Distributed Channel Selection in Time-Varying Radio Environment: Interference Mitigation Game With Uncoupled Stochastic Learning

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***Abstract*—This paper investigates the problem of distributed channel selection for interference mitigation in a time-varying radio environment without information exchange. Most existing algorithms, which were originally designed for static channels, are costly and inefficient in the presence of time-varying channels. First, we formulate this problem as a noncooperative game, in which the utility of each player is defined as a function of its experienced expected weighted interference. This game is proven to be an exact potential game with the considered network utility (the expected weighted aggregate interference) serving as the potential function. However, most game-theoretic algorithms are not suitable for the considered network, since they are coupled, i.e., the updating procedure is relying on the actions or payoffs of other players. Then, we propose a simple, completely distributed, and uncoupled stochastic learning algorithm, with which the users learn the desirable channel selections from their individual trial-payoff history. It is analytically shown that the proposed algorithm converges to pure strategy Nash equilibrium in time-varying radio environment; moreover, it achieves optimal channel selection profiles and makes the network interference-free for underloaded or equally loaded scenarios, while achieving, on average, near-optimal performance for overloaded scenarios.**

***Index Terms*—Canonical network, distributed orthogonal channel selection, exact potential game, interference mitigation, uncoupled stochastic learning.**

I. Introduction

EFFICIENT interference mitigation is key to improving the performance of wireless communication networks [1]–[7]. Currently, most existing works, for example [8]–[17], have assumed that the interference channel gains are static. Based on this assumption, several nongame-theoretic [8], [9] and game-theoretic [10]–[17] interference mitigation algorithms have been proposed in the literature. The assumption of static channel leads to mathematical tractability but is not true since wireless channels are always time varying in practice. To track the channel variations, an instinctive method is to reiterate the algorithms in each quasi-static block. This method, however, is costly and inefficient, particularly for fast-varying channels. Thus, it is important to develop new algorithms for interference mitigation in the presence of time-varying channels.

In this paper, we consider a multiuser multichannel distributed network, where the users choose orthogonal channels to mitigate mutual interference [9], [12], [13], [15]–[17]. Specifically, the considered network has the following three characteristics: 1) there is no centralized controller, 2) there is no information exchange among users, and 3) the channels undergo block fading. The reason for emphasizing no information exchange here is that information exchange in distributed networks always leads to unsustainable communication overhead and is not even feasible in some cases, e.g., in the presence of moving obstacles [18]. Furthermore, block fading means that the channel gains remain stationary in a slot and change randomly and independently in the next slot. This kind of time-varying channel model is realistic and has been extensively used in past literature.

Following similar ideas proposed in [9], [14], and [17], wherein the authors considered minimizing the weighted aggregate interference for static channels, the network utility in this paper is naturally extended to the *expected weighted aggregate interference* for block-faded channels. As a result, the optimization objective is to achieve channel selection profiles that minimize this network utility. It is seen that the channel selections are made by the users distributively and autonomously; moreover, the users have conflicting objectives, i.e., every user selfishly minimizes its experienced interference. This motivates us to formulate the problem of channel selection for interference mitigation as a noncooperative game [19].

Game model, as a powerful tool, can reveal the underlying structures of multiuser decision problems, e.g., the existence of Nash equilibrium (NE), and the performance gap between NE and the global optimum. However, game-theoretic solutions for the interference mitigation problem cannot be straightforwardly obtained, since most existing game-theoretic algorithms, e.g.,

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best response dynamic [20], no-regret learning [12], fictitious play [21], and spatial adaptive play [22], require the environment to be static. More importantly, these algorithms are *coupled*, which means that the updating procedure is relying on the actions or payoffs of other players. Notably, these coupled algorithms are not suitable for the considered network, and hence, we need to develop *uncoupled* [23] algorithms in time-varying environment, which is a challenging task.

To cope with the given difficulty, we resort to learning technologies, which are characterized by having the ability to observe the response from the environment, adjust the decision strategy, and, finally, learn desirable solutions from historical information. Specifically, we incorporate stochastic learning automata [30] into a game model to solve the interference mitigation problem. The main contributions of this paper are summarized as follows.

1) We formulate an interference mitigation game, which is further proved to be an exact potential game with the considered network utility as its potential function. The formulated game exhibits several promising features. Most importantly, it is proved that all pure strategy NE points lead to interference-free channel selection profiles for underloaded or equally loaded networks, and there exists at least one pure strategy NE point that minimizes the expected weighted aggregate interference for overloaded networks.
2) We propose an uncoupled stochastic learning algorithm, which converges to pure strategy NE points of the interference mitigation game in time-varying environment. It is *simple*, *completely distributed*, and *uncoupled*. Specifically, 1) the updating rule is linear; moreover, it requires no prior information of the system, e.g., the distance among nodes, the characteristics of channels, and the number of nodes; 2) it does not need central control and information exchange; and 3) the updating procedure is only relying on the individual trial-payoff history of each user; in fact, each user is even not aware of other users.
3) Simulation-based performance evaluation shows that the proposed learning solution averagely achieves near-optimal performance.

The rest of this paper is organized as follows: In Section II, we review the related work. In Section III, we present the system model and problem formulation. In Section IV, we formulate the interference mitigation game, prove it to be an exact potential game, and investigate its properties. In Section V, we propose a stochastic learning algorithm to converge toward pure strategy NE points in fading environment. In Section VI, simulation results and discussion are presented. Finally, we provide conclusions in Section VII.

II. Related Work

Distributed interference mitigation is a timely research topic with enormous number of wireless devices currently in use. Most existing work focused on the problem of power control and/or waveform adaptation for interference mitigation, e.g., [1], [8], and [14]. Our work is differentiated from these studies in that we consider orthogonal channel selection for mitigating interference, which is suitable for future broadband wireless networks, e.g., orthogonal frequency-division multiple-access systems.

It should be mentioned that a game-theoretic interference avoidance algorithm for dynamic networks was proposed in [14]. The dynamics considered in this reference is triggered by some events, e.g., a user becoming in-active, or *vice versa*. The network then remains static for a while after a triggered event. The considered dynamics is different since the channels are always time varying. In addition, there also exists some work using potential games to solve the problem of distributed channel selection for interference mitigation [12], [13], [15]–[17]. These studies, however, only considered static channels.

There are some studies for orthogonal resource selection (one can see that channels naturally belong to orthogonal resources) in the literature. Specifically, in [10] and [11], the problem of orthogonal sequence selection for interference mitigation is investigated, and the interference mitigation is formulated as potential games. Although the formulation proposed in [10] and [11] can be extended to the channel selection problem, there are two key differences in our work: 1) The channel gains considered in the previous two studies are static, whereas we consider time-varying (block-fading) channels, and 2) the algorithms proposed in them are coupled, whereas our proposed algorithm is uncoupled. In addition, game-theoretic approaches for channel allocation (selection) in wireless networks have been extensively investigated in the literature [24]–[28]. Our work is differentiated from these works in that we consider the distributed channel selection problem in time-varying radio environment. In our recent work [29], some preliminary results for the channel selection problem in time-varying environment were reported.

It is known that learning is the core of cognitive radios [31] and draws much attention. Some learning algorithms can be found in the literature, e.g., reinforcement learning for interference control for wireless regional area networks [32], opportunistic bandwidth sharing [33], and channel access in cognitive radio networks [34]–[36]. (For a comprehensive review on learning technologies in cognitive radios, see [37] and references therein.) In addition, a survey for decision-theoretic approaches for the channel selection problem in cognitive radio networks can be found in our recent work [38].

Recently, there is a new research direction: incorporating learning technologies into game theory [39]–[42]. This topic is very interesting and important because game theory characterizes interactions among multiple users while learning technologies, particularly uncoupled learning algorithms, address the constraints of lacking information exchange and dynamic environment. Some previous studies addressing this topic can be found in the literature, e.g., multiagent Q-learning channel selection algorithms for two-user two-channel cognitive radio systems [35] and for multiuser multichannel cognitive radio systems [36], stochastic learning solution for distributed discrete power control [43], and stochastic learning algorithms for opportunistic spectrum access [18], [44]. The challenging task here is to investigate the convergence of the learning algorithms when incorporating into game theory, which greatly differs in different scenarios.

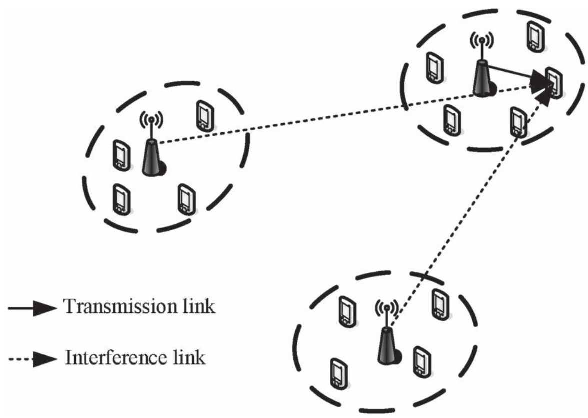

Fig. 1. Example of canonical networks.

III. System Model and Problem Formulation

A. System Model

We consider a distributed canonical network consisting of multiple autonomous nodes. It should be pointed out that each node in canonical networks is not a single communication entity; instead, it is a community of multiple entities with intranode communications [46]–[48]. The entities in a community are closely located, and there is a leading entity that manages the whole community. Generally, the leading entity chooses the operational channel, and the followers share the channel using some multiple-access control mechanisms. Examples of nodes in canonical network are given by, e.g., a wireless local area network access point with its serving clients [47] and a cluster head together with its members [9]. An example of the considered canonical networks is shown in Fig. 1. (Furthermore, see [9] for a comprehensive review on canonical networks.) In this paper, we will use node and user interchangeably. Suppose that there are N nodes autonomously competing for one of M channels. Denote the node set as $\mathcal{N} = \{1, \ldots, N\}$ and the channel set as $\mathcal{M} = \{1, \ldots, M\}$. In addition, Table I summarizes the used notations in this paper.

We assume that all the channels undergo block fading, i.e., the channel gains are block-fixed in a time slot and randomly change in the next slot. Furthermore, it is assumed that each node chooses exactly one channel for intranode communication at a time. When two nodes, for example, m and n, choose the same channel, mutual interference emerges; specifically, the instantaneous interference gain from nodes m to n in a slot can be expressed as

$$w_{mn}^s = (d_{mn})^{-\alpha} \varepsilon_{mn}^s \tag{1}$$

where superscript s is the selected channel, d_{mn} is the distance between nodes m and n, α is the path loss exponent, and ε_{mn}^s is the instantaneous random component of the path loss [49], e.g., Rayleigh fading. The instantaneous random components between two nodes in each slot may be the same or different. Their expected values, however, are assumed to be the same, which implies that we can denote the expected value of the random components between two nodes on a channel as $\bar{\varepsilon}_{mn}^s = \mathbf{E}[\varepsilon_{mn}^s] = \mathbf{E}[\varepsilon_{nm}^s], \forall m, n \in \mathcal{N}, \forall s \in \mathcal{M}$.

TABLE I
Summation of Used Notations

Notations	Description
$\mathcal{N}$	set of nodes
$\mathcal{M}$	set of available channels
d_{mn}	distance between nodes m and n
α	path loss exponent
ε_{mn}^s	instantaneous random component of the path loss between nodes m and n in channel s
$\bar{\varepsilon}_{mn}^s$	expected value of ε_{mn}^s
p_n	transmitting power of node n
N_0	noise power spectrum density
B	bandwidth of each channel
I_n	interference experienced by node n
U	expected weighted aggregate interference
$\mathcal{A}_n \equiv \mathcal{M}$	set of available actions (channels) of node (player) n
$a_n \in \mathcal{A}_n$	an action of player n
a_{-n}	an action profile of all the players except player n
u_n	utility function of player n
$S_n(a_n)$	node set excluding n which also chooses action a_n
$\Phi(a_n, a_{-n})$	potential function of the game
$q_n(k)$	a mixed strategy of player n at the kth iteration
$Q(k)$	a mixed strategy profile of all players at the kth iteration
$r_n(k)$	received payoff of player n at the kth iteration
$\tilde{r}_n(k)$	normalized received payoff of player n at the kth iteration
b	learning step size
$\hat{I}_n$	experienced interference by node n in a static network
$\hat{U}$	weighted aggregate interference in a static network
$\hat{u}_n$	utility function of player n in a static channel selection game
D	a pre-defined positive constant used in the utility function
C	a positive constant used in the proof of the algorithm

Remark 1: The above mutual interference channel model is very general and realistic, as the instantaneous random components, i.e., ε_{mn}^s, can vary from slot to slot, from channel to channel, and from user to user. More importantly, such variations may be *independent* or *correlated*. Furthermore, the expected value of random component, i.e., $\bar{\varepsilon}_{mn}^s$, can vary from channel to channel and from user to user. This makes the results obtained in this paper suitable for several scenarios. For instance, the instantaneous random component may be unit-constant, i.e., $\varepsilon_{mn}^s = 1, \forall m, n, s$, which corresponds to a no-fading scenario where only large-scale power loss is considered. In addition, it may be with lognormal distribution, which corresponds to the medium-scale power loss, and Rayleigh/Nakagami distribution, which corresponds to multiple-path power loss.

B. Problem Formulation

Denote $a_n \in \mathcal{M}$ as the selected channel of node n in a slot, then the instantaneous rate of node n is given as follows:

$$R_n = B \log\left(1 + \frac{p_n w_{nn}^{a_n}}{BN_0 + I_n}\right) \tag{2}$$

where $w_{nn}^{a_n} = (d_{nn})^{-\alpha} \varepsilon_{nn}^{a_n}$ is the intracommunication channel gain of node n, p_n is the transmitting power of n, N_0 is the noise power spectrum density, B is the bandwidth, and I_n is the interference experienced by user n. Given the network action profile being $a = \{a_1, \ldots, a_N\}$, I_n is a random variable and is given by

$$I_n = \sum_{m \in \{\mathcal{N} \backslash \{n\}\}} f(a_m, a_n) p_m w_{mn}^{a_n} \tag{3}$$

where $X\backslash Y$ means that Y is excluded from X, and $f(\cdot)$ is the Kronecker delta function defined as

$$f(x,y)=\begin{cases}1, & x=y\\ 0, & x\neq y.\end{cases} \tag{4}$$

Based on (2), the expected network rate can then be given by

$$R^{\text{sum}}=\sum_{n\in\mathcal{N}}\mathbf{E}[R_n]. \tag{5}$$

From the perspective of interference mitigation, the considered network utility in this paper is the expected weighted aggregate interference defined as

$$U=\sum_{n\in\mathcal{N}}p_n\mathbf{E}[I_n]=\sum_{n\in\mathcal{N}}\sum_{m\in\{\mathcal{N}\backslash\{n\}\}}p_m p_n \bar{w}_{mn}^{a_n} f(a_m,a_n) \tag{6}$$

where

$$\bar{w}_{mn}^{a_n}=\mathbf{E}\left[w_{mn}^{a_n}\right]=(d_{mn})^{-\alpha}\bar{\varepsilon}_{mn}^{a_n} \tag{7}$$

denotes the expected interference gain from node m to node n in channel a_n.

Note that some previous studies [9], [14], [17], [50] also considered a similar network utility metric, i.e., the weighted aggregate interference. It was shown in [14] that using such a network utility can balance the transmitting power and the experienced interference. Furthermore, it has been shown that it leads to a near-optimal network sum rate in the low-signal-to-interference-plus-noise-ratio regime [9]. The difference in this paper is that the expected version is considered to deal with the random instantaneous fading components ε_{mn}^s.

Motivated by the previous work addressing interference mitigation rather than throughput maximization, for example, [9], [12], and [13], we focus on minimizing the expected weighted aggregate interference specified by (6) and do not consider maximizing the network sum rate specified by (5) in this paper. Specifically, our goal is to find the optimal selection profile such that the network utility specified by (6) is minimized, i.e.,

$$(P1:)\quad a_{opt}\in\arg_a\min U. \tag{8}$$

It is seen that $P1$ is a combinatorial optimization problem, which can be solved in a centralized manner, on the condition that all the system parameters, i.e., p_n, d_{mn}, and $\bar{\varepsilon}_{mn}^s$, $\forall m,n\in\mathcal{N}$, $s\in\mathcal{M}$, are *a priori* known. However, in a case that there is no centralized control and these parameters are unknown, which is exactly the one considered in this paper, solving $P1$ is challenging. Thus, we need to find a distributed solution that is able to cope with the lack of centralized control, the constraint of not knowing system parameters, and the random nature of interference channels.

IV. Interference Mitigation Game

A. Game Model

We formulate the problem of distributed channel selection for interference mitigation as a noncooperative game. Formally, the interference mitigation game is denoted as $\mathcal{G}_c=[\mathcal{N},\{\mathcal{A}_n\}_{n\in\mathcal{N}},\{u_n\}_{n\in\mathcal{N}}]$, where $\mathcal{N}=\{1,\ldots,N\}$ is the set of players (nodes), $\mathcal{A}_n=\{1,\ldots,M\}$ is the set of available actions (channels) for each player n, and u_n is the utility function of player n. Notably, the experienced interference is a random variable in a slot. That is, the players receive random payoffs in each play. We then consider the following utility function, which is defined as the expected experienced interference of each node, i.e.,

$$\begin{aligned}u_n(a_n,a_{-n})&=D-p_n\mathbf{E}[I_n]\\&=D-\sum_{m\in\{\mathcal{N}\backslash\{n\}\}}p_n p_m \bar{w}_{mn}^{a_n} f(a_m,a_n)\end{aligned} \tag{9}$$

where a_{-n} is the channel selection profile of all the players except player n, I_n is the experienced interference specified by (3), and D is a predefined positive constant, which will be illustrated later. Then, the proposed interference mitigation game can be expressed as

$$(\mathcal{G}_c):\quad \max_{a_n\in A_n} u_n(a_n,a_{-n}),\quad \forall n\in\mathcal{N}. \tag{10}$$

B. Analysis of NE

In the following, we define the NE of the formulated interference mitigation game and investigate its properties.

Definition 1 (NE): A channel selection profile $a^*=(a_1^*,\ldots,a_N^*)$ is a pure strategy NE if and only if no player can improve its utility by deviating unilaterally, i.e.,

$$u_n\left(a_n^*,a_{-n}^*\right)\geq u_n\left(a_n,a_{-n}^*\right),\ \forall n\in\mathcal{N},\forall a_n\in\mathcal{A}_n, a_n\neq a_n^*. \tag{11}$$

The given definition is straightforwardly obtained from game theory [19]. The properties of the proposed interference mitigation game are characterized by the following theorem.

Theorem 1: $\mathcal{G}_c$ is an exact potential game that has at least a pure strategy NE point, and the optimal channel selection that globally minimizes the expected weighted aggregate interference constitutes a pure strategy NE point of $\mathcal{G}_c$.

Proof: To prove this theorem, we first construct a potential function as follows:

$$\Phi(a_n,a_{-n})=-\frac{1}{2}\sum_{n\in\mathcal{N}}\sum_{m\in\{\mathcal{N}\backslash\{n\}\}}p_m p_n \bar{w}_{mn}^{a_n} f(a_m,a_n) \tag{12}$$

which immediately yields the following equation:

$$\Phi(a_n,a_{-n})=-\frac{1}{2}U(a_n,a_{-n}) \tag{13}$$

where $U(a_n,a_{-n})$ is the network utility specified by (6). Denote the set of channel selection profile for all the players by $\mathcal{A}=\mathcal{A}_1\times\cdots\times\mathcal{A}_N$, we then have

$$a_{opt}\in\arg\max_{a\in\mathcal{A}}\ \Phi(a) \tag{14}$$

where a_{opt}, as specified by (8), is the optimal selection profile that minimizes the expected weighted aggregate interference.

For presentation, denote $S_n(a_n)$ as the node set excluding n, which also chooses a_n, i.e., $S_n(a_n) = \{m \in \{\mathcal{N}\backslash\{n\}\} : a_m = a_n\}$. Then, we have

$$f(a_m, a_n) = \begin{cases} 1, & \forall m \in S_n(a_n) \\ 0, & \forall m \notin S_n(a_n) \end{cases} \tag{15}$$

which can be directly derived from (4).

If an arbitrary player n unilaterally changes its selection from a_n to $\tilde{a}_n$, the change in its utility function is given by

$$\begin{aligned} u_n(\tilde{a}_n, a_{-n}) &- u_n(a_n, a_{-n}) \\ &= -\sum_{m\in\{\mathcal{N}\backslash n\}} p_n p_m \left(\bar{w}_{mn}^{\tilde{a}_n} f(a_m, \tilde{a}_n) - \bar{w}_{mn}^{a_n} f(a_m, a_n)\right) \\ &= -\sum_{m\in S_n(\tilde{a}_n)} p_m p_n \bar{w}_{mn}^{\tilde{a}_n} + \sum_{m\in S_n(a_n)} p_m p_n \bar{w}_{mn}^{a_n}. \end{aligned} \tag{16}$$

Then, the change in the potential function caused by the unilateral change in n's selection is given by

$$\begin{aligned} \Phi(\tilde{a}_n, a_{-n}) &- \Phi(a_n, a_{-n}) \\ &= \frac{1}{2}\left(-\sum_{m\in S_n(\tilde{a}_n)} p_m p_n \bar{w}_{mn}^{\tilde{a}_n} + \sum_{m\in S_n(a_n)} p_m p_n \bar{w}_{mn}^{a_n}\right) \\ &\quad - \frac{1}{2}\sum_{m\in S_n(a_n)} \left(p_n p_m \left(\bar{w}_{nm}^{\tilde{a}_n} f(\tilde{a}_n, a_m) - \bar{w}_{nm}^{a_n} f(a_n, a_m)\right)\right) \\ &\quad - \frac{1}{2}\sum_{m\in S_n(\tilde{a}_n)} \left(p_n p_m \left(\bar{w}_{nm}^{\tilde{a}_n} f(\tilde{a}_n, a_m) - \bar{w}_{nm}^{a_n} f(a_n, a_m)\right)\right) \\ &\quad - \frac{1}{2}\sum_{m\in \mathcal{N}_0} \left(p_n p_m \bar{w}_{nm} \left(\bar{w}_{nm}^{\tilde{a}_n} f(\tilde{a}_n, a_m) - \bar{w}_{nm}^{a_n} f(a_n, a_m)\right)\right) \end{aligned} \tag{17}$$

where $\mathcal{N}_0 = \mathcal{N}\backslash\{S_n(a_n) \cup S_n(\tilde{a}_n) \cup \{n\}\}$. By applying (15) into (17), we have

$$\begin{aligned} \Phi(\tilde{a}_n, a_{-n}) &- \Phi(a_n, a_{-n}) \\ &= \frac{1}{2}\left(-\sum_{m\in S_n(\tilde{a}_n)} p_m p_n \bar{w}_{mn}^{\tilde{a}_n} + \sum_{m\in S_n(a_n)} p_m p_n \bar{w}_{mn}^{a_n}\right) \\ &\quad + \frac{1}{2}\left(-\sum_{m\in S_n(\tilde{a}_n)} p_m p_n \bar{w}_{nm}^{\tilde{a}_n} + \sum_{m\in S_n(a_n)} p_m p_n \bar{w}_{nm}^{a_n}\right). \end{aligned} \tag{18}$$

Now, applying $\bar{w}_{nm}^s = \bar{w}_{mn}^s = (d_{mn})^{-\alpha}\bar{\varepsilon}_{mn}^s$, $\forall n, m \in \mathcal{N}$, $\forall s \in \mathcal{M}$, into (16) and (18), respectively, we have

$$\Phi(\tilde{a}_n, a_{-n}) - \Phi(a_n, a_{-n}) = u_n(\tilde{a}_n, a_{-n}) - u_n(a_n, a_{-n}) \tag{19}$$

which means that the change in the individual utility function caused by any player's unilateral deviation is the same as the change in the potential function. Thus, according to the definition given in [20], it is known that $\mathcal{G}_c$ is an exact potential game with Φ serving as the potential function.

Exact potential game is a special kind of potential games, which exhibits several attractive properties, and the following is the most important one: *any global or local maxima of the potential function constitutes a pure strategy NE point of the game* [20]. Thus, according to the connection between the formulated potential function and network utility specified by (13) and (14), it is known that the optimal channel selection profile that minimizes the expected weighted aggregate interference constitutes a pure strategy NE point of $\mathcal{G}_c$. Therefore, Theorem 1 is proved. ■

Theorem 1 generally characterizes the relationship between the formulated interference mitigation game $\mathcal{G}_c$ and the considered network utility. We first classify the network into three scenarios [11]: 1) underloaded scenario, wherein the number of nodes is less than the number of channels, i.e., $N < M$; 2) equally loaded scenario, wherein the number of nodes is equal to the number of channels, i.e., $N = M$; and 3) overloaded scenario, wherein the number of nodes is greater than the number of channels, i.e., $N > M$. The following propositions show the properties for the three scenarios, respectively.

Proposition 1: In underloaded or equally loaded scenarios, all pure strategy NE points of $\mathcal{G}_c$ lead to interference-free channel selection profiles.

Proof: It can be straightforwardly shown that in the two scenarios, all pure strategy NE points correspond to orthogonal channel selection profiles, i.e., no more than one node chooses a channel. This argument can be obtained by the fact that no node has incentive to deviate, as it experiences zero interference from other nodes. Thus, all pure strategy NE points are optimal solutions of $P1$ and make the network interference-free. As a result, Proposition 1 holds. ■

Proposition 2: In an overloaded scenario, there exists at least one pure strategy NE point that minimizes the expected weighted aggregate interference.

Proof: There may exist multiple pure strategy NE points in an overloaded scenario, but this number is hard to obtain. However, according to Theorem 1, there must be at least one pure strategy NE that minimizes the expected weighted aggregate interference. In addition to the optimal one, other pure strategy NE points only locally minimize the expected weighted aggregate interference. ■

The achievable expected aggregate interference at a pure strategy NE $a^* = (a_1^*, \ldots, a_N^*)$ is denoted by

$$U_{NE} = \sum_{n\in\mathcal{N}} p_n \mathbf{E}[I_n] = \sum_{n\in\mathcal{N}} \sum_{m\in\{\mathcal{N}\backslash\{n\}\}} p_m p_n \bar{w}_{mn}^{a_n^*} f\left(a_m^*, a_n^*\right). \tag{20}$$

Since optimality is not guaranteed in the overloaded scenario, it is important to study the performance of NE solutions in this scenario. Generally, price of anarchy (PoA) [45] is used to characterize the performance ratio between the worst NE and the social optimum. However, we note that it is hard to derive PoA for the formulated game. Instead, we get an upper bound for the NE solutions in the overloaded scenario, which is shown in the following proposition.

Proposition 3: If the values of expected random components of all channels are the same, i.e., $\bar{\varepsilon}_{mn}^{s} = \bar{\varepsilon}_{mn}^{0}$, $\forall m,\ n \in \mathcal{N}$, then the expected aggregate interference of NE solutions in an overloaded scenario is upper bounded by $U_{NE} \leq U_0/M$, where M is the number of available channels, and

$$U_0 = \sum_{n\in\mathcal{N}} \sum_{m\in\{\mathcal{N}\backslash\{n\}\}} p_n p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{0}$$

is the expected aggregate interference if all players choose the same channel.

Proof: According to the definition of pure strategy NE, the following equation always holds:

$$\mathbf{E}\left[I_n\left(a_n^*, a_{-n}^*\right)\right] \leq \mathbf{E}\left[I_n\left(a_n, a_{-n}^*\right)\right] \quad \forall n\in\mathcal{N}, \forall a_n \in \mathcal{A}_n, a_n \neq a_n^*. \tag{21}$$

Summarizing the right side of the given equation over $\mathcal{A}_n$, we have

$$M\mathbf{E}\left[I_n\left(a_n^*, a_{-n}^*\right)\right] \leq \sum_{a_n\in\mathcal{A}_n} \mathbf{E}\left[I_n\left(a_n, a_{-n}^*\right)\right], \forall n \in \mathcal{N} \tag{22}$$

where M is the number of available channels. Interestingly, $\sum_{a_n\in\mathcal{A}_n} \mathbf{E}[I_n(a_n, a_{-n}^*)]$ can be regarded as the experienced interference of player n if all other players keep their channel selections unchanged while player n would choose all channels simultaneously, which implies the following equation:

$$\begin{aligned}
&\sum_{a_n\in\mathcal{A}_n} \mathbf{E}\left[I_n\left(a_n, a_{-n}^*\right)\right] \\
&= \sum_{a_n\in\mathcal{A}_n} \left(\sum_{m\in\{\mathcal{N}\backslash\{n\}\}} p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{a_n} f\left(a_m^*, a_n\right)\right) \\
&= \sum_{m\in\{\mathcal{N}\backslash\{n\}\}} \left(\sum_{a_n\in\mathcal{A}_n} p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{a_n} f\left(a_m^*, a_n\right)\right) \\
&= \sum_{m\in\{\mathcal{N}\backslash\{n\}\}} \left(\sum_{a_n\in\mathcal{A}_n} p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{0} f\left(a_m^*, a_n\right)\right) \\
&= \sum_{m\in\{\mathcal{N}\backslash\{n\}\}} p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{0} \left(\sum_{a_n\in\mathcal{A}_n} f\left(a_m^*, a_n\right)\right).
\end{aligned} \tag{23}$$

The following equation can be easily verified:

$$\sum_{a_n\in\mathcal{A}_n} f\left(a_m^*, a_n\right) = 1, \quad \forall m \in \{\mathcal{N}\backslash\{n\}\} \tag{24}$$

which means that (23) can be simplified as

$$\sum_{a_n\in\mathcal{A}_n} \mathbf{E}\left[I_n\left(a_n, a_{-n}^*\right)\right] = \sum_{m\in\{\mathcal{N}\backslash\{n\}\}} p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{0}. \tag{25}$$

Applying (25) into (22) yields the following:

$$\mathbf{E}\left[I_n\left(a_n^*, a_{-n}^*\right)\right] \leq \frac{\sum_{m\in\{\mathcal{N}\backslash\{n\}\}} p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{0}}{M}. \tag{26}$$

Thus, the expected aggregate interference of any NE solution is upper bounded by

$$U_{NE} \leq \frac{\sum_{n\in\mathcal{N}} \sum_{m\in\{\mathcal{N}\backslash\{n\}\}} p_n p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{0}}{M}. \tag{27}$$

Mathematically, $\sum_{n\in\mathcal{N}} \sum_{m\in\{\mathcal{N}\backslash\{n\}\}} p_n p_m (d_{mn})^{-\alpha} \bar{\varepsilon}_{mn}^{0}$ can be regarded the expected aggregate interference when all players choose the same channel since interference exists between any two players. Therefore, Proposition 3 is proved. ■

For an arbitrary channel selection profile, it is seen that U_0 is the worst-case expected aggregate interference of the network. According to Proposition 3, an interesting result is that increasing the number of channels, i.e., M, would decrease the aggregate interference in the network.

Remark 2: It is seen that different network scenarios exhibit different properties. Specifically, all NE points in underloaded or equally loaded scenarios are optimal solutions and make the network interference-free. For an overloaded scenario, however, there may also exist some suboptimal NE points except the optimal one; moreover, the expected weighted aggregate interference for overloaded scenarios is greater than zero.

V. Achieving Nash Equilibrium Using Uncoupled Stochastic Learning

According to the given analysis, if there exists a distributed algorithm that achieves pure strategy NE points of the formulated interference migration game, desirable solutions for problem $P1$ can be obtained. Thus, we focus on developing such an algorithm in the following.

It is known that potential game enjoys good convergence property. Specifically, a large number of learning algorithms in the literature can converge to pure strategy NE points of potential games, e.g., best response dynamic [20], no-regret learning [12], fictitious play [21], and spatial adaptive play [22]. Although these algorithms are distributively implemented, there are two strict constraints: 1) They are *coupled*, i.e., the updating procedures are relying on information of other players in terms of chosen actions and/or received payoffs, and 2) it requires the environment to be static.

It is seen that the given learning algorithms are not suitable for the considered network in our work, because 1) obtaining information of other players is not feasible, and 2) the interference channel gains randomly vary from slot to slot. We then propose a simple, completely distributed, and uncoupled stochastic learning algorithm. With the proposed algorithm, the nodes learn the desirable channel selections from their individual action-payoff experiences and finally adjust their behaviors toward a pure strategy NE point.

To characterize the stochastic learning algorithm, we extend the game to a mixed strategy form. Let the mixed strategy for player n at iteration k be denoted by probability distribution $q_n(k) \in \Delta(\mathcal{A}_n)$, where $\Delta(\mathcal{A}_n)$ denotes the set of probability distributions over the available action set $\mathcal{A}_n$. In the stochastic learning algorithm, the game is played only once in a slot according to the mixed strategies. After each play, each player receives a random payoff; then, the players update their mixed

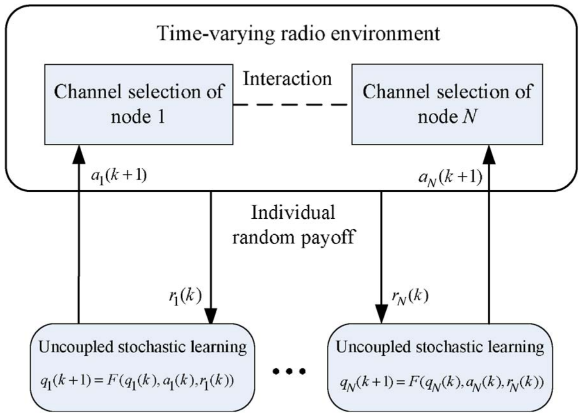

Fig. 2. Diagram of the proposed uncoupled stochastic learning algorithm.

strategy based on the received payoffs. The update rule is simple. Specifically, if an action is selected and a positive payoff is received, then the probability of choosing this channel in the next iteration increases. An illustrative diagram of the proposed uncoupled stochastic learning algorithm is shown in Fig. 2.[1] We first describe the received payoff in each slot in the following.

1) Received Random Payoff: Suppose that at the kth slot, the channel selection profile is given by $a(k) = \{a_1(k), \dots, a_N(k)\}$. Then, player n receives the following random payoff:

$$r_n(k) = D - \sum_{m \in \{\mathcal{N} \backslash \{n\}\}} p_m p_n (d_{mn})^{-\alpha} \varepsilon_{mn}^{a_n(k)} f\left(a_m(k), a_n(k)\right) \tag{28}$$

where $f(\cdot)$ is the Kronecker delta function specified by (4), and $\varepsilon_{mn}^{a_n(k)}$ is the instantaneous channel gain, which are randomly generated according to a specific fading model, e.g., Rayleigh fading. Note that the above payoff is random but can be directly measured by node n [9].

The purpose of the predefined positive constant D is to keep the received payoff positive. Even so, the received payoff may be negative due to the nature of random channel fading. Thus, the following modified received payoff is used instead:

$$r_n(k) = \max\left\{r_n(k), 0\right\}. \tag{29}$$

2) Uncoupled Stochastic Learning Algorithm for Converging Toward NE: The proposed stochastic learning solution is described in Algorithm 1, and the schematic can be described in Fig. 3. It is seen that the proposed learning algorithm is online, as the users learn the channel selections from their action-payoff history. That is, the users simultaneously transmit packets and learn desirable selections. Note that the stop criterion can be one of the following [44]: 1) the maximum iteration number is reached, 2) the variation of the network utility during a period is trivial, or 3) for each player n, $\forall n \in \mathcal{N}$, there is a component of the channel selection probability sufficiently approaching 1, for example, 0.99.

Algorithm 1: the uncoupled stochastic learning algorithm

Initialization: set $k = 0$ and set the initial mixed strategy of each node to $q_{ns}(k) = 1/|\mathcal{A}_n|$, $\forall n \in \mathcal{N}$, $\forall s \in \mathcal{M}$.
Loop for $k = 0, 1, 2, \dots,$

1. **Selecting channels stochastically:** In the kth slot, node n stochastically selects channel $a_n(k)$ according to its current channel selection probability vector $q_n(k)$.
2. **Measuring received payoffs:** The game is played once with the channel selection profile being $\{a_1(k), \dots, a_N(k)\}$, and then, the nodes measure the received payoffs $r_n(k)$ using (29).
3. **Updating mixed strategy:** All the nodes update their mixed strategies according to the following rules:

$$\begin{aligned} q_{ns}(k+1) &= q_{ns}(k) + b\tilde{r}_n(k)\left(1 - q_{ns}(k)\right), \quad & s = a_n(k) \\ q_{ns}(k+1) &= q_{ns}(k) - b\tilde{r}_n(k) q_{ns}(k), \quad & s \neq a_n(k) \end{aligned} \tag{30}$$

where $0 < b < 1$ is the learning step size, $\hat{r}_n(k)$ is the normalized received payoff defined as follows:

$$\tilde{r}_n(k) = r_n(k)/D. \tag{31}$$

End loop

It is seen that the proposed stochastic learning algorithm is suitable for the considered network, as it is simple and completely distributed. Specifically, 1) the updating rule, as specified by (30), is linear with the received payoff $\tilde{r}_n(k)$; moreover, it does not require any prior information of the system, e.g., the distance among nodes, the characteristics of channels, and the number of nodes, and 2) it does not need information exchange and central control.

The proposed algorithm is also called linear reward-inaction (L_{R-I}), which is a special case of linear learning automata [30]. The updating rules for linear learning automata are generally expressed as follows:

$$q_n(k+1) = q_n(k) + bF\left(q_n(k), a_n(k), r_n(k)\right) \tag{32}$$

where $F(\cdot, \cdot, \cdot)$ is a learning function. In addition, other forms of update schemes can be used, e.g., linear reward-penalty and linear reward-ε-penalty [30]. The reason for using L_{R-I} is that it has analytical tractability when being incorporated with game theory, which will be discussed below. Moreover, it is noted from (32) that the proposed stochastic learning algorithm is uncoupled, since the update procedure is only relying on the

[1]We assume that the system is fixed, where the number of nodes stays unchanged. Then, all nodes start to update their selections at the same time, which means that all the nodes perform learning simultaneously. For a dynamic system, the proposed learning algorithm in this work cannot be directly applied, and new mechanisms, which address dynamic node joining or leaving, are desirable.

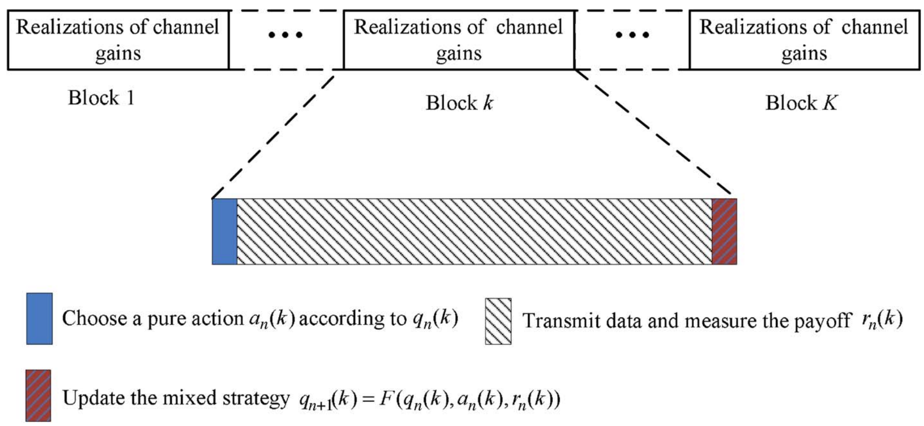

Fig. 3. Schematic of the stochastic-learning-automata-based channel selection algorithm.

individual trial-payoff history of a player; in fact, each user is not even aware of other users.

Denote $a(k) = \{a_1(k), \ldots, a_N(k)\}$ as the channel selection profile of all the nodes at the kth slot, and $Q(k) = \{q_1(k), \ldots, q_N(k)\}$ as a mixed strategy profile of $\mathcal{G}_c$, where $q_n(k) = (q_{n1}(k), \ldots, q_{ns}(k))$, $\forall n \in \mathcal{N}$, $s \in \mathcal{A}_n$, represents the channel selection probability of player n at the kth slot. Moreover, let $g_{nm}(Q)$ denote the expected utility function of player n if it employs pure strategy s (i.e., $a_n = s$) and other users k, $\forall k \in \mathcal{N}$, $k \neq n$, employ mixed strategy Q_{-n}. Formally

$$g_{nm}(Q) = \sum_{a_k, k \neq n} u_n(a_1, \ldots, a_{n-1}, s, a_{n+1} \ldots, a_N) \prod_{k \neq n} q_{ka_k}. \tag{33}$$

According to the update rule of the proposed algorithm, it is known that sequence $Q(k)$ is a discrete Markov chain. Furthermore, its behavior can be described by an ordinary differential equation on the condition that the learning step size b is sufficiently small [39]. The investigation on the convergence of L_{R-I} has been done well for single-player systems. Recently, the convergence of L_{R-I} for multiple-player systems is receiving attention, and some results can be found in the literature [18], [43]. Specifically, the following theorem presents a sufficient condition to achieve pure strategy NE points of a game using L_{R-I}.

Theorem 2: Suppose that there is a nonnegative function $G(Q) : Q \to R$ for some positive constant C such that

$$\begin{aligned} &G(s_1, Q_{-n}) - G(s_2, Q_{-n}) \\ &\quad = C\left[g_{ns_1}(Q) - g_{ns_2}(Q)\right], \quad \forall n, s_1, s_2, Q \end{aligned} \tag{34}$$

where $G(s, Q_{-n})$ is the value of G on the condition that q_n is a unit vector with the sth component unity, and $g_{ns}(Q)$ is specified by (33). Then, the L_{R-I}-based learning algorithm converges to a pure strategy NE point of a game.

Proof: See Theorem 5 in our recent work [44]. ∎

Based on Theorem 5, the asymptotic convergence behavior of the proposed stochastic learning algorithm is determined by the following theorem.

Theorem 3: With a sufficiently small step size b, the proposed uncoupled algorithm asymptotically converges to a pure strategy NE point of $\mathcal{G}_c$.

Proof: Take $G(Q) = \mathbf{E}[\Phi(Q)]$, where Φ is the potential function specified by (12). Then, we have

$$\begin{aligned} &G(s, Q_{-n}) \\ &\quad = \sum_{a_k, k \neq n} \Phi(a_1, \ldots, a_{n-1}, s, a_{n+1}, \ldots, a_N) \prod_{k \neq n} q_{ka_k}. \end{aligned} \tag{35}$$

Combining (33) and (35) yields the following equation:

$$G(s_1, Q_{-n}) - G(s_2, Q_{-n}) = g_{ns_1}(Q) - g_{ns_2}(Q) \tag{36}$$

where we apply $\Phi(\bar{a}_n, a_{-n}) - \Phi(a_n, a_{-n}) = u_n(\bar{a}_n, a_{-n}) - u_n(a_n, a_{-n})$, which is specified by (19).

Thus, the convergence of the proposed algorithm toward a pure strategy NE is validated by setting $C = 1$ in (34). ∎

Based on Theorem 3, the aggregate interference performance of the proposed algorithm for different network scenarios is characterized by the following propositions.

Proposition 4: In underloaded or equally loaded scenarios, the proposed uncoupled algorithm asymptotically converges to an optimal channel selection profile that makes the network interference-free.

Proof: Combining Theorem 3 and Proposition 1 straightforwardly proves this proposition. ∎

Proposition 5: In an overloaded scenario, the proposed uncoupled algorithm asymptotically converges to a pure strategy channel selection profile and minimizes the expected weighted aggregate interference globally or locally.

Proof: The convergence toward pure strategy NE points follows from Theorem 3. In addition, according to Proposition 2, among the converging channel selection profiles, there is at least an optimal one, while others are suboptimal. Thus, Proposition 5 is proved. ∎

It is important to study the achievable performance for different fading characteristics, since there are various fading models in practice, e.g., Rayleigh, Nakagami, and Lognormal.

Proposition 6: For a given distributed network, the achievable performance of the proposed algorithm is determined by the expected interference gain but not the specific fading model.

Proof: It is seen from (6) that expected weighted aggregate interference is jointly determined by the network topology, the transmitting power of the users, the converging channel selection profile, and the expected interference gain $\bar{\varepsilon}^s_{mn}$. Thus, for a given network, the achievable performance is only determined by the expected interference gain but the specific fading model. ■

According to Proposition 6, two different fading models with the same expected fading gain, for example, Rayleigh and Nakagami, will lead to the same expected weighted aggregate interference. Moreover, if a given fading model is with unit-mean, then the resulting expected weighted aggregate interference would be equal to a no-fading scenario, where only large-scale power loss is considered.

It is known that static environment is an extreme case of time-varying case, which implies that the proposed stochastic learning algorithm would also converge in static environment. Specifically, the following proposition characterizes its convergence in static environment.

Proposition 7: In a static system with symmetrical interference channels, the proposed uncoupled stochastic learning algorithm also asymptotically converges to a pure strategy NE point of the channel selection game.

Proof: In a static system, the experienced interference of a user is degenerated as

$$\hat{I}_n = \sum_{m \in \{\mathcal{N} \setminus \{n\}\}} f(a_m, a_n) p_m \hat{w}_{mn}^{a_n} \tag{37}$$

where $\hat{w}_{mn}^{a_n}$ is the fixed interference gain from users m to n on channel a_n, satisfying $\hat{w}_{mn}^{a_n} = \hat{w}_{nm}^{a_n}$. Then, the aggregate weighted interference can be given by

$$\hat{U} = \sum_{n \in \mathcal{N}} p_n \hat{I}_n = \sum_{n \in \mathcal{N}} \sum_{m \in \{\mathcal{N} \setminus \{n\}\}} p_m p_n \hat{w}_{mn}^{a_n} f(a_m, a_n). \tag{38}$$

We define a static channel selection game $\hat{\mathcal{G}}_c$ with the following utility function:

$$\hat{u}_n(a_n, a_{-n}) = D - p_n \hat{I}_n. \tag{39}$$

Now, applying similar lines of proof for Theorem 1, it can be proved that the channel selection game in static environment is also a potential game with $-(1/2)\hat{U}$ serving as the potential game. Based on this argument, using the same methodology in Theorem 3 proves this proposition. ■

Finally, it should be pointed out that there exists a speed-accuracy conflict for the proposed algorithm. Denote the event $A = \{\text{It converges to a pure strategy NE}\}$. Based on Theorem 3, it follows that $\Pr\{A|(b, Q(0))\} = 1$, as $b \to 0$. However, smaller step size b implies slower convergence speed, whereas larger step size leads to faster convergence speed and low accuracy. More importantly, there is a probability that it converges to pure strategies but not NE points when step size b is not sufficiently small. Hence, the choice of step size b in the proposed algorithm is application dependent, which should be done by practical experiment or training [43]. We will later study the impact of the parameters on the proposed learning algorithm through simulation.

Remark 3: The achievable performance of the proposed algorithm is summarized as follows: It achieves the optimal channel selection profiles for underloaded or equally loaded scenarios and can achieve optimal or suboptimal solutions for overloaded scenarios. This is promising as the proposed algorithm is simple, completely distributed, and uncoupled.

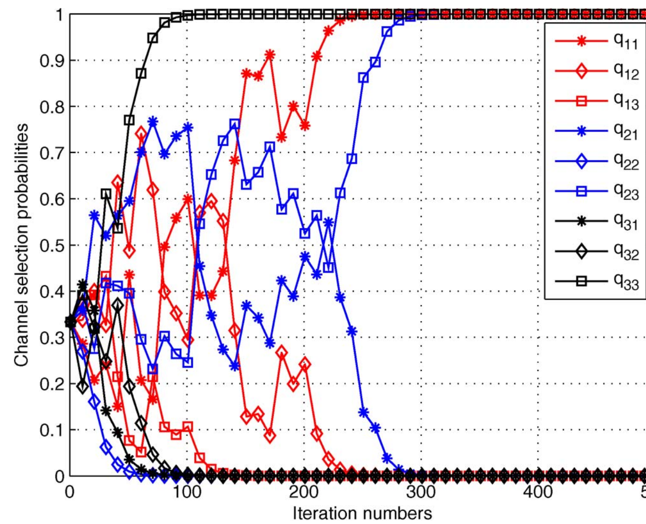

Fig. 4. Evolution of channel selection probabilities for three arbitrarily selected nodes in Rayleigh fading environment ($N = 5$, $M = 3$, $D = 0.005$, and $b = 0.1$).

VI. Simulation Results and Discussion

Following the similar simulation setting used in [9], the nodes in the simulation study are randomly located in a 100 m × 100 m region. For presentation, we set the transmitting power of all the users as $p_n = 0$ dB, $\forall n \in \mathcal{N}$, the path loss exponent as $\alpha = 2$, and the noise power as $N_0 = -130$ dB. For simplicity of analysis, the transmission distance for each intracommunication is set to 1 m, i.e., $d_{nn} = 1, \forall n \in \mathcal{N}$; in addition, the instantaneous random components of intracommunications are assumed to be unit-constant. The channels are with equal bandwidth 1 MHz. The fading of interfering channels is assumed to undergo block fading, i.e., they remain stationary within a slot and randomly changes in the next slot. Three fading models, e.g., Rayleigh, Nakagami, and Lognormal, are considered.

A. Convergence Behavior

1) Convergence Behavior in Dynamic Environment: Here, we consider dynamic environment. For presentation, we consider a network involving three channels and five nodes, which are randomly located. The channels are assumed to undergo Rayleigh fading with unit mean. Moreover, the positive constant used in (9) and (28) is set to $D = 0.005$, and the step size of the proposed algorithm is set to $b = 0.1$. We will discuss the impact of these parameters later.

The convergence behavior of three arbitrarily selected nodes is shown in Fig. 4. Let us take node 1 as an illustrative example. It chooses the channels randomly with equal probabilities at the beginning $(q_{11} = 0.33, q_{12} = 0.33, q_{13} = 0.33)$. As the algorithm iterates, it finally chooses channel 3 $(q_{11} = 1, q_{12} = 0, q_{13} = 0)$. It is seen that their channel selection probabilities converge to pure strategy in about 100, 250, and 290 iterations, respectively. Moreover, the evolution of the number of nodes on the channels is shown in Fig. 5. It is noted that the number of

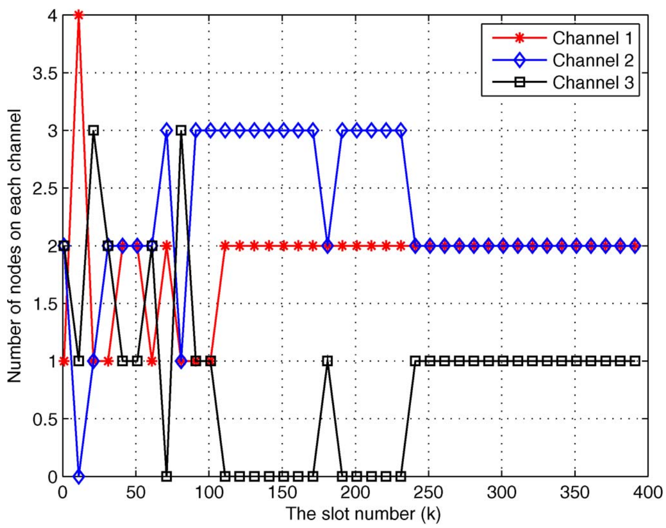

Fig. 5. Evolution of the number of nodes choosing the channels in Rayleigh fading environment ($N = 5$, $M = 3$, $D = 0.005$, and $b = 0.1$).

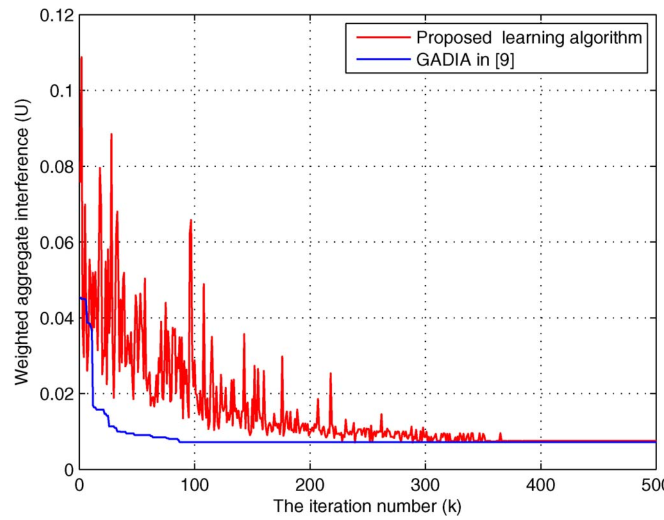

Fig. 6. Convergence behavior comparison in static environment ($N = 20$, $M = 5$, $D = 0.005$, and $b = 0.1$).

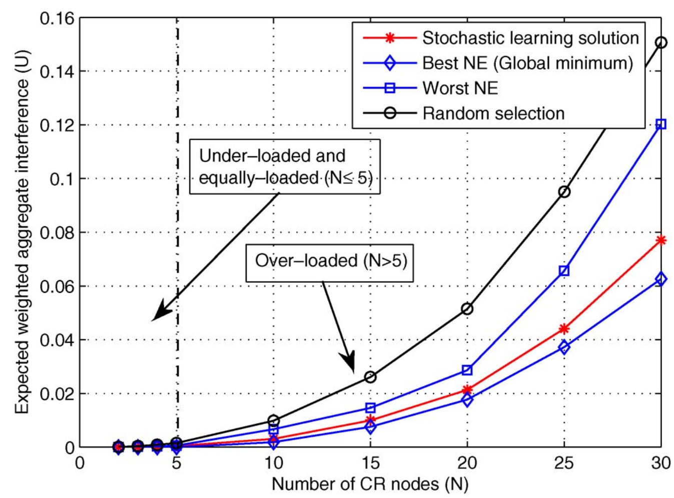

Fig. 7. Performance evolution for a distributed network involving a Rayleigh fading environment ($D = 0.005$, $b = 0.08$, and $M = 5$).

nodes selecting different channels becomes invariant in about 250 iterations, which again means that Algorithm 1 converges. These results validate the convergence of the proposed stochastic learning algorithm for the interference mitigation game.

2) Convergence Behavior in Static Environment: Here, we consider static environment and compare with an existing static algorithm. There is an existing channel selection algorithm proposed by Babadi and Tarokh, which is called GADIA [9], that has been shown to achieve good performance in static distributed systems with symmetrical interference channels. As illustrated by Proposition 7, the proposed algorithm in our work also converges in static environment. The convergence behaviors of an arbitrary network topology with 20 users and five channels are shown in Fig. 6. It is noted that the proposed algorithm converges in the static environment, as the existing GADIA algorithm. However, it is also noted that GADIA converges relatively rapidly and smoothly. The reasons are as follows: 1) The GADIA algorithm measures the interference on all channels before updating the channel selection of a user, and the selection update is implemented in a deterministic manner, whereas 2) the proposed learning algorithm only measures the interference on the current channel, and the selection update is implemented in a stochastic manner.

B. Performance Evaluation

1) Performance Comparison for Different Solutions: Here, we evaluate the performance of the proposed stochastic learning algorithm in terms of the expected weighted aggregate interference. We consider a network involving five channels and the number of nodes increasing from 2 to 30. The learning parameters in the algorithm are set as $D = 0.005$ and $b = 0.08$. For comparison, we additionally present the performance evaluation for a random selection scheme, the worst NE, and the best NE. In the random selection scheme, each node randomly chooses a channel in each slot. Due to the restriction that the channel gains randomly vary and that there is no information exchange, random channel selection seems to be an instinctive method. The best and worst NE solutions are obtained in a quasi-centralized manner. Specifically, we assume that there is an omnipotent genie, which perfectly knows the channel characteristics and the distance among every pair of nodes. We run the proposed learning algorithm 10^3 times and then choose the best (worst) result, respectively. According to Theorem 1, it is known that best NE also serves as global minimum for the expected weighted aggregate interference.

Then, we compare the performance of the proposed learning solution, the random selection, the worst NE, and the best NE in Fig. 7. The presented results are obtained by simulating 10^3 independent trials and then taking the expected value. It is noted that in the underloaded and equally loaded network scenarios, i.e., $N \leq 5$, the difference between the performance of the learning solution and that of the best NE is trivial. The reason is that the learning solution asymptotically achieves the global optimum, as characterized by Proposition 4. On the other hand, in the overloaded network scenarios, i.e., $N > 5$, it is shown that the performance gap between the learning solution and the best NE is small. The reason is that the learning

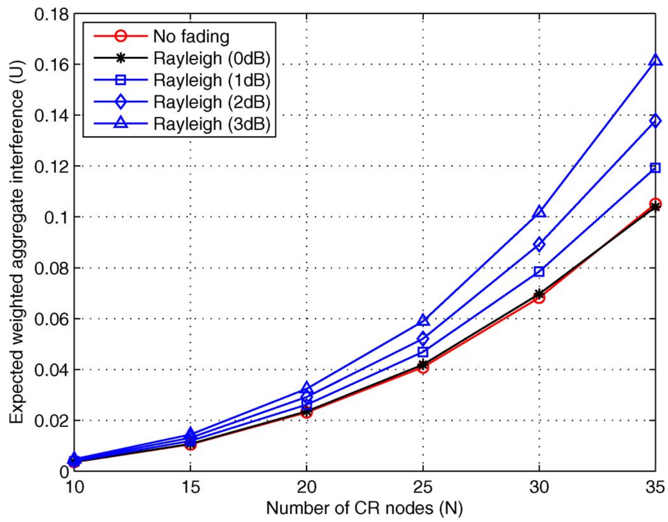

Fig. 8. Comparison results of expected aggregate interference for different Rayleigh fading parameters ($D = 0.005$, $b = 0.08$, and $M = 5$).

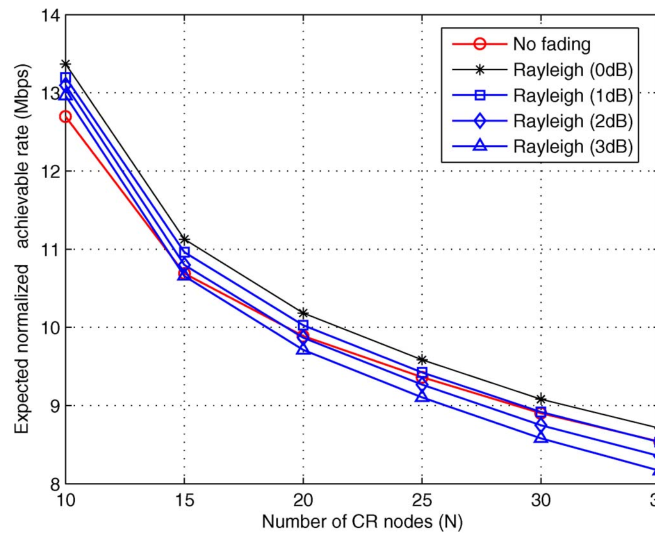

Fig. 9. Comparison results of expected normalized achievable throughput for different Rayleigh fading parameters ($D = 0.005$, $b = 0.08$, and $M = 5$).

solution may converge to an optimal or a suboptimal channel selection profile, as characterized by Proposition 5, and, hence, on average, achieves near-optimal performance.

In addition, it is noted in Fig. 7 that even the worst NE leads to less aggregate interference than the random selection scheme. In the random selection scheme, there is a probability that some channels are crowded, whereas others are unoccupied. On the contrary, the nodes are spread over different channels in the pure strategy NE, which leads to less interference. It is also noted in the figure that as the number of nodes increases, the performance gap between the learning solution and the best NE slightly increases, whereas that between the random selection between the best NE significantly increases. In addition, it is noted in the figure that as the number of nodes increases, the aggregate interference increases accordingly, as can be expected in any multiuser systems.

2) Performance Evaluation for Different Fading Parameters: Here, we present the performance evaluation for different fading parameters. Specifically, the comparison results of expected aggregate interference are shown in Fig. 8. The results are obtained by simulating 20 independent topologies with 10^3 independent trials and then taking the average value. No-fading means only large-scale power loss is considered. It is noted that the performance gap between no-fading and Rayleigh with 0-dB mean is trivial. The reason is that 0-dB mean implies unit-mean. Thus, according to Proposition 6, its performance should coincide with that of the no-fading scenario. Moreover, it is noted that as the mean value of Rayleigh fading increases, e.g., varying from 1 to 3 dB, the expected weighted aggregate interference increases, as can be expected in any interference mitigation systems.

In addition, we present the comparison results of expected normalized achievable throughput in Fig. 9. It is noted that as the number of nodes increases, the expected normalized achievable rate decreases. The reason is that large numbers of nodes results in heavy mutual interference, as can be expected in any multiuser system. Moreover, it is noted that as the mean value of Rayleigh fading increases, e.g., varying from 1 to 3 dB, the expected normalized achievable rate decreases, as can be expected in any interference mitigation systems. An interesting result shown in the figure is that No-fading achieves the lowest throughput performance when the number of channels is small $(M < 15)$, whereas it achieves medium performance when the number of channels increases. This is due to multiuser diversity. Specifically, the fluctuation in the channel gains leads to higher throughput.

3) Performance Evaluation for Different Fading Models: Here, we present the performance evaluation for different fading models. We consider the well-known models, including Rayleigh, Nakagami, and Lognormal. Specifically, in the Rayleigh model, the channel gains are exponentially distributed with unit-mean. In the Nakagami model, the probability distribution function of the channel gains is given as $f(x) = (m^m x^{m-1}/\Gamma(m))e^{-mx}$, $x \geq 0$. In the Lognormal model, the channel gains can be modeled by a random variable e^X, where X is a Gaussian variable with zero mean and variance σ^2. Lognormal fading is usually characterized in the decibel-spread form, which is related to σ, by $\sigma = 0.1 \log(10)\sigma_{\mathrm{dB}}$. The decibel-spread of Lognormal fading typically ranges from 4 to 12 dB, as indicated by the empirical measurements [49].

The comparison results of expected aggregate interference for different fading models are shown in Fig. 10. The results are obtained by simulating 20 independent topologies with 10^3 independent trials and then taking the average value. It is noted that the performance gap is trivial. The reason is that all the presented fading models are with unit-mean. Thus, according to Proposition 6, the presented results hold. Moreover, since all the presented fading models are with unit-mean, their performance coincides with that of a system with no fading, as can be noted in this figure.

In addition, we present the comparison results of expected normalized achievable throughput for different fading models in Fig. 11. It is noted that as the number of nodes increases, the expected normalized achievable rate decreases, as can be expected. It is interesting to see that Rayleigh fading outperforms Nakagami fading and Lognormal fading. In addition,

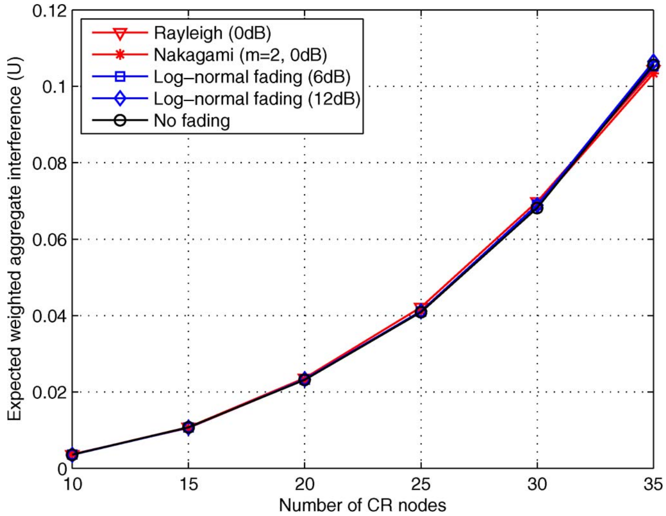

Fig. 10. Comparison results of expected aggregate interference for different fading models ($D = 0.005$, $M = 5$, and $b = 0.08$).

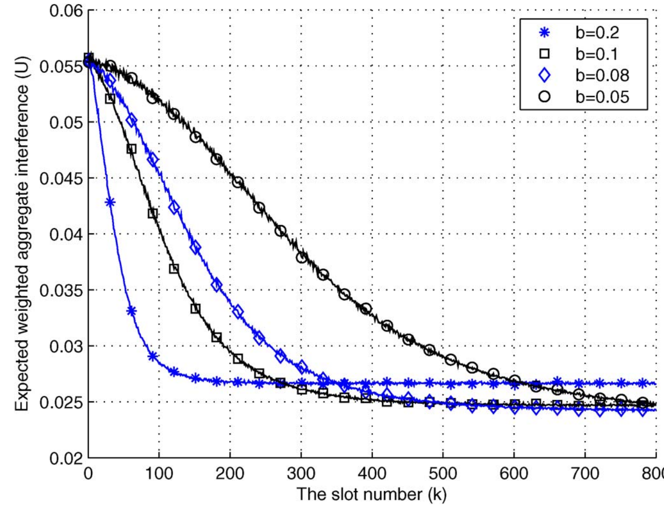

Fig. 12. Impact of learning step size b on the learning algorithm in Rayleigh fading environment ($N = 20$, $M = 5$, and $D = 0.005$).

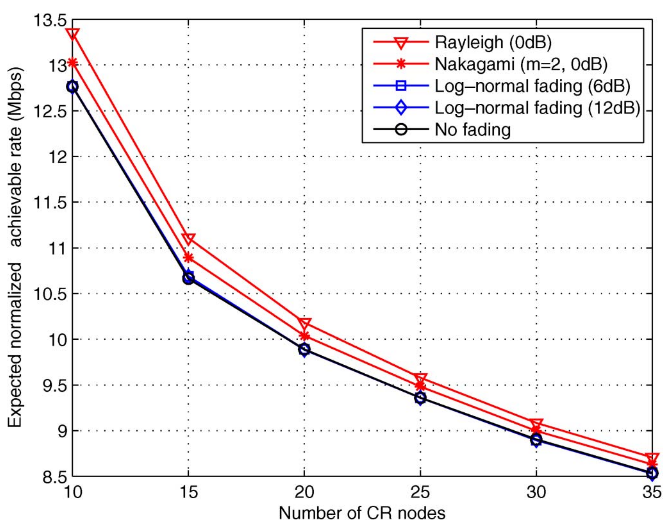

Fig. 11. Comparison results of expected normalized achievable throughput for different fading models ($D = 0.005$, $M = 5$, and $b = 0.08$).

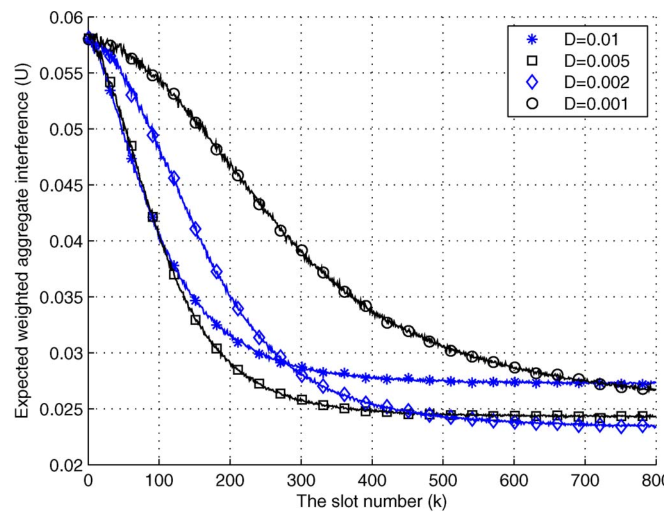

Fig. 13. Impact of positive constant D on the learning algorithm in Rayleigh fading environment ($N = 20$, $M = 5$, and $b = 0.1$).

the performance of Lognormal fading is almost the same as that of No-fading. The reasons can be as follows: 1) Multiuser diversity for Rayleigh fading is stronger than those for other fading models, and 2) the multiuser diversity of Lognormal fading is weak.

C. Impact of Parameter Selection

Here, we study the impact of the learning parameters on the convergence and performance of the proposed stochastic learning algorithm. We study the case of the Rayleigh fading model. All the results presented below are obtained by simulating 5000 independent trials and then taking the expected value.

First of all, the impact of the learning step size b is shown in Fig. 12. It is noted that larger b, e.g., $b = 0.2$, leads to faster convergence speed, while resulting in relatively higher aggregate interference. The reason is that larger b makes the proposed algorithm rapidly converge toward a pure strategy but not necessarily to an NE point. In other words, this decreases the exploration opportunities for the proposed algorithm and, hence, increases the probability of converging to local optimum. Therefore, the result presented in Fig. 12 follows.

Second, the impact of the predefined positive constant D is shown in Fig. 13. It is noted that larger D, e.g., $D = 0.01$, leads to faster convergence speed, while resulting in relatively higher aggregate interference. On the other hand, smaller D, e.g., $D = 0.001$, leads to slower convergence speed (since its expected convergence iteration is greater than 800, we do not completely plot its convergence behavior in Fig. 13), while resulting in relatively lower aggregate interference. The reason is that smaller D increases the probability of receiving a zero payoff, as characterized by (29). A player with zero received payoff will keep its mixed strategy unchanged, as can be seen from (30), which eventually implies that the mixed strategy remains unchanged. Thus, the result presented in Fig. 13 follows.

To summarize, the most appealing feature of the proposed algorithm is that it converges in time-varying environment, and it is simple, completely distributed, and uncoupled. Such a feature makes it suitable for a large number of wireless distributed optimization problems, particularly the one involving time-varying environment, e.g., distributed discrete power control and distributed rate adaptation. Furthermore, it should be noted that the choice of the parameters has great impact on the algorithm in terms of both convergence speed and performance. Thus, the choice is application dependent, which should be done by practical experiment or training.

VII. Conclusion

We have investigated the problem of distributed channel selection for interference mitigation in time-varying radio environment without information exchange. First, we formulated this problem as a noncooperative game in which the utility of each player is defined as a function of its experienced expected weighted interference. We proved that the formulated game is an exact potential game with the considered network utility, i.e., the expected weighted aggregate interference, serving as the potential function. Then, we proposed a simple, completely distributed, and uncoupled stochastic learning algorithm, without needing information exchange among users and requiring prior information of the network. Using the proposed algorithm, the users learn the desirable channel selections from their trial-payoff history. It was analytically shown that the proposed learning algorithm converges to pure strategy NE in the fading environment and averagely achieves near-optimal performance. The results provide with a desirable solution for distributed wireless optimization problems, particularly the one involving time-varying channels.

However, it is seen that the convergence speed of the proposed stochastic learning algorithm is still relatively slower, particularly when the number of users becomes large. Thus, we attempt to investigate algorithms with fast convergence speed in future work. In addition, although the proposed learning algorithm copes with the time-varying channels well, it does not take into account a time-varying network topology, i.e., a user becoming in-active, or *vice versa*, or a user joining in or leaving the network dynamically. Considering such a variation is more realistic and would be studied in the future.

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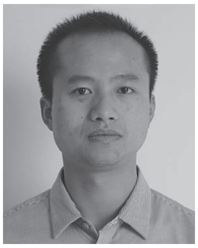

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