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Power Allocation and Relay Assignment for Shared-Band Nonregenerative Relaying in Cognitive Radio Systems

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***Abstract*—The use of multiple relays can increase the performance of a cooperative communication system. A well-designed multiple-relay-assignment and power-allocation scheme can be helpful in reducing interference induced to primary users (PUs) in a cognitive radio system (CRS). In this correspondence, we propose a joint multiple-relay-assignment and power-allocation (JRAPA) scheme in shared-band nonregenerative relaying in CRSs. The main aim of JRAPA in CRSs is to maximize sum capacity under the constraint that the interference to the PUs is below their specified levels. The proposed JRAPA is an NP-hard nonconvex mixed-integer nonlinear optimization problem (NC-MINLP). We propose a greedy iterative JRAPA (IJRAPA) scheme for CRSs. The complexity of the proposed IJRAPA is quadratic with the number of relays. The effectiveness of IJRAPA is verified through simulation results satisfactorily.**

***Index Terms*—Cognitive radio, cooperative communication, nonconvex optimization.**

I. Introduction

Recent research in wireless communication systems shows that relaying techniques can offer significant benefits in throughput enhancement and range extension. The performance of a cooperative communication system can be improved by using multiple relays rather than a single relay, which convey the same information to the destination via a different and possibly uncorrelated path [1], [2]. However, using all the relays available in the system for secondary users (SUs) in a cognitive radio system (CRS) may not be a viable idea because the interference caused by relays to primary users (PUs) may exceed the prescribed limit. The use of multiple relays in a network comprising single source and multiple receivers brings forth the issue of how best to assign the relays to the receivers. In this paper, we focus on the problem of optimally assigning the relays to the SUs in a CRS such that the sum capacity of CRSs is maximized under the constraint that the interference to the PUs is below their specified levels.

The broadband standard IEEE 802.16j has been proposed for supporting mobility and relaying in vehicular networks [7]. The use of cognitive radio with broadband technologies can help in increasing the data rate in vehicular networks such as highways and railways. The relays can be deployed on the infrastructures near the roads. A simple scenario of relay-assisted vehicular cognitive radio is shown in Fig. 1, where we can see that a vehicle can get data from multiple relays. The

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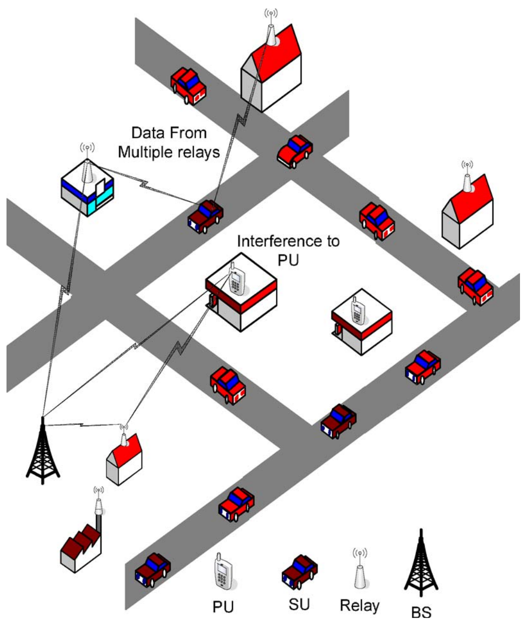

Fig. 1. Vehicular network with multiple relays.

use of multiple relays is also useful in coping with connectivity issues in vehicular cognitive radio networks.

A. Literature Review

In [1], joint bandwidth and power-allocation strategies for a Gaussian relay network are investigated. Orthogonal and shared-band amplify-and-forward (AF) and decode-and-forward (DF) schemes are analyzed for joint bandwidth and power allocation. The main objective of joint bandwidth and power allocation is to maximize the signal-to-noise ratio (SNR) at the receiver using AF and DF schemes. The study in [2] proposes a centralized framework that selects multiple relays for transmission in a two-hop network. The aim of multiple-relay selection is to maximize the SNR at the destination using binary power allocation at the relays. Optimal relay assignment and power allocation in a cooperative cellular network is discussed in [3]. Using sum-rate maximization as a design metric, Kadloor and Adve proposed a convex optimization problem that provides an upper bound on the performance of the cooperative communication network. A heuristic water-filling algorithm is also suggested to find near-optimal relay assignment and power allocation. In [4], a linear-marking mechanism is investigated for relay assignment in a multihop network with multiple source–destination pairs. The aim of the proposed linear-marking mechanism is to maximize the worst user capacity. A decoupling-based method for joint relay selection and link scheduling in a relay-assisted cellular network is proposed in [5]. An uplink base relay assignment to improve the worst user capacity is described in [6]. The relay selection/assignment schemes in the literature [1]–[6] are not applicable in CRSs because the relay selection/assignment obtained from these schemes may generate more interference to the PUs than allowed, thus violating the interference constraint. Recently, relay selection/assignment in CRSs has drawn the attention of the wireless research community [8]–[12]. A dynamic spectrum-sharing protocol for bidirectional relaying for CRSs is proposed in [8]. Li *et al.* analytically derived the outage probabilities for both the primary and secondary systems under their proposed dynamic spectrum-sharing protocols for bidirectional CRSs. In [10], Asghari and Aissa proposed end-to-end performance analysis in a spectrum-sharing cooperative relaying system under the PUs QoS constraint. The authors investigated the overall average bit error rate, ergodic capacity, and outage probability of the SUs subject to appropriate constraints on the interference power at the primary receivers. In [11], Hou *et al.* investigated the mathematical formulation with the objective of minimizing the required network-wide radio spectrum resources for a set of user sessions in CRSs. A relay-assignment scheme with a binary power control scheme is presented in [12].

Finding an optimal interference-aware multiple-relay assignment is computationally expensive. An exhaustive search algorithm (ESA) evaluates all possible relay assignments, which is computationally inefficient. Therefore, we need an efficient algorithm for joint multiple-relay assignment and power allocation (JRAPA) in CRSs.

B. Contributions

To avoid delay in communication, instead of multiple orthogonal relays or multihop communication, in this paper, we use two-hop shared-band AF communication. In two-hop shared-band AF communication, the receiver will get the same data simultaneously from multiple relays at the same frequency and time slot. To the best of the authors' knowledge, there exists no JRAPA scheme for shared-band relaying in multiuser CRSs in the literature. The motivation of this work is to fill the gap, which is particularly important for multiple-relay assignment in shared-band relaying in CRSs. The main contributions of this paper are summarized as follows.

1) We propose and formulate a novel constrained optimization problem for JRAPA in shared-band AF relaying in CRSs.
2) The proposed optimization problem is a nonconvex mixed-integer nonlinear optimization problem (NC-MINLP), which is generally NP-hard.
3) In the context of the proposed optimization problem, we propose an efficient greedy iterative algorithm that jointly assigns multiple relays and allocates power to the SUs while satisfying the interference constraint to the primary network. The complexity of the proposed algorithm is quadratic with the number of relays.
4) We compare the proposed greedy algorithm with an ESA and a branch-and-bound-based outer-approximation algorithm. We also examine the effect of different system parameters (e.g., interference threshold level, number of PUs, and number of relays) on the performance of the proposed scheme.

Throughout this paper, we use $\boldsymbol{A}$, $\boldsymbol{a}$, and a to represent a matrix, a vector, and an element of a vector, respectively. The rest of this paper is organized as follows: The system model is presented in Section II. In Section III, we present our low-complexity interference-aware iterative JRAPA (IJRAPA) algorithm and its computational complexity. Simulation results are presented in Section IV.

II. System Model and Problem Formulation

We consider a two-hop uplink wireless network with K SUs, L relay nodes, M PUs, and a base station (BS). We denote by h_l^{BS} the channel from the lth relay to the BS, by h_k^l the channel from the kth SU to the lth relay, by g_m^k the channel from the kth SU to the mth PU, and by g_m^l the channel from the lth relay to the mth PU. We denote by p_l the lth relay's transmission power, by $p_l^{\max}$ the maximum power of the lth relay, by p_k^s the source power in the kth user band, by $p_s^{\max}$ the maximum source power of the kth SU, and by $I_{m,k}^{\max}$ the maximum tolerable interference at the mth PU in the kth SU's band. In our system model, each user will receive the data on a separate frequency band. Each relay will transmit and receive in the same frequency band. We consider a two-step shared-band AF scheme [1]. In shared-band AF, any user can get data from multiple relays at the same time and in the same frequency band. We assume that the BS knows the channel measurement of the relay and SU links through a reliable feedback channel.

The IEEE 802.22 WRAN standard recommends two schemes for PU protection. These are listen-before-talk (spectrum sensing) and geo-location/database schemes [13]. In the listen-before-talk scheme, the SU senses the presence of primary network signals to select the channels that are not in use. In the geo-location/database scheme, the locations of PUs and SUs are stored in a central database. The central controller/spectrum manager (also called as a BS) of the SUs has access to the location database. In this paper, we assume that the secondary network's BS gets the location information on each PU from the central database. We also assume that the BS knows the active PU's channel gains. Knowledge of the PU's location and channel gains adds overhead on CRS operations. The overhead refers to any extra sensing time, delay, energy, and operations devoted to cooperative/noncooperative spectrum sensing [14]. A number of low-overhead channel sensing methods are in the literature [14]–[17]. In this paper, we will mainly focus on the relay-assignment and power-allocation problem.

We consider a half-duplex protocol in which each symbol is transmitted in two time slots: in the first time slot by the source and in the second time slot by the relays. In the first time slot, the signal received by the lth relay (after listening to the kth SU's band) can be written as $\sqrt{p_k^s} h_l^{\mathrm{BS}} s + Z_l$, where complex-valued s represents the transmitted symbol, and Z_l represents the complex-valued white Gaussian noise at the lth relay. The symbol value s is normalized so that $E(|s|^2) = 1$ and $(N_o/2)$ is the power spectral density of noise Z_l. Noise power N (in watts) in each SU band can be written as $N = (N_o/2)2W$, where W is the bandwidth of each SU band [19]. In the second time slot, the relays amplify the received signal and retransmit the amplified signal. The channel capacity of the kth user for shared-band AF is [1]

$$C_k = \frac{1}{2}\log\left[1 + \frac{p_k^s}{N}\left(|h_k^s|^2 + \frac{\left(\sum_{l=1}^{L} |h_l^{\mathrm{BS}} h_k^l|\beta_l\sqrt{p_l}\right)^2}{1 + \sum_{l=1}^{L}\left(|h_k^l|\beta_l\sqrt{p_l}\right)^2}\right)\right]$$

where $\beta_l = (\sqrt{p_k^s |h_l^{\mathrm{BS}}|^2 + (N/2)})^{-1}$. For relay assignment, we define as a binary assignment indicator

$$\varepsilon_k^l = \begin{cases} 1, & \text{if the } l\text{th relay is assigned to the } k\text{th SU} \\ 0, & \text{otherwise.} \end{cases}$$

The channel capacity of the kth user for AF relaying with a binary assignment indicator is

$$C_{k,\varepsilon} = \frac{1}{2}\log\left[1 + \frac{p_k^s}{N}\left(|h_k^s|^2 + \frac{\left(\sum_{l=1}^{L} \varepsilon_k^l |h_l^{\mathrm{BS}} h_k^l|\beta_l\sqrt{p_l}\right)^2}{1 + \sum_{l=1}^{L}\varepsilon_k^l\left(|h_k^l|\beta_l\sqrt{p_l}\right)^2}\right)\right].$$

The JRAPA problem in a CRS is to determine the assignment of relays and power allocation to maximize the sum capacity under the constraint that interference to the PUs is within a specified value. Mathematically, we can write the JRAPA problem as

$$\begin{aligned} &\max_{\boldsymbol{\varepsilon}, \boldsymbol{p_k}, \boldsymbol{p_l}} \quad \sum_{k=1}^{K} C_{k,\boldsymbol{\varepsilon}} \\ &\text{subject to } C1: \sum_{k=1}^{K} \varepsilon_k^l \le 1 \forall l \\ &C2: 0 \le p_k^s \le p_s^{\max} \forall k \\ &C3: ; 0 \le p_l \le \sum_{k=1}^{K} \varepsilon_k^l p_l^{\max} \forall l \\ &C4: p_k^s |g_m^k|^2 \le I_{m,k}^{\max} \forall (m,k) \\ &C5: \sum_{l=1}^{L} \varepsilon_k^l p_l |g_m^l|^2 \le I_{m,k}^{\max} \forall (m,k) \\ &C6: \varepsilon_k^l \in \{0,1\} \forall (l,k). \end{aligned} \tag{1}$$

In (1), constraint $C1$ assures that a relay can only be assigned to one SU, and $C2$ and $C3$ are power constraints. Constraint $C3$ ensures that if the lth relay is not assigned to any SU, then the transmission power of the lth relay should be zero. Constraints $C4$ and $C5$ are interference constraints. The given JRAPA problem is an NC-MINLP. Even for given subsets of relay assignment, the objective in (1) is not a concave function of the source and relay power levels. This is due to the term

$$\Gamma_k = \left(\frac{\left(\sum_{l=1}^{L}\left|h_l^{\mathrm{BS}} h_k^l\right|\beta_l\sqrt{p_l}\right)^2}{1 + \sum_{l=1}^{L}\left(|h_l^{\mathrm{BS}}|\beta_l\sqrt{p_l}\right)^2}\right) \tag{2}$$

which means that we cannot apply standard convex optimization techniques on (1). The optimization problem in (1) is NP-hard as it is easy to reduce the multiple multidimensional knapsack problem to the joint relay and power-allocation problem. NP-hardness means that there exists no algorithm that can solve problem (1) in polynomial time.

An ESA for (1) would evaluate all possible relay assignments and, for each assignment, determine the power of each relay. The number of different relay assignments exponentially increases with the number of relays and the number of SUs. High-speed communications demand an assignment scheme with low computational complexity. In the following section, we will present a low-complexity algorithm for JRAPA.

III. Proposed Approach to a Solution

A. Decoupling of Source Power

We first note a special structure of the optimization problem in (1). For any choice of relay assignment represented by $\{\varepsilon_k^l\}$ and relay transmission power represented by $\{p_1, p_2, \ldots, p_L\}$, the objective function is an increasing function of variable p_k^s, which is the source transmission power. In addition, the only constraints on variable p_k^s are $0 \le p_k^s \le p_s^{\max} \forall k$ and $p_k^s |g_m^k|^2 \le I_{m,k}^{\max} \forall (m,k)$, which can be simplified to

$$0 \le p_k^s \le \min\left(p_s^{\max}, \frac{I_{1,k}^{\max}}{\left|g_1^k\right|^2}, \ldots, \frac{I_{M,k}^{\max}}{\left|g_M^k\right|^2}\right)$$

and variable p_k^s does not appear in any other constraints in (1). Therefore, for any choice of relay assignment $\{\varepsilon_k^l\}$ and relay transmission power $\{p_1, p_2, \ldots, p_L\}$, maximizing source power p_k^s is equivalent to

$$\min\left(p_s^{\max}, \frac{I_{1,k}^{\max}}{\left|g_1^k\right|^2}, \ldots, \frac{I_{M,k}^{\max}}{\left|g_M^k\right|^2}\right)$$

for each k.

Algorithm 1 : Iterative algorithm for Joint multiple-Relay Assignment and Power Allocation (IJRAPA)

Initialization: $k \leftarrow 1$
$\Theta(l) \leftarrow \mathbf{argmax}_{k \in \{1,2,\ldots,K\}} |h_k^l|^2, \forall l$
while $\{k \le K\}$ **do**
  $L_k \leftarrow \{l | \Theta(l) = k\}$
  if $\{L_k \ne \emptyset\}$ **then**
    $\alpha \leftarrow \mathbf{sortindex}((|h_l^{\mathrm{BS}}|^2 |h_l, k|^2 / \max(|g_1^l|^2, \ldots, |g_m^l|^2)), \forall l \in L_k)$
    $i = i + 1, \Omega \leftarrow \emptyset, I_m^{\mathrm{sum}} = 0$
    while $i \le |L_k|$ **do**

```
9:          Ω ← Ω ⋃ {α(i)}
10:         p_Ω ← (|h^s_{α(i)} h_k^{α(i)}|β_{α(i)}(1 + Σ_{l≠α(i),l∈Ω} β_l^2 |h_k^l|^2 p_l)/
            Σ_{l≠α(i),l∈Ω} |h_l^{BS} h_k^l|β_l √p_l))^2
11:         p_{α(i)} ← min((I_m^max − I_m^sum/|g_m^{α(i)}|^2), p_{α(i)}^max, p_Ω)
12:         I_m^sum ← I_m^sum + p_{α(i)}|g_m^{α(i)}|^2
13:         L_k ← L_k \ {α(i)}
14:       end while
15:    end if
16:    k ← k + 1
17: end while
```

B. Greedy Heuristic

Now, we will present a greedy IJRAPA for CRSs. Algorithm 1 shows the pseudo code of the proposed IJRAPA.

Relay Assignment: The channel gains between the relays and the SUs play a key role in relay assignment. For each relay l, the algorithm compares the channel gains from it to all SUs. Then, the algorithm assigns relay l to the SU to which relay l has the best channel. At the start, the algorithm generates vector Θ that contains the indexes of the SUs that have the highest channel gains with the relays. Vector Θ is obtained using the expression $\Theta(l) \leftarrow \mathbf{argmax}_{k\in\{1,2,\ldots,K\}} |h_k^l|^2 \forall l$. After deciding the relay and SU pairs, the algorithm iteratively allocates power to the relays that are assigned to the SUs.

Power Allocation: For developing a power-allocation algorithm, we can view the channel gain from the lth relay to the kth SU as profit taken from investing a unit transmission power to relay l. We also view the channel gain from the lth relay to its PUs as loss. In particular, our algorithm views $\max(|g_1^l|^2, \ldots, |g_m^l|^2)$ as loss incurred from investing unit transmission power to relay l. The algorithm allocates maximum power to a relay that has the maximum ratio of channel gains $|h_k^l|$ to the channel gain with the worst PU. Now, we will describe the algorithm.

At the kth iteration, based on Θ, the algorithm gets L_k subset of relays that are assigned to the kth user, that is, $L_k \leftarrow \{l|\Theta(l) = k\}$. At the start of each iteration, we initialize iteration counter $i \leftarrow 0$, relay-assignment set $\Omega \leftarrow \emptyset$, and interference due to assigned relays as $I_m^{\mathrm{sum}} \leftarrow 0$. We define function $sortindex$, which sorts the relays according to the ratio $sortindex((|h_l^{\mathrm{BS}}|^2 |h_l, k|^2 / \max(|g_1^l|^2, \ldots, |g_m^l|^2)), \forall l \in L_k)$ in descending order. Function $sortindex$ returns a vector that consists of the relay indexes according to the sorted values. The sorted indexes are stored in vector α. The algorithm iterates over the relay indexes in the sorted order. Since source power is already decoupled from (1), with known relay assignment, the objective function is monotonically increasing with $\Gamma_k = ((\sum_{l=1}^L |h_l^{\mathrm{BS}} h_k^l|\beta_l \sqrt{p_l})^2 / 1 + \sum_{l=1}^L (|h_l^{\mathrm{BS}}|\beta_l \sqrt{p_l})^2)$. On the other hand, we can observe that Γ_k is not increasing with respect to the relay powers. By taking $(\partial \Gamma_k / \partial p_j)$, we observe that Γ_k is nondecreasing with respect to p_j when $p_j \leq ((|h_j^s h_k^j|\beta_j (1 + \sum_{l\neq j} \beta_l^2 |h_k^l|^2 p_l) / \sum_{l\neq j} |h_l^{\mathrm{BS}} h_k^l|\beta_l \sqrt{p_l}))^2$ (see Appendix A). With the help of the given observation, the power of the selected relay is determined using the formula as

$$p_{\alpha(i)} \leftarrow \mathbf{min}\left(\frac{I_m^{\max} - I_m^{\mathrm{sum}}}{g_m^{\alpha(i)}}, p_{\alpha(i)}^{\max}, p_\Omega\right) \tag{3}$$

where $p_\Omega \leftarrow (|h_{\alpha(i)}^s h_k^{\alpha(i)}|\beta_{\alpha(i)} (1 + \sum_{l\neq\alpha(i), l\in\Omega} \beta_l^2 |h_k^l|^2 p_l) / \sum_{l\neq\alpha(i), l\in\Omega} |h_l^{BS} h_k^l|\beta_l \sqrt{p_l})^2$.

The minimum of all three entries not only satisfies the interference and maximum power constraint but also ensures that the allocated power lies within the range of values for which the cost function is nondecreasing with respect to relay powers. Then, the algorithm determines the total interference generated by this allocated power. The algorithm continues until set L_k becomes empty. The algorithm will terminate when all the users get their assigned relays.

C. Complexity Analysis

The complexity of IJRAPA is measured in terms of flops, and the assignment operator takes one flop [23]. Lines 2, 4, 6, and 7 of the algorithm need approximately $LK + 2K + LMK\log(L)$ flops. Similarly, lines 9–11 require $2MK + L^2K$ flops, and lines 12 and 13 require $2KL$ flops. The number of flops required by the IJRAPA algorithm is $4LK + 2K + LMK\log(L) + 2MK + L^2K \approx O(L^2K)$. From the complexity analysis, we can observe that the complexity of the proposed algorithm is quadratic with the number of relays (L). The complexity of exhaustive search is $O(L^K)$.

IV. Simulation Results

Here, we present some simulation results to demonstrate the performance and numerical convergence of the proposed iterative schemes. The impact of network parameters (e.g., number of SUs, number of PUs, and interference threshold) is also analyzed.

A. Simulation Setup

The simulation setup for cooperative communication is similar to the noncooperative case. In all the simulations, channel gain h is modeled as $h = \Phi K_o (d_o/d)^\beta$ [24], where K_o is a constant that depends on the antenna characteristic and average channel attenuation, d_o is the reference distance for the antenna far field, d is the distance between the transmitter and the receiver, β is the path loss constant, and Φ is the Rayleigh random variable. Since this formula is not valid in the near field, in all the simulation results, we assume that d is greater than d_o. In all the results, $d_o = 10$ m, $K_o = 50$, and $\beta = 3$.

B. Results and Discussions

As mentioned in Section II, for a given realization of integer variables, the optimization problem in (1) is not a concave function of the relay powers. Thus, even for a given realization of integer variables, convex optimization techniques cannot be applied to the resulting optimization problem. For comparison, we provide an upper bound on the sum-rate capacity of JRAPA. The proposed upper bound is concave for a given realization of integer variables. The concave upper bound can be derived by using the Cauchy–Schwarz inequality as

$$\Gamma_k = \frac{\left(\sum_l \left|h_l^{\mathrm{BS}} h_k^l\right| \beta_l \sqrt{p_l}\right)^2}{1 + \sum_l \left(\left|h_l^{\mathrm{BS}}\right| \beta_l \sqrt{p_l}\right)^2} < \frac{\left(\sum_l \left|h_l^{\mathrm{BS}}\right|^2\right) \sum_l \left(\left|h_k^l\right|^2 \beta_l^2 p_l\right)}{1 + \sum_l \left(\left|h_k^l\right|^2 \beta_l^2 p_l\right)}. \tag{4}$$

The proposed upper bound is a concave function of relay powers. The proof is given in Appendix B. Fig. 2 shows an exemplary scenario for a nonconcave objective function and its concave upper bound. For this example, $K = 1$ and $L = 2$. The channel gains are set to $h_l^{\mathrm{BS}} = \{0.05, 1.25\}$ and $h_k^l = \{1.5, 1\}$. We can easily observe the concavity of the objective upper bound.

We compare the proposed IJRAPA with an ESA (for relay assignment) that uses an upper bound (ESA-UB). In ESA-UB, for each set of assigned relays, we run convex optimization to allocate the relay powers. We also include a lower bound on the ESA by substituting the values of power obtained by ESA-UB into the original objective

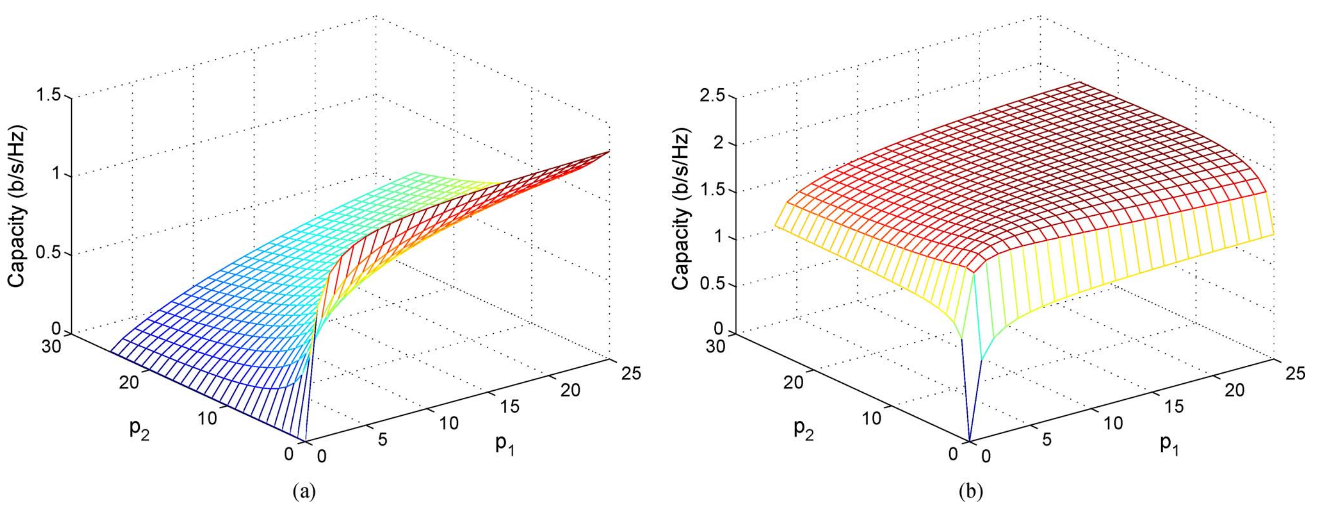

Fig. 2. Example scenario for a nonconcave objective function and its concave upper bound with $K = 1$ and $L = 2$. (a) Original objective. (b) Upper bound.

function. This will give a lower bound on the ESA. We call this as ESA-LB. The optimal value is in between ESA-UB and ESA-LB. The main drawback is the complexity of ESA, which exponentially increases with the number of relays and SUs. In this paper, we also compare our proposed algorithm with the combination of branch-and-bound algorithm with outer approximation (BB-OA). BB-OA is an algorithm that solves nonlinear mixed-integer problems. For outer approximation, with fixed discrete variables, the objective function and constraints should be a continuously differentiable function, and constraint qualification (e.g., Slater's condition) must hold at the solution of every nonlinear programming subproblem resulting from the original nonlinear MINLP [20], [21]. We use the Basic Open-Source Nonlinear Mixed-Integer Programming software for BB-OA.[1] The main drawback of BB-OA is that its worst-case complexity is exponential.

An analytical proof for convergence is generally applicable when an optimal solution is known. Since the JRAPA problem in (1) is a nonconvex MINLP NP-hard problem, an exact optimal solution is unknown. In this paper, we numerically compare the results of the proposed greedy IJRAPA scheme with the ESA and the branch-and-bound-based outer-approximation algorithm. The simulation results show that the proposed greedy IJRAPA is always within 99% of the exhaustive search and BB-OA, as shown in the performance figures.

Fig. 3 shows a plot of sum capacity versus interference threshold with the following parameters: $L = 6, K = 2, M = 16, p_s^{\max} = 1$ W, $p_l^{\max} = 1$ W. The value of $I_m^{\max}$ is in watts. The results show that the performance of IJRAPA is very close to that of the ESAs and BB-OA. Fig. 3 also shows that the sum capacity increases with the interference threshold because the feasible set of the optimization problem with lower interference threshold is a subset of the feasible set of the optimization problem with higher interference threshold. With the increase in the interference threshold, the optimizer has the freedom to allocate more power to the source and the relays, which, in turn, increases the sum-rate capacity.

In Fig. 4, we present a plot of sum capacity versus number of relays with the following parameters: $M = 16, K = 2, I_m^{\max} = 100\ \mu$W, $p_s^{\max} = 1$ W, and $p_l^{\max} = 1$ W. We observe that the sum capacity increases with the number of relays as more relays mean more degrees of freedom in relay assignment. By sending data from more number of relays to any SU, the diversity of the system will increase.

[1]The optimization software is available at http://www.i2c2.aut.ac.nz/Wiki/OPTI/

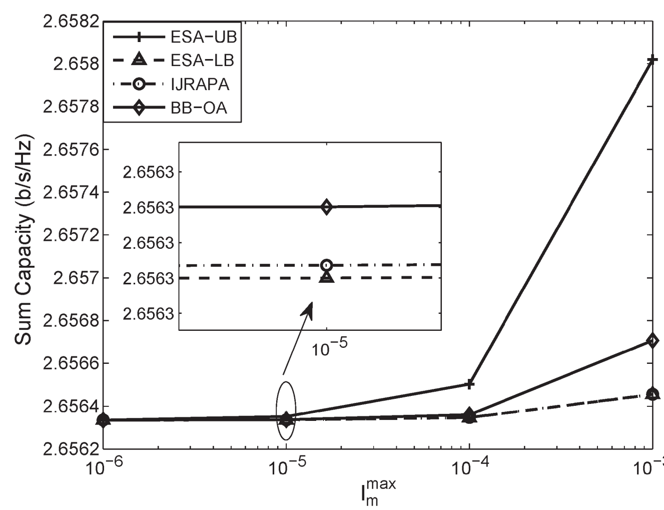

Fig. 3. Sum-rate capacity versus interference threshold (in watts). The parameters are $L = 6, K = 2, M = 16, p_s^{\max} = 1$ W, and $p_l^{\max} = 1$ W.

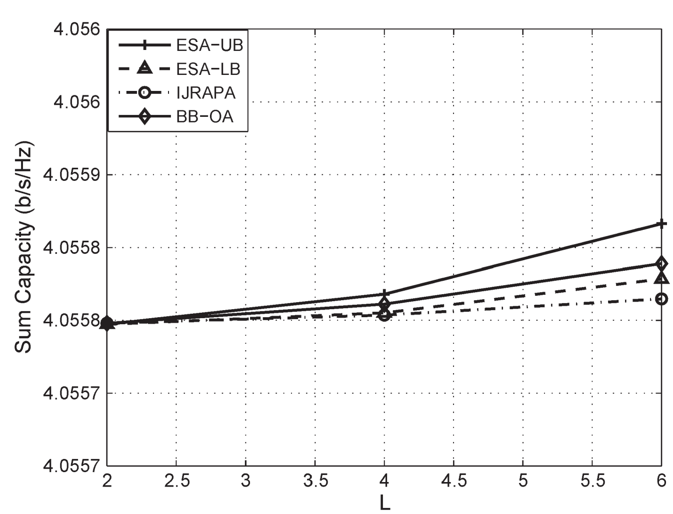

Fig. 4. Sum-rate capacity versus number of relays. The parameters are $M = 16, K = 2, I_m^{\max} = 100\ \mu$W, $p_s^{\max} = 1$ W, and $p_l^{\max} = 1$ W.

Fig. 5 shows the variation in sum capacity with the increase in the number of PUs with the following parameters: $L = 4, K = 2, I_m^{\max} = 1$ mW, $p_s^{\max} = 1$ W, $p_l^{\max} = 1$ W. In this result, we observe that

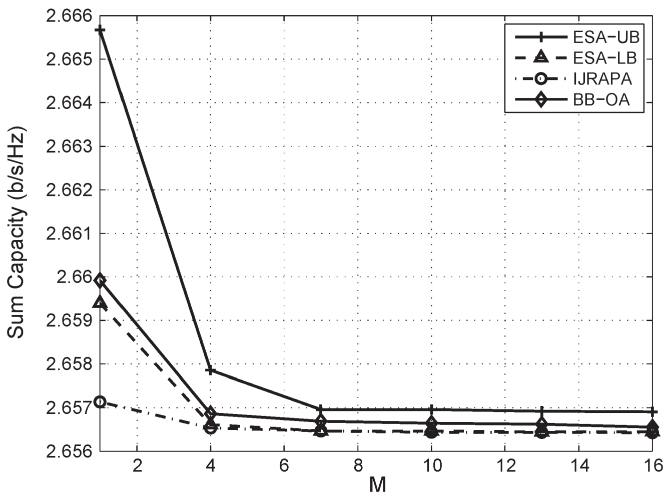

Fig. 5. Sum-rate capacity versus number of PUs. The parameters are $L = 4, K = 2, I_m^{\max} = 1$ mW, $p_s^{\max} = 1$ W, and $p_l^{\max} = 1$ W.

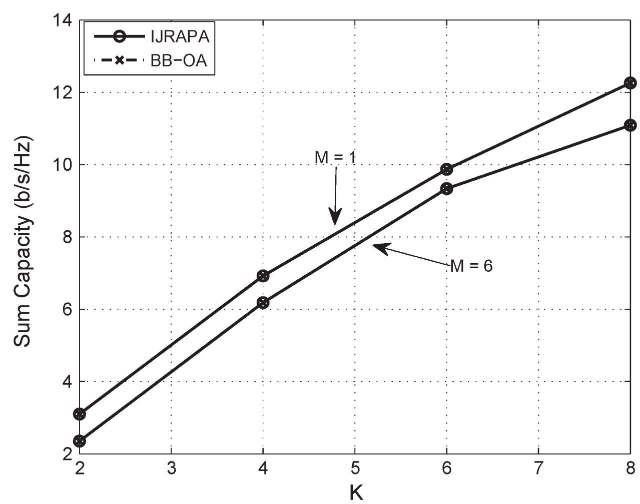

Fig. 6. Sum-rate capacity versus number of SUs. The parameters are $L = 16, I_m^{\max} = 1$ μW, $p_s^{\max} = 1$ W, and $p_l^{\max} = 1$ W.

the sum capacity decreases as the number of PUs increases. This is because relay assignment needs to satisfy more interference constraints as the number of PUs increases. In Fig. 6, we plot sum-rate capacity versus number of SUs. For this scenario, we set $L = 16, M = \{1, 6\}, I_m^{\max} = 1$ μW, $p_s^{\max} = 1$ W, and $p_l^{\max} = 1$ W. Due to the computational load of the ESA, for this result, we only compare IJRAPA with BB-OA. The result shows that the average sum-rate capacity increases with the increase in the number of SUs and decreases with the number of PUs. We can also observe that the sum-rate capacity achieved by the proposed IJRAPA is close to the BB-OA for a wide range of SUs.

V. Conclusion

In this paper, we have presented an IJRAPA algorithm for multiuser CRSs. The proposed iterative algorithm has low computational complexity, and its performance is close to that of the ESA. Its simple underlying concept and ease of implementation with low complexity make this iterative algorithm a suitable candidate for the multiple-relay-assignment and power-allocation problem in CRSs. In the future, we will extend this algorithm to CRSs with imperfect channel gain knowledge at the central controller.

Appendix A
Proof of Nondecreasing Relay Power

By taking $(\partial\Gamma_k/\partial p_j)$ and with some mathematical simplifications, we will get

$$\frac{\partial\Gamma_k}{\partial p_j} = \left(\frac{\sum_l A_l\sqrt{p_l}}{B_l}\right)\left(\frac{A_jB_l}{\sqrt{p_j}} - B_j\sum_l A_l\sqrt{p_l}\right)$$

where $A_l = |h_l^{\text{BS}}h_k^l|^2\beta_l$, and $B_l = 1 + \sum_l \beta_l^2|h_k^l|^2p_l$. We observe that for any fixed set of values of $p_1, \ldots, p_{j-1}, p_{j+1}, \ldots, p_L$, Γ_k is nondecreasing with respect to p_j when $((A_jB_l/\sqrt{p_j}) - B_j\sum_l A_l\sqrt{p_l}) \geq 0$. We can rewrite the inequality as

$$\left|h_j^{\text{BS}}\right|B_l - \sqrt{p_j}\beta_j\left|h_k^j\right|\left(\sum_l A_l\sqrt{p_l}\right) \geq 0$$

which can be further written as

$$\left|h_j^{\text{BS}}\right|\left(1+\sum_{l\neq j}\beta_l^2\left|h_k^l\right|^2p_l\right) \geq \sqrt{p_j}\beta_j\left|h_k^j\right|\left(\sum_{l\neq j}A_l\sqrt{p_l}\right). \quad (5)$$

Finally, after rearranging the terms, we obtain

$$p_j \leq \left(\frac{\left|h_j^{\text{BS}}\right|\left(1+\sum_{l\neq j}\beta_l^2\left|h_k^l\right|^2p_l\right)}{\beta_j\left|h_k^j\right|\left(\sum_{l\neq j}A_l\sqrt{p_l}\right)}\right)^2. \quad (6)$$

$\square$

Appendix B
Proof of Concavity of Upper Bound

We need to establish the concavity of function $f : \mathbb{R}^L \to \mathbb{R}$, which is defined as $f(p_1, \ldots, p_l) = (\sum_l(|h_k^l|^2\beta_l^2p_l)/1 + \sum_l(|h_k^l|^2\beta_l^2p_l)) = (\sum_l x_l/1 + \sum_l x_l)$, where $x_l = |h_k^l|^2\beta_l^2p_l$. From the definition, function f is concave if $\mathbf{dom}\ f$ is a convex set and if for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbf{dom}f$, and λ with $0 \leq \lambda \leq 1$, we have

$$f(\lambda\boldsymbol{x} + (1-\lambda\boldsymbol{y})) \geq \lambda f(\boldsymbol{x}) + (1-\lambda)f(\boldsymbol{y}).$$

Let us define a linear function $g : \mathbb{R}^L \to \mathbb{R}$ as $g(x_1, \ldots, x_L) = \sum_l x_l$ and a concave function $h : \mathbb{R} \to \mathbb{R}$ as $h(z) = (z/1 + z)$. We know that the composition of a concave function with affine mapping is concave, that is, $h(g(\boldsymbol{x}))$ is concave. Therefore

$$h\left(g(\lambda\boldsymbol{x}) + g\left((1-\lambda\boldsymbol{y})\right)\right) \geq \lambda h\left(g(\boldsymbol{x})\right) + (1-\lambda)h\left(g(\boldsymbol{y})\right). \quad (7)$$

Further, we observe that

$$\begin{aligned} h\left(g(\lambda\boldsymbol{x}) + g\left((1-\lambda\boldsymbol{y})\right)\right) &= \frac{\lambda\sum_l x_l + (1-\lambda\sum_l y_l)}{1+\lambda\sum_l x_l + (1-\lambda\sum_l y_l} \\ &= f\left(\lambda\boldsymbol{x} + (1-\lambda\boldsymbol{y})\right) \\ \lambda h\left(g(\boldsymbol{x})\right) + (1-\lambda)h\left(g(\boldsymbol{y})\right) &= \lambda f(\boldsymbol{x}) + (1-\lambda)f(\boldsymbol{y}). \end{aligned}$$

Hence, from (7), we conclude that $f(\lambda\boldsymbol{x} + (1-\lambda\boldsymbol{y})) \geq \lambda f(\boldsymbol{x}) + (1-\lambda)f(\boldsymbol{y})$, which establishes that the function is concave. $\square$

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Analysis of Transmit Antenna Selection With Switch-and-Examine Combining With Postselection at the Receiver Over Rayleigh Fading Channels

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***Abstract*—A diversity-combining system using transmit antenna selection at the transmitter and switch-and-examine combining with postselection (TAS/SECPS) at the receiver is proposed. This system has lower processing complexity as compared with the existing TAS schemes with maximal-ratio combining and selection combining (TAS/SC) at the receiver. The performance of the TAS/SECPS system with different modulation schemes over independent and nonidentically distributed Rayleigh fading channels is examined. The expressions of average output signal-to-noise ratio (SNR) and symbol error rate (SER) are derived and validated by simulation. It is shown that the proposed TAS/SECPS is able to achieve similar average output SNR and SER performance as compared with TAS/SC when the predetermined SNR threshold is optimized. Moreover, the TAS/SECPS requires less number of channel estimates.**

***Index Terms*—Rayleigh fading, switch-and-examine combining (SEC), transmit antenna selection (TAS).**

I. Introduction

Diversity-combining techniques such as maximal-ratio combining (MRC), equal-gain combining, selection combining (SC), and switch combining have been used to mitigate the effects of multipath fading [1], [2]. In switch combining, several variants such as the switch-and-stay combining (SSC), switch-and-examine combining (SEC), SEC with postselection (SECPS), and scan-and-wait combining have been proposed [1]–[6]. Similar to SC, only one branch is selected among the available L branches in switch combining. However, instead of selecting the branch with the largest signal-to-noise ratio (SNR), the selection criterion for switch combining is based on a predetermined SNR threshold [3]–[7]. As compared with SC, switch combining has lower complexity, since it does not require estimation of all the diversity branches. Hence, switch diversity is often used in mobile units due to its simplicity [1].

The transmit antenna selection (TAS) scheme, which selects a subset of transmit antennas for transmission, has been investigated in [8]–[10]. In particular, the single TAS based on the highest channel gain from the available N_t transmit antennas has been studied in [11]–[18]. The single TAS scheme is suitable for downlink communications in cellular radio systems, where only a single radio-frequency chain is required regardless of the number of antennas at the transmitter [11]. Furthermore, it has been shown in [11]–[13] that the TAS is able to outperform the orthogonal space-time block code of the same diversity order using the same number of receive antennas over flat-fading channels. However, it comes with additional complexity of a feedback channel. The TAS scheme with MRC at the receiver (TAS/MRC) was investigated in [11], [12], [14], and [15]. The TAS scheme with SC at the receiver (TAS/SC) was proposed in [16]. The TAS scheme with

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