Opportunistic Spectrum Access with Spatial Reuse: Graphical Game and Uncoupled Learning Solutions

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***Abstract*—This article investigates the problem of distributed channel selection for opportunistic spectrum access systems, where multiple cognitive radio (CR) users are spatially located and mutual interference only emerges between neighboring users. In addition, there is no information exchange among CR users. We first propose a MAC-layer interference minimization game, in which the utility of a player is defined as a function of the number of neighbors competing for the same channel. We prove that the game is a potential game with the optimal Nash equilibrium (NE) point minimizing the aggregate MAC-layer interference. Although this result is promising, it is challenging to achieve a NE point without information exchange, not to mention the optimal one. The reason is that traditional algorithms belong to coupled algorithms which need information of other users during the convergence towards NE solutions. We propose two uncoupled learning algorithms, with which the CR users intelligently learn the desirable actions from their individual action-utility history. Specifically, the first algorithm asymptotically minimizes the aggregate MAC-layer interference and needs a common control channel to assist learning scheduling, and the second one does not need a control channel and averagely achieves suboptimal solutions.**

***Index Terms*—Opportunistic spectrum access, cognitive radio networks, distributed channel selection, MAC-layer interference, potential game, graphical game, uncoupled learning algorithms.**

I. INTRODUCTION

COGNITIVE radio has been regarded as a promising solution to address the spectrum shortage problem [1]. In cognitive radio networks (CRNs), opportunistic spectrum access is key to improve the network throughput performance [2]. It is a current active research topic to study the problem of distributed channel selection for a kind of CRNs, where CR users are spatially located and direct interaction only emerges between neighboring users [3]–[8]. Note that a common assumption in the existing studies is that they need information exchange among neighboring users during the convergence towards a desirable solution. Such an assumption, however, does not always hold in practice, e.g., in the presence of deep channel fading or shadowing [9], and may lead to heavy communication overhead. Thus, we focus on the problem of distributed channel selection for the above-mentioned CRNs without information exchange.

We investigate this problem from an interference minimization perspective. The commonly used model in the literature is the PHY-layer interference model, in which it cares more about the amount of experienced interference [10]. However, it may not coincide with practical communications, especially when multiple access control mechanisms are involved. Specifically, recently reported experiment [11] shows that the traditional PHY-layer interference model does not coincide with the measured results (see Section III for detailed discussion and analysis). Therefore, it is important and timely to design a new interference metric to capture practical communications.

In this article, we consider a new interference metric, called the MAC-layer interference, which is defined as the number of neighboring users choosing the same channels. Unlike the traditional PHY-layer interference model, the MAC-layer interference in essence is used in determining whether two users interfere with each other or not. Such a binary-abstraction captures most multiple access control mechanisms well, e.g., CSMA and Aloha. In fact, MAC-layer interference model can be regarded as a high-level abstraction of traditional PHY-layer interference model and is more practical and useful.

We formulate a *MAC-layer interference minimization game*, in which each user selfishly minimizes its experienced MAC-layer interference. The reasons for formulating this game are threefold: (i) the decisions of CR users are interactive, (ii) minimizing the number of nearby competing users would lead to high throughput for a user, (iii) the CR users behave selfishly and distributively in essence.

However, the task of achieving Nash equilibrium (NE) solutions of the formulated game is challenging, not to mention achieving the optimal one. The reason is that most traditional game-theoretic algorithms, e.g., spatial adaptive play [3], best response [12], and fictitious play [13], are *coupled*, i.e., they need information of other users in terms of chosen actions and/or received payoffs during the convergence towards NE points.

As a result, we propose two *uncoupled* [14] learning algorithms for the considered CRNs. The key idea is that the users observe the environment, estimate their received utilities,

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intelligently learn from their individual action-payoff history, and then adjust their behaviors towards NE solutions. In fact, such a procedure is a typical form of the well-known *cognitive cycle* [1]. In comparing the two uncoupled learning algorithms,

- The first uncoupled learning algorithm (tagged as Algorithm 1) needs a common control channel to assist learning scheduling (but no exchange of information), asymptotically achieves the optimal NE solution and minimizes the aggregate MAC-layer interference.
- The second uncoupled learning algorithm (tagged as Algorithm 2) does not need a common control channel, and hence averagely converges to some suboptimal solutions for network collision minimization problem.

The rest of the article is organized as follows. In Section II, we give a brief review of the related work. In Section III, we present the motivation and definition of MAC-layer interference model. In Section IV, we present the system model and formulate the optimization problem. In Section V, we propose the MAC-layer interference minimization game and investigate its properties. In Section VI, we develop two learning algorithms that converge to pure strategy NE points of the game. Simulation results are presented in Section VII and conclusion are drawn in Section VIII.

II. Related Work

In order to study the interactions among users in the problem of distributed channel selection for CRNs, game theory has been widely applied and various versions of game models can be found [15]–[21]. Most formulated game models in these references are non-spatial, in which the CR users are located closely and hence the transmission of a user interferes with all other users. To overcome this problem, different versions of local interaction games (also called graphical games in [5], [6]) have been proposed [3]–[8]. Specifically, graphical game formulation for distributed channel selection has been established in [5], local minority game for this problem has been studied in [6], global optimization using local interaction games has been investigated in [3], spatial congestion game formulation for spectrum sharing has been established in [7], and local bargaining based spectrum allocation algorithms on a conflict graph have been proposed in [8].

However, most existing game theoretic solutions, including global and local interaction game models, mainly focus on investigating the properties of the games, e.g., the existence of NE and the convergence toward NE with explicit or implicit information exchange among users, but little attention has been paid to scenarios in the absence of information exchange.

There are some uncoupled learning solutions for CRNs, which mainly include reinforcement learning and no-regret learning. Specifically, reinforcement learning solutions for interference control in wireless regional area networks [22] and opportunistic bandwidth sharing [23] have been proposed respectively. In parallel, multi-agent reinforcement learning channel selection algorithms were proposed for CRNs [24]. In addition, no-regret learning based channel selection algorithms for CRNs have been investigated in [15], [19]. It is hard to directly apply these learning algorithms to the MAC-layer interference minimization game because the effects of the local interaction on the convergence and performance are generally hard to analyze. In addition, comprehensive and comparative survey on decision-theoretic approaches for CRNs can be found in [25].

Note that, the considered system model in this article is similar to that proposed in our previous work [3], [4]. In this article, we re-consider this problem and formulate the MAC-layer interference model, which has physical meaning for practical communication systems. More importantly, the algorithms in [3], [4] are coupled, whereas the algorithms proposed in this article are uncoupled. In our recent work [26], we also proposed a new interference metric which is very similar to the MAC-layer interference model in this work. The key difference is that partially overlapping channels were considered in [26] while we consider orthogonal channels in this article.

III. Motivation, Definition and Discussion of MAC-layer Interference

A. Motivation and definition

In a multiuser wireless system, mutual interference among users is unavoidable due to the open attribute of wireless transmissions. The commonly used model in the literature is the PHY-layer interference model, in which it cares about the amount of experienced interference [10]. However, it is noted that the PHY-layer interference models do not take into account multiple access control mechanisms, which have been commonly used in today's wireless communication systems. As a result, the PHY-layer interference models are more suitable for information-theoretic analysis and may not coincide with practical communications.

Recently reported experimental results [11] reveal that when multiple access control mechanisms are considered in interference models, some distinct and interesting features emerge. Following the same experiment setup therein, two nodes (links) equipped with 802.11a/b/g cards are considered to study the effect of mutual interference. Note that a node (link) here consists of a transmitter and a receiver which are located clearly [11]. We consider Log-normal model fading model in this article, since it captures the medium-scale path-loss well[1]. Specifically, the received signal strength (RSS) of a link from the other link is expressed as:

$$S = P_t d^{-\beta} e^X, \tag{1}$$

where P_t is the transmitting power, d is the distance between the two nodes, β is the path loss exponent and X is a Gaussian variable with zero-mean and variance σ^2. The Log-normal fading is usually characterized in the dB-spread form which

[1]It is seen that we do not include small-scale fading in the formulation, e.g., Rayleigh and Rician. The reasons are: 1) these kinds of fading are only considered when the processing time is comparable with the symbol period. In practice, these kinds of fading are generally eliminated with some PHY-layer technologies, e.g., channel equalization, and 2) the proposed MAC-layer interference model is a kind of MAC-layer scheduling, whose processing time is much larger than the symbol period. In its time scale, generally only large-scale or medium-scale path-loss are considered. However, it should be noted that one would apply the approaches in this article to study the MAC-layer interference when small-scale channel fading is eliminated in the PHY-layer.

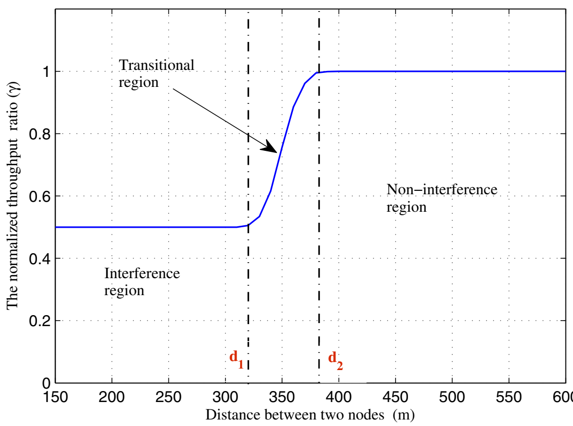

Fig. 1. The effect of mutual interference on the achievable throughput.

is related to σ by $\sigma = 0.1\log(10)\sigma_{\text{dB}}$. The dB-spread of Log-normal fading typically ranges from 4 to 12 dB as indicated by the empirical measurements [27].

The well-known CSMA is applied as the multiple access control mechanism, since it has been commonly used in practice. According to its principles, a link can hear the transmission of the other link if the RSS is great than a threshold S_{th}. In the experiment setup, the parameters are set to $P_t = 1W$, $\beta = 2$, $S_{th} = 8.1633 \times 10^{-6}W$ and $\sigma_{\text{dB}} = 6$dB. Denote s_1 and s_2 as the throughput of link1 and link2 respectively when the other link is inactive, and s_1' and s_2' as the throughput of them when both links are active. The relationship between the ratio $\gamma = \frac{s_1'+s_2'}{s_1+s_2}$ and their distance is measured to study the effect of mutual interference on the throughput [11].

By simulating 10^6 independent trials, we illustratively present the throughput ratio in Fig. 1. Similar to those shown in [11], it is noted that there are also two interesting results: (i) the measured normalized throughput changes sharply from severe interference (around 0.5) to almost no interference (around 1) with a slight increase in their physicaldistance. As a result, there are three regions with respect to distance, i.e., interference region, transitional region and non-interference region, as shown in the figure, and (ii) the normalized throughput in both interference and non-interference regions remains almost the same regardless of the change of distance, while it increases linearly with the distance in the transitional region.

It is shown in Fig. 1 that the span of the transitional region, i.e., $d_2 - d_1$, is relatively small. Thus, we use a simplified interference model for the convenience of analysis. Specifically, if the throughput ratio is less than a threshold, e.g., 0.95, mutual interfere exists, otherwise there is no interference. The distance corresponding to the interference threshold is denoted as d_0, $d_1 < d_0 < d_2$. Formally, it motivates us to define the MAC-layer interference as follows:

$$\alpha = \begin{cases} 1, & x \le d_0 \\ 0, & x > d_0, \end{cases} \tag{2}$$

where x is the distance between the two links. Based on this definition, the normalized throughput can be approximately expressed as $R = \frac{1}{1+\alpha}$. It is noted that such a binary interference model provides a good approximation for the measured results. Thus, by sacrificing a little accuracy, we are able to design efficient algorithms. However, it should be pointed out other forms of MAC-layer interference can also be defined to capture the effect of channel fading[2].

Some issues related to channel fading are discussed below: (i) in the interference region, the large-scale path loss component, i.e., $P_t x^{-\beta}$, is *strong* enough, and one link can deterministically hear the other link regardless of the realizations of channel fading. Thus, only one link can transmit successfully at a time. In this region, the impact of channel fading is concealed by strong large-scale path loss component, (ii) in the transitional region, the large-scale path loss component is *medium*, and the RSS is fluctuating around the interference detection threshold due to the randomness of channel fading gain. Thus, one link can probabilistically hear the other link, which is referred as probabilistic interference. However, we simplify the probabilistic interference into a binary model by ignoring the effect of channel fading. Such a simplification sacrifices a little accuracy but leads to mathematical tractability, and (iii) in the non-interference region, the large-scale path loss component is *weak*, and one link never hear the other link regardless of the realizations of channel fading. In other words, the impact of channel fading is eliminated by the far distance. In short, the impact of channel fading in the interference and non-interference regions is concealed by the strong and weak RSS respectively. In addition, channel fading indeed has impact on the MAC-layer interference in the transitional region but was ignored for mathematical tractability. We will include simulation results for channel fading in Section VII.

B. Extension to multiuser systems

In a multiuser system, if user n can hear the transmissions of other users, it means that mutual interference exists among user n and multiple other users. According to the principles of multiple access control mechanisms, the normalized throughput of user n can be approximated by $\frac{1}{1+\sum_m \alpha_{mn}}$, where α_{mn} is the MAC-layer interference between users n and m with distance x_{mn}. Then, following the same idea of the relationship between the two-user MAC-layer interference and normalized throughput, we can define the MAC-layer interference experienced by user n in a multiuser system as:

$$I_n = \sum_m \alpha_{mn}. \tag{3}$$

[2]For instance, in order to address the transitional region, an improved method is to extend the binary interference model to a real-valued one. In particular, one may define the MAC-layer interference as follows:

$$\alpha' = \begin{cases} 1, & x \le d_1 \\ \frac{x-d_2}{d_1-d_2}, & d_1 < x < d_2 \\ 0, & x \ge d_2 \end{cases}$$

The above defined α' is a continuous function ranging in $[0,1]$. Specifically, $\alpha' = 1$ and $\alpha' = 0$ correspond to the same meanings as those in (2), and the value of $0 < \alpha' < 1$ corresponds to probabilistic interference and addresses the transitional region well. The normalized throughput of a link can also be well expressed as $R = \frac{1}{1+\alpha'}$. It is noted that this formulation fits the measured throughput well.

The above alternative MAC-layer interference model is more precise than the binary model defined in (2); in particular, it addresses the randomness of channel fading gain in the transitional region. However, the task of dealing with the a real-valued interference leads to mathematical intractability. Thus, we use the binary interference model in this article and will consider the real-valued interference model in future.

In the following, the above defined MAC-layer interference will be used in the distributed channel selection problem. In fact, as a binary-abstraction of PHY-layer interference model, the MAC-layer interference captures most multiple access control mechanisms well, e.g., CSMA and Aloha.

IV. System Model and Problem Formulation

A. Bilateral interference CRNs

Consider a CRN consisting of N CR users. Here, a CR user represents a closely located pair of transmitter and receiver, which means that the considered network belong to canonical networks [3]. Generally, the CR users are spatially distributed and interference only occurs among nearby users [5], [6]. To capture the spatial separation, we characterize the limited range of interference by an un-directional graph $G = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \ldots, N\}$ is the vertex set and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is the edge set. Each vertex corresponds to a CR user, and the edges represent the mutual interference relationship among the users when they transmit in the same channel. Specifically, if the distance between two CR users m and n, denoted as D_{mn}, is less than a threshold D_0, it implies that they can hear each other and hence interfere with each other when simultaneously transmitting on the same channel; thus, m and n are connected by an edge. In other words, there is an edge $e_{mn} = (m, n) \in \mathcal{E}$. We assume that the interference is bilateral between any two users, i.e., user m is also interfered by user n if it interferes with n. We call this kind of networks, bilateral interference cognitive radio networks (BI-CRNs). An example of a BI-CRN with four CR users and two primary users is shown in Fig. 2.

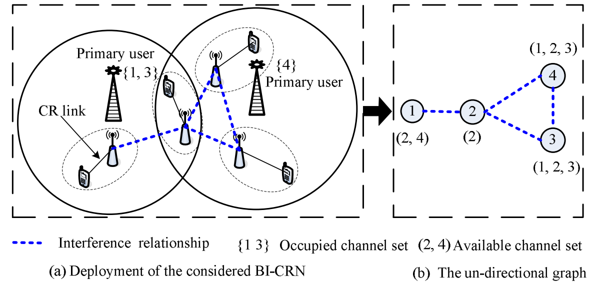

Fig. 2. An example of the considered BI-CRN with four licensed channels. Note that CR users 2 and 3 will interfere with each other when transmitting on the same channel, whereas user 1 will never interfere with user 3.

Suppose that there are M orthogonal licensed channels, which are owned by the primary users and can be opportunistically used by the CR users when not occupied by the primary users (PUs). Notice that the available spectrum opportunities vary from user to user due to their different locations. We characterize the heterogeneous spectrum opportunities by the channel availability vector C_n, $n \in \mathcal{N}$. Specifically, $C_n = \{C_{n1}, C_{n2}, \ldots, C_{nM}\}$, where $C_{nm} = 1$ means that channel m is available for player n, while $C_{nm} = 0$ means that it is not available. For simplicity of analysis, we assume that the spectrum sensing is perfect[3]; moreover, it is assumed that the spectrum opportunities are quasi-static in time. Such an assumption holds for some realistic CRNs with slow-varying spectrum opportunities, e.g., IEEE 802.22 [28].

B. MAC-layer interference minimization

It is assumed that all the CR users can sense all channels, but can transmit on only one channel due to hardware limitation [29]. Let $\mathcal{J}_n$ denote the neighboring user set of user n, i.e.,

$$\mathcal{J}_n = \{i \in \mathcal{N} : (i, n) \in \mathcal{E}\}, \tag{4}$$

and a_n denote a channel selection of user n. Specifically, $a_n \in \{1, \ldots, M\}$ if there is at least one channel available for n; otherwise $a_n = \emptyset$ if there is no available channel. We call user n as an active user for the former case, and a silent user for the latter case. In the rest of this article, we assume that all the users are active. For the considered CRNs, efficient distributed approaches such as CSMA can be applied to coordinate transmissions among neighboring and interfering users. According to the principle of CSMA, the individual throughput of user n under channel selection profile $a = \{a_1, \ldots, a_N\}$ is given by:

$$r_n(a_1, \ldots, a_N) = \frac{f(c_n + 1) R_{a_n}}{c_n + 1}, \tag{5}$$

where $f(k)$ is the throughput loss function on the condition that k users are competing for a single channel [9], which satisfies $0 < f(k) \leq 1$ and is decreasing over k, R_{a_n} is the transmission rate of channel a_n, and

$$c_n = \sum_{j \in \mathcal{J}_n} \delta(a_n, a_j) \tag{6}$$

is the number of competing neighboring users of user n. Specifically, $\delta(x, y)$ is the following *MAC-layer interference indicator function*:

$$\delta(x, y) = \begin{cases} 1, & x = y \\ 0, & x \neq y. \end{cases} \tag{7}$$

Note that the individual throughput specified by (5) implies that all mutually interfering users share the channel equally. Consequently, the network throughput can be given by:

$$R(a_1, \ldots, a_N) = \sum_{n \in \mathcal{N}} r_n. \tag{8}$$

The problem of maximizing the above network throughput is a combinatorial problem on a graph and is hence NP-hard. Motivated by the previous work addressing the topic of minimizing the aggregate PHY-layer interference [10], we use the MAC-layer interference to study this problem. For simplicity of analysis, we assume that all channels have the same transmission rate[4].

The MAC-layer interference experienced by user n in the channel selection problem is defined as c_n, i.e., the number of neighboring users choosing the same channel. Note that this definition coincides with that defined in (3) except that channel selection is considered here. From the user-side,

[3]However, the results obtained in this article can easily be extended to the scenarios with imperfect spectrum sensing.

[4]Although the channels may have different instantaneous transmission rates due to different channel quality, they would have the same transmission rate in a long-run time. The reason to make such an assumption is that by sacrificing a little accuracy, we are able to design more efficient algorithms to minimize the aggregate MAC-layer interference.

lower value of c_n is desirable, because minimizing c_n is equivalent to maximizing its throughput, as can be seen from (5). Consequently, lower aggregate MAC-layer interference is more preferable from the network-side. Based on this, a quantitative characterization of the aggregate MAC-layer interference experienced by all the users can be given by:

$$I_g(a_1, \ldots, a_N) = \sum_{n \in \mathcal{N}} c_n \tag{9}$$

Then, the network-centric goal is to find an optimal channel selection a^{opt} that minimizes the network utility, i.e.,

$$P1: \quad a^{opt} \in \arg\min \ I_g. \tag{10}$$

The above problem is a variant of general graph coloring problem for distributed channel selection [30]. However, due to the higher-order of computational complexity, $P1$ is NP-hard even in a centralized manner. Furthermore, lack of knowledge about other users makes the task of finding a good solution challenging, not to mention finding an optimal one.

V. MAC-Layer Interference Minimization Game

Since there is no central controller, the channel selections are self-determined by the CR users; moreover, their decisions are interactive. This motivates us to formulate the problem of channel selection in BI-CRNs as a non-cooperative game. Different from most existing game models, the proposed game belongs to local interaction games (also known as graphical game) [3], in which the utility of a user only depends on the action profile of its neighboring users rather than that of all the other users.

A. Graphical game model

The proposed MAC-layer interference minimization game is denoted as $\mathcal{G}_c = [\mathcal{N}, \{\mathcal{A}_n\}_{n\in\mathcal{N}}, \{\mathcal{J}_n\}_{n\in\mathcal{N}}, \{u_n\}_{n\in\mathcal{N}}]$, where $\mathcal{N} = \{1, \ldots, N\}$ is the set of players (CR users), $\mathcal{A}_n = \{m \in \mathcal{M} : C_{nm} = 1\}$ is the set of available actions (channels) for player n, $\mathcal{J}_n$ is the neighboring user set of player n, and u_n is the utility function of player n. Generally, the utility function is defined as $u(a_n, a_{-n})$, where $a_n \in \mathcal{A}_n$ is the channel selection of player n and $a_{-n} \in \mathcal{A}_1 \otimes \cdots \mathcal{A}_{n-1} \otimes \mathcal{A}_{n+1} \cdots \mathcal{A}_N$ denotes the channel selection profile of all the players except n. However, due to limited interference in BI-CRNs, the achievable throughput of a player is only dependent on its own action and the action profile of its neighboring player set; then, the utility function can be reduced to $u_n(a_n, a_{\mathcal{J}_n})$, where $a_{\mathcal{J}_n}$ is the selection profile of n's neighboring player set. Notably, lower MAC-layer interference is desirable for a CR user, which motivates us to define the utility function as follows:

$$u_n(a_n, a_{\mathcal{J}_n}) = L_n - c_n(a_n, a_{\mathcal{J}_n}), \tag{11}$$

where $c_n(a_n, a_{\mathcal{J}_n}) \equiv c_n(a_1, \ldots, a_N)$ represents the MAC-layer interference experienced by user n, as specified by (6). L_n is a predefined positive constant for player n which satisfies $L_n > |\mathcal{J}_n|$, where $|X|$ denotes the cardinality of set X. Then, the proposed game can be expressed as:

$$\mathcal{G}: \qquad \max_{a_n \in A_n} u_n(a_n, a_{\mathcal{J}_n}), \forall n \in \mathcal{N} \tag{12}$$

That is, each user in the game selfishly maximizes its individual utility. Thus, the questions remain to be answered include: (i) does the game have some stable states in the local interactive and competitive environment? (ii) if there exist some stable states, what are the achievable performance and how to achieve them in the absence of information exchange?

Remark 1: The reason for adding L_n in (11) is to keep the utility function positive, which makes the received payoffs compatible with the learning algorithms proposed later. Moreover, user n can choose L_n independently and autonomously, as long as the condition is satisfied.

B. Analysis of Nash equilibrium (NE)

In this subsection, we first define the Nash equilibrium (NE) of $\mathcal{G}$, which are the most well-known stable states in game models, and then investigate its properties.

Definition 1 (Nash equilibrium [31]):. A channel selection profile $a_{\text{NE}} = (a_1^*, \ldots, a_N^*)$ is a pure strategy NE if and only if no user can improve its utility by deviating unilaterally, i.e.,

$$u_n(a_n^*, a_{\mathcal{J}_n}^*) \geq u_n(\bar{a}_n, a_{\mathcal{J}_n}^*), \forall n \in \mathcal{N}, \forall \bar{a}_n \in \mathcal{A}_n, \bar{a}_n \neq a_n^* \tag{13}$$

Theorem 1. *$\mathcal{G}$ is an exact potential game which has at least one pure strategy NE point.*

Proof: To prove this theorem, we need to prove that there exists a potential function such that the change in individual utility function caused by any player's unilateral deviation is the same as the change in the potential function. We construct the potential function as follows:

$$\Phi(a_n, a_{-n}) = -\frac{1}{2} \sum_{n \in \mathcal{N}} c_n(a_1, \ldots, a_N). \tag{14}$$

Suppose that an arbitrary player n unilaterally changes its channel selection from a_n to $\bar{a}_n$. Following the similar lines given in [3], [4][5], it can be proved that the following equation holds:

$$\Phi(\bar{a}_n, a_{-n}) - \Phi(a_n, a_{-n}) = u_n(\bar{a}_n, a_{\mathcal{J}_n}) - u_n(a_n, a_{\mathcal{J}_n}), \tag{15}$$

which shows that $\mathcal{G}$ is an exact potential game. According to [12], exact potential game has at least one pure strategy NE point. Thus, Theorem 1 follows. ■

The achieved aggregate MAC-layer interference of a pure strategy NE point $a_{\text{NE}} = \{a_1^*, \ldots, a_N^*\}$ is given by:

$$U(a_{\text{NE}}) = \sum_{n \in \mathcal{N}} c_n(a_n^*, a_{\mathcal{J}_n}^*) \tag{16}$$

It is seen from (12) that the users in the game are selfish, which may lead to the well-known *tragedy of commons* [32]. Generally, $\mathcal{G}_c$ may have multiple pure strategy NE points and the number is hard to achieve [31]. The following theorems characterize the achievable performance bounds of the game.

Theorem 2. *For any network topology and spectrum opportunities, the aggregate MAC-layer interference at any NE point is bounded by $U(a_{NE}) \leq \sum_{n=1}^{N} \frac{|\mathcal{J}_n|}{|\mathcal{A}_n|}$.*

[5] For brevity, we omit the detailed proof in this article, which can be found in our previous work [3], [4].

Proof: For any pure strategy NE $a_{\rm NE} = (a_1^*, \ldots, a_N^*)$, the following inequality holds for any user n, $\forall n \in \mathcal{N}$:

$$c_n(a_n^*, a_{\mathcal{J}_n}^*) \leq c_n(\bar{a}_n, a_{\mathcal{J}_n}^*), \forall \bar{a}_n \in \mathcal{A}_n, \bar{a}_n \neq a_n^*, \quad (17)$$

which can be naturally obtained from the definition given in (13). Summing the two-sides of (17) yields the following:

$$|\mathcal{A}_n| \times c_n(a_n^*, a_{\mathcal{J}_n}^*) \leq \sum_{\bar{a}_n \in \mathcal{A}_n} c_n(\bar{a}_n, a_{\mathcal{J}_n}^*), \quad (18)$$

where $|\mathcal{A}_n|$ denotes the number of the available channels of user n. We can re-write (18) as follows:

$$c_n(a_n^*, a_{\mathcal{J}_n}^*) \leq \frac{\sum_{\bar{a}_n \in \mathcal{A}_n} c_n(\bar{a}_n, a_{\mathcal{J}_n}^*)}{|\mathcal{A}_n|}. \quad (19)$$

It is seen that $\sum_{\bar{a}_n \in \mathcal{A}_n} c_n(\bar{a}_n, a_{\mathcal{J}_n}^*)$ represents the experienced MAC-layer interference of user n as if it would transmit on all available channels simultaneously while its neighbors still only transmit on one channel. This immediately implies the following equation:

$$\sum_{\bar{a}_n \in \mathcal{A}_n} c_n(\bar{a}_n, a_{\mathcal{J}_n}^*) = |\mathcal{J}_n|, \quad (20)$$

where $|\mathcal{J}_n|$ is the number of neighboring users of user n. Accordingly, it follows that

$$U_{NE}(a^*) = \sum_{n \in \mathcal{N}} c_n(a_n^*, a_{\mathcal{J}_n}^*) \leq \sum_{n=1}^{N} \frac{|\mathcal{J}_n|}{|\mathcal{A}_n|}, \quad (21)$$

which proves Theorem 2. ■

Theorem 2 characterizes the upper bound of achieved performance at any NE point. It is shown that in order to achieve lower aggregate MAC-layer interference, larger number of available channels ($|\mathcal{A}_n|$) and smaller number of neighboring users ($|\mathcal{J}_n|$) are preferable. In particular, this bound can be further refined in some special kinds of systems.

Proposition 1. *In a network with all the channels being available to each user, the aggregate MAC-layer interference at any NE point is bounded by* $U(a_{NE}) \leq \frac{2N}{M}$.

Proof: All the channels being available to each user implies $|\mathcal{A}_n| = M, \forall n \in \mathcal{N}$. Consequently, we have:

$$U(a_{\rm NE}) \leq \sum_{n=1}^{N} \frac{|\mathcal{J}_n|}{M}, \quad (22)$$

which is obtained by applying Theorem 2 directly. Moreover, it can be verified that

$$\sum_{n=1}^{N} |\mathcal{J}_n| = 2N \quad (23)$$

is true for any network topology. Substituting (23) into (22) proves this proposition. ■

According to the refined result characterized by Proposition 1, large number of licensed channels can lead to low aggregate MAC-layer interference, as can be expected. The reason is that as the number of channels increases, the users can choose different channels to avoid mutual interference. The above analysis gives lower bounds of the performance, while we investigate its best achievable performance in the following.

Theorem 3. *The best pure strategy NE point of $\mathcal{G}$ is a global minimum of problem P1.*

Proof: The defined potential function and the aggregate MAC-layer interference are connected by $\Phi(a_n, a_{-n}) = -\frac{1}{2} I_g(a_n, a_{-n})$. According to (10), we then have:

$$a^{opt} \in \arg\max\ \Phi(a_n, a_{-n}). \quad (24)$$

That is, the optimal channel selection profile that minimizes the aggregate MAC-layer interference also maximizes the potential function. According to the properties of potential game, it is known that any global or local maximizer of the potential function constitutes a pure strategy NE point [12]. Thus, we can see that the best pure strategy NE point is a global minimum of $P1$, which proves this theorem. ■

Theorem 3 shows that the best NE point of $\mathcal{G}$ lies in the global minimum of the formulated MAC-layer interference minimization problem $P1$. This result is interesting and promising, since the global optimality emerges as the result of distributed and selfish decisions.

VI. Uncoupled Learning Algorithms for Achieving Best Nash Equilibrium

With the distributed channel selection problem now formulated as an exact potential game, there are large number of learning algorithms available in the literature to achieve pure strategy NE, e.g., best response dynamic [12], spatial adaptive play [3], and fictitious play [13]. These existing algorithms, however, belong to coupled algorithms which require to know the information of choosing actions and/or utilities of other users in each iteration. However, obtaining information of other players consumes resources (time, power or bandwidth), or it is not even feasible in some cases. In the following, we propose two uncoupled learning algorithms for the MAC-layer interference minimization problem.

Clearly, the only available information in the game is the individual action-payoff history of each player, which implies that utilizing this information to adjust the players' behaviors is the only approach[6]. Accordingly, the basic framework for the proposed learning algorithms is structured as follows: (i) all the users adhere to their channel selections in a certain amount of time, which is referred to as a decision period, and then estimate their individual utility function, (ii) based on the individual action-utility information, the users update their channel selections according to some learning rules.

A. Log-linear learning based uncoupled algorithm

The proposed log-linear learning based uncoupled algorithm is described in Algorithm 1. The key idea is that a player is randomly and autonomously selected to update its channel selection in an iteration; furthermore, the selected player randomly explores other channels, based on which it determines its channel selection stochastically in the next iteration.

In the player selection step of Algorithm 1, the selection of an autonomous player can be implemented through 802.11

[6]One would also apply some swarm intelligence inspired algorithms, which have been shown to be the most promising approach in distributed wireless networks [33].

Algorithm 1: *log-linear learning based uncoupled algorithm*

Initialization: Set the iteration $k = 0$, and let each player i, $\forall i \in \mathcal{N}$, randomly select a channel $a_i(1) \in A_i$.
Loop for $k = 0, 1, 2, \ldots,$
1. Player selection: A player, say n, is randomly selected in an autonomous fashion through a common control channel. Then, all the users adhere to their selection in an estimation period and the selected user estimates its received utility $u_n(k)$.
2. Exploration: The selected player n randomly chooses a channel $m \in \mathcal{A}_n$ with equal probability $\frac{1}{|\mathcal{A}_n|}$, where $|\mathcal{A}_n|$ is the number of available channels. The users adhere to their selections in an estimation period and the selected player estimates its received utility in channel m, denoted as v_m.
3. Updating channel selection: The selected player n updates its channel selection according to the following rule:

$$\begin{aligned} \Pr[a_n(k+1) = m] &= \frac{\exp\{v_m \beta\}}{Y} \\ \Pr[a_n(k+1) = a_n(k)] &= \frac{\exp\{u_n(k)\beta\}}{Y}, \end{aligned} \tag{25}$$

where $Y = \exp\{v_m\beta\} + \exp\{\hat{u}_n(k)\beta\}$ and β is a learning parameter. Meanwhile, all other players keep their selections unchanged, i.e., $a_{-n}(k+1) = a_{-n}(k)$.
End loop

DCF like contention mechanisms over a common control channel (CCC) [3], [19]. The stop criterion can be one of the following: (i) the maximum iteration number is reached, (ii) the variation of the network utility during a period is trivial.

Denote $\mathcal{A}$ as the set of available channel profiles of all nodes, i.e., i.e., $\mathcal{A} = \mathcal{A}_1 \otimes \cdots \otimes \mathcal{A}_N$. The properties of Algorithm 1 are determined by the following theorems.

Theorem 4. *In the log-linear learning based uncoupled algorithm, the unique stationary distribution $\mu(a) \in \Delta(\mathcal{A})$ of any channel selection profile $a \in \mathcal{A}$, $\forall \beta > 0$, is given as:*

$$\mu(a) = \frac{\exp\{\beta\Phi(a)\}}{\sum_{s\in\mathcal{A}} \exp\{\beta\Phi(s)\}}, \tag{26}$$

where $\Phi(\cdot)$ is the potential function given in (14).

Proof: The following proof follows similar lines of proof given in [3], [34]–[36].

Firstly, denote the channel selection state at the kth iteration by $s(k) = \{s_1(k), \ldots, s_N(k)\}$, where $s_n(k)$ is the action selection of node n. It is seen that $s(k)$ is only determined by $s(k-1)$, which means that $a(k)$ is a discrete time Markov process. Furthermore, it has a unique stationary distribution since it is irreducible and aperiodic [34].

Let us consider any two arbitrary network states at any two successive iteration, which are denoted as a and b respectively. Denote the transition probability from a to b by $P_{ab} = \Pr[s(k+1) = b|s(k) = a]$. According to the procedure of Algorithm 1, there is at most one element that can be changed between a and b. Therefore, we only consider two nontrivial cases: (i) $a = b$, or (ii) a and b differ by one element.

Second, we show that the unique distribution of Algorithm 1 must be (26) by verifying that the distribution (26) satisfies the following balanced equations [35]:

$$\mu(a)P_{ab} = \mu(b)P_{ba} \tag{27}$$

The above equation always holds for $a = b$. Then, we focus on the case of $a \neq b$. Specifically, suppose that a and b differ by the nth element. Since an arbitrary node n has probability $1/N$ of being chosen to update in any iteration and any channel m has probability $1/|\mathcal{A}_n|$ of being chosen to transmit, it follows that:

$$\begin{aligned} \mu(a)P_{ab} = &\left[\frac{\exp\{\beta\Phi(a)\}}{\sum_{s\in\mathcal{A}}\exp\{\beta\Phi(s)\}}\right] \\ &\times\left[\left(\frac{1}{N|\mathcal{A}_n|}\right)\left(\frac{\exp\{u_n(b)\beta\}}{\exp\{u_n(a)\beta\}+\exp\{u_n(b)\beta\}}\right)\right], \end{aligned} \tag{28}$$

where the iteration index k is omitted for presentation and $u_n(a)$ is the utility function of node n under the network state a.

Setting

$$\lambda = \frac{(1/N)(1/|\mathcal{A}_n|)}{\Big(\sum\limits_{s\in\mathcal{A}}\exp\{\beta\Phi(s)\}\Big)\Big(\exp\{u_n(a)\beta\} + \exp\{u_n(b)\beta\Big)}, \tag{29}$$

we obtain the following equation:

$$\mu(a)P_{ab} = \lambda \exp\{\beta\Phi(a) + \beta u_n(b)\} \tag{30}$$

Due to symmetry, we also have

$$\mu(b)P_{ba} = \lambda \exp\{\beta\Phi(b) + \beta u_n(a)\}. \tag{31}$$

Considering that $\Phi(a) - \Phi(b) = u_n(a) - u_n(b)$, as specified by (15), equations (30) and (31) immediately yield the balanced equation (27). Thus, we have

$$\sum_{a\in\mathcal{A}} \mu(a)P_{ab} = \sum_{a\in\mathcal{A}} \mu(b)P_{ba} = \mu(b)\sum_{a\in\mathcal{A}} P_{ba} = \mu(b), \tag{32}$$

which is exactly the balanced stationary equation of the Markov process $s(k)$ [34]. Then, its stationary distribution is characterized by (26) since Algorithm 1 has a unique stationary distribution and the distribution given by (26) satisfies the balanced equations of its Markov process [34], [35]. Therefore, Theorem 4 is proved. ∎

Note that the stationary distribution of Algorithm 1, as characterized by Theorem 4, is the same with that of an existing coupled algorithm with explicit information exchange between neighboring players [3]. This result is very interesting since the latter has been proved to admit asymptotic optimality.

Theorem 5. *With a sufficiently large β, Algorithm 1 asymptotically minimizes the aggregate MAC-layer interference I_g.*

Proof: Based on Theorem 4 and the methodology presented in [3] (see Theorem 4 therein), it can be proved that with a sufficiently large β, Algorithm 1 asymptotically converges to an action profile that maximizes the potential function of a potential game[7]. Now, applying again the property $\Phi(a) = -\frac{1}{2}I_g(a)$, Theorem 5 can be obtained. ∎

[7]We only apply this statement in this article, but detailed proof can be found in [3].

Algorithm 2*: learning-automata based uncoupled algorithm*

Initialization: set $k = 0$ and the initial channel selection probability vector to $q_{nm}(0) = 1/|\mathcal{A}_n|, \forall n \in \mathcal{N}, \forall m \in \mathcal{A}_n$.
Loop for $k = 0, 1, 2, \ldots,$
1. Selecting channels stochastically: Each CR player n selects a channel $a_n(k)$ according to its current channel selection probability vector $\mathbf{q}_n(k)$.
2. Estimating payoffs: All the CR players adhere to their selections in a decision period. After that, they estimate the current received payoff $u_n(k)$.
3. Updating mixed strategy: All the CR users update their mixed strategies according to the following rules:

$$\mathbf{q}_n(k+1) = \mathbf{q}_n(k) + br_n(k)\big(\mathbf{I}_{a_n(k)} - \mathbf{q}_n(k)\big), \tag{33}$$

where $\mathbf{I}_{a_n(k)}$ a unit vector with the $a_n(k)$th component unity, $0 < b < 1$ is the learning step size and $r_n(k)$ is the normalized received payoff defined as follows:

$$r_n(k) = u_n(k)/L_n. \tag{34}$$

End loop

It is seen that Algorithm 1 is distributed and uncoupled, since it is only relying on the individual action-utility history of each user. However, the procedure of player selection needs a CCC, which may not be available in some scenarios. This motivates us to propose the second uncoupled algorithm which is suboptimal but does not require a CCC.

B. Learning-automata based uncoupled algorithm

First, we extend $\mathcal{G}$ to a mixed strategy form [31]. Let the mixed strategy for player n at iteration k be denoted by a vector probability distribution $\mathbf{q}_n(k) \in \Delta(\mathcal{A}_n)$, where $\Delta(\mathcal{A}_n)$ denotes the set of probability distributions over the available action set $\mathcal{A}_n$.

The mixed strategy is repeatedly played in every iteration. Then, the users adopt a learning-automata rule [37], [38] to update their mixed strategies based on the estimated received payoffs. This process is repeated until the mixed strategies converge or other stop criterion is met. Specifically, the learning-automata based uncoupled algorithm is described in Algorithm 2 and the stop criterion can be one of the following [9]: (i) the maximum iteration number is reached, or (ii) for each player n, $\forall n \in \mathcal{N}$, there is a component of the channel selection probability sufficiently approaching one, say 0.99.

Theorem 6. *With a sufficiently small step size b, the learning-automata based uncoupled algorithm converges to a pure strategy NE point of $\mathcal{G}$, and hence minimizes the aggregate MAC-layer interference I_g globally or locally.*

Proof: In our recent work [9], it was proved that the learning-automata based algorithm converges to a pure strategy NE point of any exact potential game as the step size b goes sufficient small. Thus, although the analysis presented in [9] is originally for a global interactive game, Algorithm 1 converges to a pure strategy NE point of $\mathcal{G}$ since it is an exact potential game. Now, applying again the connection between the MAC-layer aggregate interference minimization problem $P1$ and the pure strategy NE points of $\mathcal{G}$, Theorem 6 is proved. ■

It is seen that Algorithm 2 is also distributed and uncoupled, since it is only relying on the individual action-utility history of each user. However, it may converge to an optimal or sub-optimal pure strategy NE point, which means that Algorithm 2 averagely achieves the suboptimal solution for the aggregate MAC-layer interference minimization problem $P1$.

Although the convergence of Algorithm 2 seems straightforwardly proved by applying the sufficient condition proposed in [9], it is emphasized that this is due to the careful design of utility function in this article. Specifically, traditional learning automata requires positive payoff, while the received payoffs in the MAC-layer interference minimization game $\mathcal{G}_c$ would be negative if we do not add a constant L_n, as characterized by (11). Furthermore, our previous work [9] mainly focused a global interactive game in dynamic and unknown environment, while the game investigated in this article is local interactive.

C. Distributed implementation and complexity analysis

For a better understanding of the proposed uncoupled learning algorithms, we discuss some issues related to distributed implementation and analyze their complexity cost. Denote the number of convergence iterations of the two algorithms as N_{it1} and N_{it2} respectively. Table I shows the number of channel switching, the storage size and the complexity.

For both Algorithms 1 and 2, the complexity of estimating the number of interfering users can be expressed as $\mathcal{O}(C_1)$, where C_1 is a constant determined by the duration of the estimation period and the used estimation approaches. Since an iteration of Algorithm 1 consists of two estimation period, its complexity is given by $\mathcal{O}(2C_1)$. The procedure of updating channel selection in Algorithm 1 only involves the operations of 2 exponents, 1 sum and 2 divisions. Thus, its complexity can be expressed as $\mathcal{O}(C_2)$, where C_2 is a small constant. In addition, the procedure of user selection has complexity of $\mathcal{O}(C_3)$, where C_3 is also a small constant. Thus, the total complexity of Algorithm 1 can be expressed as:

$$\mathcal{C}_{\text{Algorithm1}} = N_{it1}\big(\mathcal{O}(2C_1) + \mathcal{O}(C_2) + \mathcal{O}(C_3)\big) \tag{35}$$

In addition, since an iteration of Algorithm 1 consists of two estimation periods, the number of channel switching of Algorithm 1 is upper bounded by $2N_{it1}$.

Since the procedure of updating the channel selection vectors of Algorithm 2 involves the operations of 2 vector-vector sums and 1 scalar-vector product, its complexity can be expressed as $\mathcal{O}(2M+1)$, where M is the number of licensed channels. Thus, the total complexity of Algorithm 2 can be expressed as:

$$\mathcal{C}_{\text{Algorithm2}} = N_{it2}\big(\mathcal{O}(C_1) + \mathcal{O}(2M+2)\big) \tag{36}$$

It is shown that the complexity of Algorithm 2 scales with $2M$. In addition, since an iteration of Algorithm 2 consists of exactly one estimation period, the number of channel switching of Algorithm 2 is upper bounded by N_{it2}.

It is noted that the complexities of both algorithms are irrespective of N, i.e., the number of CR users. The reason is

TABLE I
COMPLEXITY ANALYSIS OF THE TWO UNCOUPLED ALGORITHMS.

	Number of Channel Switching	Computation	Operation	Storage Size	Complexity
Algorithm 1	$\leq 2N_{it1}$	$u_n(k), v_m$	—	—	$\mathcal{O}(2C_1)$
		$\Pr[a_n(k+1)=m]$ and $\Pr[a_n(k+1)=a_n(k)]$	2 exponents, 1 sum, 2 divisions	2	$\mathcal{O}(C_2)$
Algorithm 2	$\leq N_{it2}$	$u_n(k)$	—	—	$\mathcal{O}(C_1)$
		$\mathbf{q}_n$	(a). 2 vector-vector sums (b). 1 scalar-vector product	$2M+2$	$\mathcal{O}(2M+2)$

that they are uncoupled, as emphasized before. In comparison, Algorithm 2 has relatively heavy complexity since it evolves vector operations, while Algorithm 1 leads to relatively large number of channel switching since it has two estimation periods in an iteration.

D. Comparison of the two uncoupled algorithms

The two distributed learning algorithms are uncoupled. Specifically, the users observe the environment, learn from history information and then adjust their behavior towards a desirable solution (NE points in game models). This makes them suitable for distributed networks and scalable for large networks. Furthermore, the comparison results of the two algorithms are summarized as follows:

- Algorithm 1 asymptotically minimizes the aggregate MAC-layer interference, while Algorithm 2 averagely achieves suboptimal solutions.
- Algorithm 1 needs a common control channel to assist learning scheduling, while Algorithm 2 does not.
- Algorithm 1 is implemented in sequence, i.e., only a CR user learns at a time, while Algorithm 2 is implemented in parallel, i.e., all the CR players learn simultaneously.

Based on the above comparison, it is clear that the choice of the algorithm should be scenario-dependent. Furthermore, there exists speed-accuracy conflict for the two algorithms, which implies that the choice of the learning parameters (β in Algorithm 1 and b in Algorithm 2) should be application-dependent and be chosen by practical experiment or training.

Remark 2: Besides the MAC-layer interference metric, there are also many other optimization metrics designed for distributed opportunistic spectrum access, for example, reducing PU disruptions, reducing number of channel switching and maximizing expected idle time on a channel. These metrics can lead to quite different behaviors and SU/PU performance. In essence, the proposed uncoupled algorithms are suitable for optimization problems which can be formulated as potential games. Thus, the method of formulating these optimization metrics as potential games and applying the proposed algorithms is interesting and would be studied in future.

VII. SIMULATION RESULTS AND DISCUSSION

In this section, simulation results are presented to validate the proposed game-theoretic channel selection algorithms. Although the proposed algorithms are only theoretically analyzed for scenarios with no fading, it is shown that they are not only suitable for scenarios with no fading but also for scenarios with fading.

A. Scenario setup

In the simulation study, the CR users are randomly located in a region. We construct CRNs using MATLAB. It is assumed that the licensed channels are independently occupied by PUs with the same probability θ, $0 < \theta < 1$; and the spectrum opportunities are randomly generated according to the occupied probability. However, note that the spectrum opportunities vary slowly in time, or are at least static during the convergence of the algorithms. To capture this, we assume that different channels support the same transmission rate $R = 1\text{Mbps}$ for the users.

It is assumed that the CR users use the perfect CSMA/CA to share the idle channels. Specifically, time is divided into slots with equal length and a user with the minimum backoff time accesses an idle channel. The throughput of a user is calculated by $\frac{R}{c_n+1}$, which is approximately obtained by setting the throughput loss function as $f(c_n+1) \approx 1$ in (5).

We propose the following method to estimate the MAC-layer interference experienced by a user[8]. Specifically, suppose that each estimation period consists of H slots and denote T_n as the number of slots in which user n successfully access the channel. According to the principle of perfect CSMA/CA, the maximum likelihood estimation (MLE) of the MAC-layer interference experienced by user n is given by:

$$\hat{s}_n = \frac{H}{T_n} - 1, \tag{37}$$

which also means that MLE of the received utility in an estimation period is given by:

$$\hat{u}_n = L_n + 1 - \frac{H}{T_n} \tag{38}$$

B. Scenario with no fading

In this subsection, we simulate scenarios with no fading, where only the simplified binary-interference is considered. The learning parameter in Algorithm 1 is set to $\beta = 10 + k/50$, where k is the iteration number. The step size in Algorithm 2 is set to $b = 0.05$, which has been optimized by experiment. In all simulations, the estimation period is set to $H = 100$.

1) Convergence behavior: We study a small network involving nine CR users and three licensed channels as shown in Fig. 3. We consider a scenario with all channels being available for the users. The reason for considering such a scenario is that it provides with the same spectrum opportunities and thus we can investigate the expected convergence

[8] Other methods for estimating the number of competing users can also be found in the literature, e.g., [39], [40]. We use this simple method since we focus on uncoupled channel selection algorithms but not those estimation algorithms

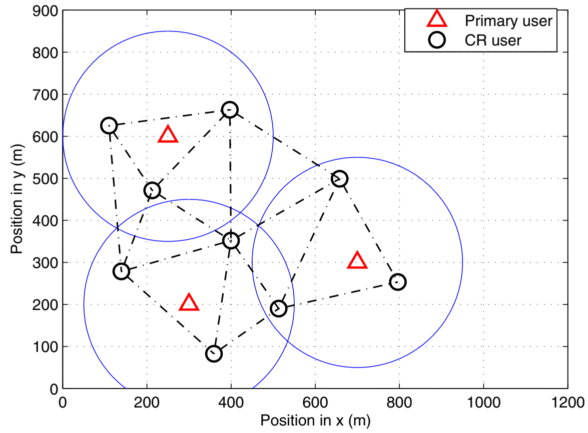

Fig. 3. The simulated small CRN. (Each circle represents a CR link, and the dashed lines represent the bilateral interferences between the CR users).

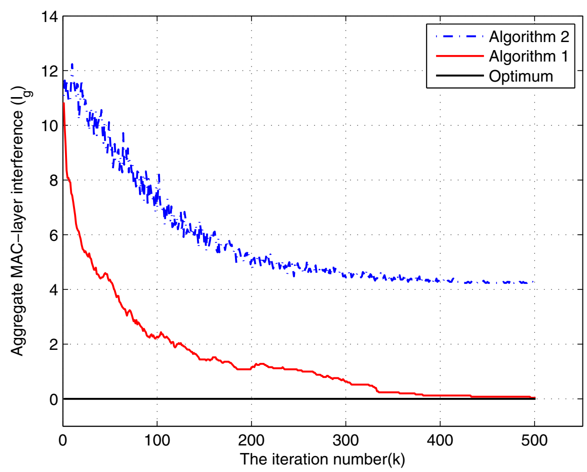

Fig. 4. The expected convergence behaviors of the two algorithms for the simulated CRN.

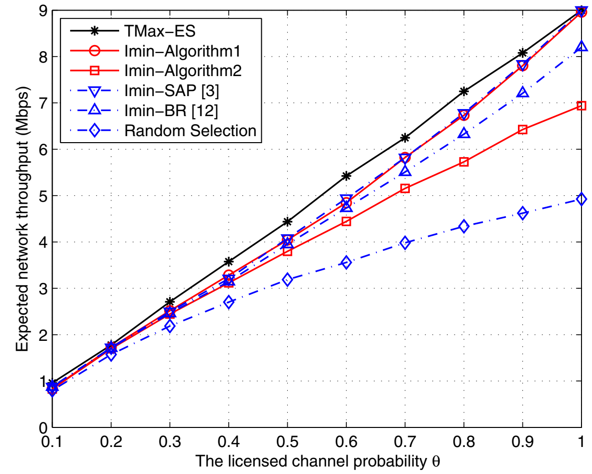

Fig. 5. Companion results of six channel selection methods for the simulated small network with no fading (For the learning algorithms, we set $\beta = 10 + k/50$ for Algorithm 1 and $b = 0.05$ for Algorithm 2).

behavior of the algorithms by taking the average results of several independent trials. In other simulation studies, the occupancy of PUs is considered. For the simulated bilateral interference CRNs, the expected convergence behaviors of Algorithms 1 and 2 are shown in Fig. 4. The results are obtained by simulating 1000 independent trials and then taking the average. It is noted that Algorithm 1 converges in about 300 iterations, while Algorithm 2 converges in about 400 iterations. Furthermore, Algorithm 1 asymptotically minimizes the aggregate MAC-layer interference while Algorithm 2 converges to some suboptimal solutions. The results validate the asymptotical optimality of Algorithm 1, and the convergence of both Algorithms.

2) Throughput performance: In this subsection, we compare the throughput performance of six methods, which are throughput maximization using exhaustive search (TMax-ES), random selection, interference minimization using the proposed Algorithm 1 (Imin-Algorithm1), interference minimization using the proposed Algorithm 2 (Imin-Algorithm2), interference minimization using spatial adaptive play (Imin-SAP) [3] and interference minimization using best response (Imin-BR) [12]. In the TMax-ES, the aggregate network throughput specified by (8) is directly maximized using an exhaustive search in a centralized manner. In the random selection scheme, each CR user chooses a channel from its available channel set with equal probability. SAP and BR are two well-known coupled learning algorithms for potential games; specifically, they need the information of actions chosen by other users in each iteration. Moreover, SAP asymptotically achieves the global maxima of the potential function while BR achieves its global or local maxima.

i) Small networks The comparison results of expected network throughput for the small network (see Fig. 3) are shown in Fig. 5. It is noted that Algorithm 1 achieves higher network throughput than Algorithm 2; moreover, both algorithms always perform better than the random selection, and the throughput gap increases as the licensed channel idle probability θ increases. The reason is that it converges to a pure strategy NE of the game, in which the users are spread over different channels and hence it leads to less collision. In other words, this is due to the fact that all pure strategy NE points of the game minimize the aggregate MAC-layer interference globally or locally as characterized by Theorems 1, 2 and 3.

It is noted that Algorithm 1 achieves the same performance with that of Imin-SAP, while there is a slight gap between Algorithm 2 and Imin-BR. These results are interesting since SAP asymptotically maximizes the potential function [3], i.e., minimizing the aggregate MAC-layer interference in our game. These results validate the usefulness of the two proposed uncoupled algorithms, especially considering the fact that both of them do not need information exchange.

Although Algorithm 1 asymptotically minimizes the aggregate MAC-layer interference, it is noted from Fig. 5 that there is a throughput gap between Imin-Algorithm1 and TMax-ES. The reason is that although a lower aggregate MAC-layer interference would lead to higher network throughput as can be expected, there lacks a quantitative characterization between minimizing the MAC-layer interference and maximizing the network throughput. In fact, such a characterization is dependent on network topology as well as spectrum opportunities and hence hard to obtain. Even so, it is shown

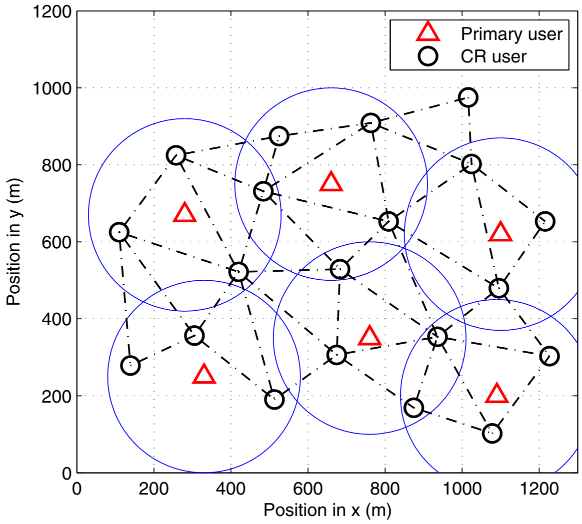

Fig. 6. The simulated large CRN.

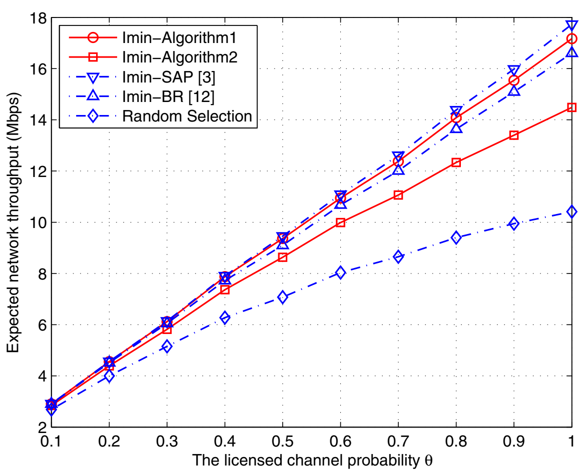

Fig. 7. Companion results of five methods for the simulated large network with no fading.

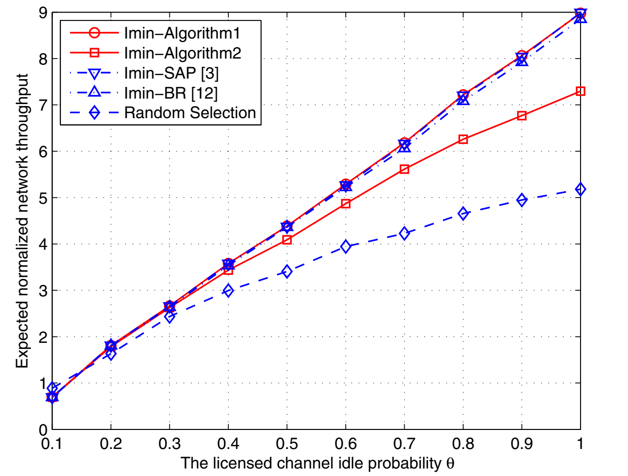

Fig. 8. Companion results for the simulated small network with Log-normal fading ($P_t = 1W$, $\beta = 2$ and $\sigma_{\mathrm{dB}} = 6$dB).

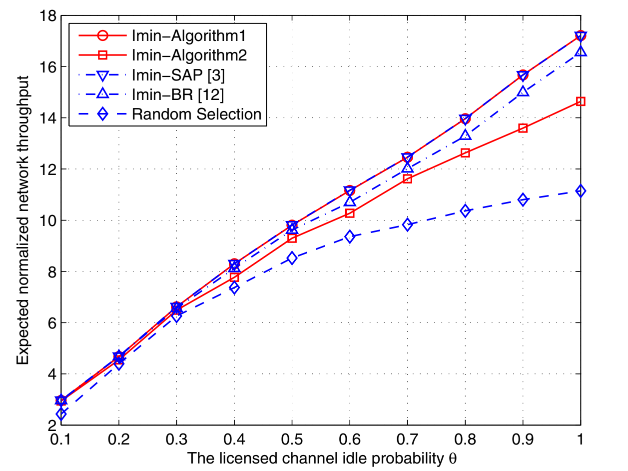

Fig. 9. Companion results for the simulated large network with Log-normal fading ($P_t = 1W$, $\beta = 2$ and $\sigma_{\mathrm{dB}} = 6$dB).

that the proposed two algorithms are desirable for realistic applications, as they both achieve higher network throughput.

ii) Large networks We consider a relatively large network consisting 20 CR users and three licensed channels as shown in Fig. 6. The comparison results of expected network throughput are shown in Fig. 7. Note that the TMax-ES method can not be applied due to intolerable complexity. It is noted that Algorithm 1 achieves nearly the same throughput performance with Imin-SAP, while the throughput gap between Algorithm 2 and both Imin-SAP and Imin-BR is small. These results validate that the proposed algorithms are not only suitable for small networks but also for large networks.

C. Scenario with fading

The above simulation results validate the proposed game-theoretic channel selection algorithms for scenarios without fading. However, every practical wireless channel has fading, which motivates us to consider the performance results for scenarios with fading. In this part, we consider the networks as shown in Fig. 3 and Fig. 6 for small and large networks respectively. As stated before, we consider Log-normal model fading in the simulation. Specifically, the transmitting power of all nodes is set to $P_t = 1W$, the path loss exponent is set to $\beta = 2$ and the dB-spread of Log-normal fading is set to 6dB. Moreover, the detection threshold of CSMA is set to $8.1633 \times 10^{-6} W$. It is shown that the proposed two algorithms also converge for practical channels with fading. Since their convergence behaviors are similar to those shown in Fig. 4, we do not present them here due to space limitation.

The comparison results for the achievable throughput of the small and large networks are shown in Fig. 8 and Fig. 9 respectively. It is noted that the same trends observed in Fig. 5 and Fig. 7 also holds in the scenarios with fading. Specifically, Algorithm 1 achieves higher throughput than Algorithm 2 and both Algorithms 1 and 2 outperform the random selection scheme. Moreover, Algorithm 1 achieves the same performance with that of Imin-SAP, while there is a slight gap between Algorithm 2 and Imin-BR. These results validate that although the presented mathematical analysis is just for scenarios with no fading, the proposed algorithms are suitable for scenarios with fading as well.

D. Discussion and extension to unilateral interference CRNs

As stated before, the learning parameters involve an accuracy-speed conflict. We also evaluated the effect of the learning parameters on the convergence and performance of the learning algorithms by simulation results, which are not presented here. The simulation results reveal that larger learning parameters lead to faster convergence speed, while resulting in higher aggregate MAC-layer interference, as can be expected in any learning algorithm.

In addition, we would like to point out that there are some scenarios involving unilateral interference relationships among the users, e.g., cognitive ad hoc networks. For those networks, the mutual interference relationships can be characterized by a directional graph rather than an undirected graph. We call this kind of networks, unilateral interference cognitive radio networks (UI-CRNs). Following the same methodology used for BI-CRNs, similar definitions for UI-CRNs can also be given. Then, a similar aggregate MAC-layer interference minimization game can be established accordingly. We also carried out simulation study for UI-CRNs. Simulation results show that the proposed two uncoupled algorithms also converge to pure strategy profile and achieve higher network throughput for UI-CRNs. Therefore, we point out that the proposed uncoupled algorithms are not only suitable for BI-CRNs, but also are suitable for UI-CRNs.

VIII. Conclusion

We investigated the problem of distributed channel selection for opportunistic spectrum access with spatial reuse, where multiple cognitive radio (CR) users are spatially located and mutual interference only emerges between neighboring users. Considering that the MAC-layer interference model is more desirable and useful for practical communications, we first proposed a MAC-layer interference minimization game. We proved that the proposed game is a potential game with the optimal Nash equilibrium (NE) point minimizing the aggregate MAC-layer interference. Although this result is promising, it is challenging to achieve a NE point without information exchange, not to mention the optimal one. We proposed two uncoupled learning algorithms, with which the CR users intelligently learn the desirable actions from their individual action-utility history. Specifically, the first algorithm asymptotically minimizes the aggregate MAC-layer interference and needs a common control channel to assist learning scheduling. The second one does not need a control channel and averagely achieves suboptimal solutions. The convergence and performance of the algorithms were analytically proved.

However, there are some issues that are still open and would be further investigated in future. First, although the proposed uncoupled algorithms do not need information exchange among CR users, they implicitly requires some mechanisms to assist learning scheduling, e.g., the update of player selection in Algorithm 1 and the notification of updating for all the players after every estimation period in Algorithm 2. Thus, fully distributed and asynchronous learning algorithms should be investigated. Second, analytical investigation on the real-valued MAC-layer interference model by taking into account the randomness of channel fading should be considered.

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