = V \begin{pmatrix} \frac{(\lambda_1)^3}{1-P_{II}\lambda_1} & 0 & 0 \\ 0 & \frac{(\lambda_2)^3}{1-P_{II}\lambda_1} & 0 \\ 0 & 0 & \frac{(\lambda_3)^3}{1-P_{II}\lambda_1} \end{pmatrix} V^{-1}.$$

Similarly, we can write

$$\begin{aligned}P_2 &= \left(W_{(1,2)} P_{2_B} + W_{(3,2)} P_{2_U}\right) (P_{IB} + P_{IU}) \\ &+ (\delta W_{(3,2)} + (1-\delta) W_{(1,2)}) (P_{IB} + P_{IU}) P_2\end{aligned} \tag{21h}$$

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Sivasothy Senthuran received the B.Sc. degree in Electronic and Telecommunication Engineering with first class honors from the University of Moratuwa, Sri Lanka in 2003, M.Eng. degree in Telecommunications from Asian Institute of Technology (AIT), Thailand, and M.Sc. degree in Communication Networks and Services from Télécom SudParis, France, in 2006. He is currently working towards the Ph.D. degree at the Department of Electrical and Computer Engineering in Ryerson University, Toronto, Canada. From September 2006 to August 2010, Senthuran was a Research Assistant with the WINCORE Lab at Ryerson University, Canada and he is currently with Research In Motion Ltd., Canada.

Alagan Anpalagan received the B.A.Sc. (H), M.A.Sc. and Ph.D. degrees in Electrical Engineering from the University of Toronto, Canada. He joined the ELCE Department of at Ryerson University in 2001 and was promoted to Full Professor in 2010. He served the department as Graduate Program Director (2004-09) and the Interim Electrical Engineering Program Director (2009-10). During his sabbatical (2010-11), he was a Visiting Professor at Asian Institute of Technology and Visiting Researcher at Kyoto University. Dr. Anpalagan's industrial experience includes working at Bell Mobility on 1xRTT system deployment studies (2001), at Nortel Networks on SECORE R&D projects (1997) and at IBM Canada as IIP Intern (1994).

Dr. Anpalagan directs a research group working on radio resource management (RRM) and radio access & networking (RAN) areas within the WINCORE Lab that mainly focuses on cross layer design, analysis and optimization of wireless systems. His current research interests include cognitive radio RRM, wireless cross layer design and optimization, collaborative communication, green communications technologies and QoE-aware femtocells. Dr. Anpalagan serves as Associate Editor for the IEEE COMMUNICATIONS LETTERS (2010-12) and *Springer Wireless Personal Communications* (2009-12), and past Editor for *EURASIP Journal of Wireless Communications and Networking* (2004-2009). He also served as EURASIP Guest Editor for two special issues in RRM in 3G+ Systems (2006) and Fairness in RRM for Wireless Networks (2008). Dr. Anpalagan served as TPC Co-Chair of: IEEE PIMRC'11 Track on Cognitive Radio and Spectrum Management, IEEE IWCMC'11 Workshop on Cooperative and Cognitive Networks, IEEE CCECE'04/'08 and WirelessCom'05 Symposium on RRM.

Dr. Anpalagan served as IEEE Toronto Section Chair (2006-07), ComSoc Toronto Chapter Chair (2004-05), Chair of IEEE Canada Professional Activities Committee (2009-11). He is the recipient of the Dean's Teaching Award (2011), Faculty Scholastic, Research and Creativity Award (2010), Faculty Service Award (2010) at Ryerson University. Dr. Anpalagan also completed a course on Project Management for Scientist and Engineers at the University of Oxford CPD Center. He is a registered Professional Engineer in the province of Ontario, Canada.

Olivia Das received the BSc and MSc degrees in mathematics from the University of Calcutta, India, in 1993 and 1995, respectively, and the master's degree in information and system sciences and the PhD degree in electrical engineering in the area of dependability evaluation of software architectures from Carleton University, Canada, in 1998 and 2004, respectively. Currently, she is an assistant professor in electrical and computer engineering at Ryerson University, Canada. She is an active researcher in the area of dependability and performance evaluation of distributed systems. Her work exploits knowledge of functional layering to identify failure dependencies in complex systems and create a scalable analysis. She is a member of the IEEE and the IEEE Computer Society.