A Game-Theoretic Approach to Exploit Partially Overlapping Channels in Dynamic and Distributed Networks

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***Abstract*—In this letter, we investigate the partially overlapping channels for interference mitigation in dynamic and distributed networks. The interference mitigation problem is formulated as a dynamic game, which is proved to be an exact potential game. Furthermore, the performance bounds of the Nash equilibrium (NE) are theoretically derived and analyzed. Finally, we design a fully distributed, stochastic learning algorithm to converge to the NE of the formulated game. Simulation results validate the effectiveness of the proposed algorithm.**

***Index Terms*—Interference mitigation, partially overlapping channels, dynamic and distributed networks, potential game.**

I. Introduction

EFFICIENT channel allocation plays an important role in improving the performance of communications networks [1], [2]. Existing work mainly focuses on assigning non-overlapping channels (NOCs) to interfering nodes, in which different channels are completely orthogonal [3], [4]. However, the number of NOCs is limited, leading to low spectrum efficiency. Recently, more attention has been drawn into the investigation of partially overlapping channels (POCs) [5]–[8], which are not necessary to be orthogonal, but can increase the spectrum utilization as well as improve the network throughput [6]–[8].

In most work, the POCs are allocated in a centralized manner, using e.g., graph coloring [5] or genetic algorithm [6]. However, these approaches need a central controller for information exchange among users. Authors in [7], [8] solve this problem in a distributed manner by using game theory. Although their results are promising, they (as well as the existing research on channel allocation) assume that the communication nodes participate in the communication process all the time. In practice, nodes may not always need to occupy the channels for communication due to their specific service demands [2]. That is, the nodes participating in the competition for communication are variable. Although the dynamics on users has been addressed in [2], it studied the NOCs, without considering multiple access control mechanisms.

Manuscript received May 19, 2014; revised October 12, 2014 and October 17, 2014; accepted October 19, 2014. Date of publication October 30, 2014; date of current version December 8, 2014. This work was supported by the Project of Natural Science Foundations of China under Grant 61301163, Grant 61301162 and by the Jiangsu Provincial Nature Science Foundation of China under Grant BK20130067. The associate editor coordinating the review of this paper and approving it for publication was E. Liu.

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Digital Object Identifier 10.1109/LCOMM.2014.2366093

In this paper, we adopt game theory to exploit the POCs for interference mitigation in dynamic and distributed networks. Specifically, the main contributions of this paper are:

- A dynamic game is formulated. The change of the active nodes will lead to the change of players in the game model, which causes a large difference and intractability to the existing game framework.
- The dynamic game is proved to be an exact potential game. Moreover, the upper and lower performance bounds of the Nash equilibrium (NE) are both analyzed.
- A fully distributed and stochastic learning algorithm is designed to find the NE solution when the active players dynamically vary

II. System Model and Problem Formulation

This work studies an IEEE 802.11b-based canonical communication network consisting of multiple autonomous nodes. The set of nodes and the set of all available channels are denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$, $\mathcal{M} = \{1, 2, \ldots, M\}$, respectively. Each node chooses one channel for its communication. Recent measurement results in [6] report that interference are jointly determined by the following two factors: (i) the physical distance, and (ii) the channel separation. Let a_n and a_k denote the channel selected by two different nodes n and k, respectively. Let $\delta_{nk} = |a_n - a_k|$ denote their channel separation. The interference indicator α_{kn} between nodes k and n is thus given by [8]:

$$\alpha_{nk} = \begin{cases} 1, & d_{nk} \leq R_I(\delta_{nk}), \\ 0, & d_{nk} > R_I(\delta_{nk}), \end{cases} \tag{1}$$

where d_{nk} denotes the physical distance between nodes n and k, $R_I(\delta_{nk})$ denotes the interference range for a specific channel separation δ_{nk}. That is, given the channel separation of two different nodes δ_{nk}, they interfere with each other only when their physical distance d_{nk} is less than $R_I(\delta_{nk})$. According to [6], $R_I(\delta_{nk}) = 0$ when $\delta_{nk} \geq 5$, and $R_I(4) < R_I(3) < R_I(2) < R_I(1) < R_I(0)$. Then, the set of nodes that potentially interferes with n is denoted by $B_n = \{k \in \mathcal{N} : d_{nk} \leq R_I(0)\}$, which is also referred to as node n's neighbors.

Considering the specific service requirements of different nodes, we assume that nodes are active/inactive in probability at each time, and denote the active probability for node n by θ_n. In general, the active probabilities for different nodes are different due to their different service requirements, i.e., $\theta_n \neq \theta_m$ when $n \neq m$. Moreover, each node does not know the active probabilities of other nodes, since the considered network is distributed. We define a probability space as $(\Omega, \mathcal{H}, \mathbb{P})$, where Ω is a sample space, $\mathcal{H}$ is a minimal σ-algebra on subsets of Ω, and $\mathbb{P}$ is a probability measure on $(\Omega, \mathcal{H})$. Let ω denote an event in the sample space Ω. $\mathbf{C}(\omega) : \Omega \to 2^N$ is a random vector, where $\mathbf{C} = [c_n]_{\forall n \in \mathcal{N}}$, and $c_n \in \{0, 1\}$ denotes the state of node n (0 for silent, and 1 for active).

Notably, the experienced interference is a random variable in a slot and can vary from slot to slot due to the dynamic variation of active nodes. For a specific realization $\omega[t] \in \Omega$ at time t, the experienced interference (i.e., the number of neighboring users choosing the same channel) by node n under channel selection profile $a = (a_1, \ldots, a_N)$ is given by:

$$\hat{s}_n(a_n, a_{\mathcal{B}_n}, \omega[t]) = \sum_{k \in \mathcal{B}_n} c_k^t \alpha_{kn}, \tag{2}$$

where c_k^t denotes the state of node n at time t, and $a_{\mathcal{B}_n}$ denotes the channel selection profile of node n's neighbors. It is seen that the experienced interference of each node depends only on its own action and the action profile of its neighboring node set. To maximize network throughput, lower aggregate interference is preferable. Similar to [1], [2], [8], this work investigates the optimal channel selection from the perspective of minimizing interference, and a quantitative characterization of the aggregate interference experienced by all the users is expressed as:

$$\hat{I}(a_1, \ldots, a_N, \omega[t]) = \sum_{n \in \mathcal{N}} \hat{s}_n. \tag{3}$$

Then, in the dynamic case, the network-centric goal is to find an optimal channel selection a^{opt} that minimizes the expected network interference defined by $I(a_1, \ldots, a_N) = \mathbb{E}_{\mathbf{C}}[\hat{I}(a_1, \ldots, a_N, \mathbf{C})]$, where $\mathbb{E}_{\mathbf{C}}[\cdot]$ is the expectation operator. The optimization problem can be formulated by:

$$(P1): \quad a^{\text{opt}} \in \arg\min_{a \in \mathcal{A}} I, \tag{4}$$

where $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \ldots \times \mathcal{A}_N$ is the joint channel allocation strategy space, $\mathcal{A}_n$ denotes the set of available channels for node n, and $\times$ represents the Cartesian product. Notably, in the studied network, $\mathcal{A}_n = \mathcal{M}, \forall n \in \mathcal{N}$.

III. Interference Mitigation Game

In this section, distributed study on problem $P1$ is modeled as a dynamic game, where each node is a game player.

For a specific realization $\omega[t] \in \Omega$ at time t, the state-based utility function is defined as:

$$\hat{u}_n(a_n, a_{\mathcal{B}_n}, \omega[t]) = L - \hat{s}_n(a_n, a_{\mathcal{B}_n}, \omega[t]), \tag{5}$$

where $L > |\mathcal{B}_n|$ is a predefined constant for player n to guarantee the utility non-negative and compatible with the learning algorithm to be proposed later, where $|\mathcal{B}_n|$ denotes the cardinality of set $\mathcal{B}_n$. In the dynamic environment, we define the expected interference received by player n as $s_n(a_n, a_{\mathcal{B}_n}) = \mathbb{E}_{\mathbf{C}}[\hat{s}_n(a_n, a_{\mathcal{B}_n}, \mathbf{C})]$. Accordingly, the expected utility function of player n is given by:

$$u_n(a_n, a_{\mathcal{B}_n}) = \mathbb{E}_{\mathbf{C}}[\hat{u}_n(a_n, a_{\mathcal{B}_n}, \mathbf{C})] = L - s_n(a_n, a_{\mathcal{B}_n}), \tag{6}$$

which reflects the expected interference level experienced by node n in the dynamic environment.

We formulate the dynamic game as $\mathcal{G} = [\mathcal{N}, \mathbf{C}, \{\mathcal{A}_n\}_{n \in \mathcal{N}}, \{\mathcal{B}_n\}_{n \in \mathcal{N}}, \{u_n\}_{n \in \mathcal{N}}]$, where $\mathcal{A}_n = \mathcal{M}$ is the set of available actions (channels) for each player n. Then, the proposed dynamic game can be expressed as:

$$(\mathcal{G}): \quad \max_{a_n \in \mathcal{A}_n} u_n(a_n, a_{\mathcal{B}_n}), \forall n \in \mathcal{N}. \tag{7}$$

Definition 1 (Nash Equilibrium): A channel selection profile $a_{NE} = (a_1^*, \ldots, a_N^*)$ is a pure strategy NE if and only if no player can improve its utility by deviating unilaterally, i.e.,

$$u_n(a_n^*, a_{\mathcal{B}_n}^*) \geq u_n(a_n, a_{\mathcal{B}_n}^*), \quad \forall n \in \mathcal{N}, \forall a_n \in \mathcal{A}_n. \tag{8}$$

Theorem 1: $\mathcal{G}$ is an exact potential game which has at least one pure-strategy NE point, and the best pure-strategy NE point globally minimizes the expected aggregate interference.

Proof: It has been proved in [8] ((14) and (15) therein) that the following equation holds for an arbitrary realization $\omega[t]$:

$$\begin{aligned}\hat{s}_n(a_n', a_{\mathcal{B}_n}, \omega[t]) &- \hat{s}_n(a_n, a_{\mathcal{B}_n}, \omega[t]) \\ &= \frac{1}{2}\left[\hat{I}(a_n', a_{-n}, \omega[t]) - \hat{I}(a_n, a_{-n}, \omega[t])\right],\end{aligned} \tag{9}$$

where a_{-n} represents a channel selection profile of all the players excluding n. By taking expectation for the above equation, we can derive

$$u_n(a_n', a_{\mathcal{B}_n}) - u_n(a_n, a_{\mathcal{B}_n}) = \Phi(a_n', a_{-n}) - \Phi(a_n, a_{-n}), \tag{10}$$

where $\Phi(a_n, a_{-n}) = -\frac{1}{2} I(a_n, a_{-n})$.

According to the definition given in [10], it is known that $\mathcal{G}$ is an exact potential game with Φ serving as the potential function. Any global or local maxima of the potential function constitutes a pure strategy NE point of the game $\mathcal{G}$ [10], [11]. According to the relationship between the potential function Φ and the aggregate interference I, we can conclude that the best NE that maximizes the potential function also minimizes the aggregate interference. Therefore, Theorem 1 is proved. ∎

Theorem 1 gives the existence as well as the best achievable performance of NE. This result is interesting and promising, since the competitive and selfish decisions lead to global optimality. Next, we will further investigate the lower bound of the performance following the similar idea of proof in [8]. For convenience of analysis, we make further division of the $\mathcal{B}_n$ by defining $\mathcal{B}_n^{(0)} = \{k \in \mathcal{N} : R_I(1) < d_{nk} \leq R_I(0)\}$, $\mathcal{B}_n^{(1)} = \{k \in \mathcal{N} : R_I(2) < d_{nk} \leq R_I(1)\}$, $\mathcal{B}_n^{(2)} = \{k \in \mathcal{N} : R_I(3) < d_{nk} \leq R_I(2)\}$, $\mathcal{B}_n^{(3)} = \{k \in \mathcal{N} : R_I(4) < d_{nk} \leq R_I(3)\}$, and $\mathcal{B}_n^{(4)} = \{k \in \mathcal{N} : d_{nk} \leq R_I(4)\}$. Notably, $\mathcal{B}_n = \mathcal{B}_n^{(0)} \cup \mathcal{B}_n^{(1)} \cup \mathcal{B}_n^{(2)} \cup \mathcal{B}_n^{(3)} \cup \mathcal{B}_n^{(4)}$.

Theorem 2: For any network topology and spectrum opportunities, the aggregate interference at any NE point is bounded by $\frac{\sum_{n \in \mathcal{N}} \sum_{i=0}^{4} (2i+1) \sum_{k \in \mathcal{B}_n^{(i)}} \theta_k}{M}$.

Proof: According to the definition of utility function (6) and the definition of NE (8), for any pure strategy NE point $a_{NE} = (a_1^*, \ldots, a_N^*)$, the following inequality holds:

$$s_n(a_n^*, a_{\mathcal{B}_n}^*) \leq s_n(a_n, a_{\mathcal{B}_n}^*), \quad \forall n \in \mathcal{N}, a_n \in \mathcal{A}_n. \tag{11}$$

Summing the two-sides of (11) yields the following inequality:

$$|\mathcal{A}_n| \cdot s_n(a_n^*, a_{\mathcal{B}_n}^*) \leq \sum_{a_n \in \mathcal{A}_n} s_n(a_n, a_{\mathcal{B}_n}^*), \quad \forall n \in \mathcal{N}. \tag{12}$$

Under the assumption of ergodicity, we have

$$\begin{aligned}\sum_{a_n \in \mathcal{A}_n} s_n(a_n, a_{\mathcal{B}_n}^*) &= \sum_{a_n \in \mathcal{A}_n} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \hat{s}_n(a_n, a_{\mathcal{B}_n}^*, \omega[t]) \\ &= \sum_{a_n \in \mathcal{A}_n} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=0}^{4} \hat{s}_n^{(i)}\left(a_n, a_{\mathcal{B}_n^{(i)}}^*, \omega[t]\right) \\ &= \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=0}^{4} \sum_{a_n \in \mathcal{A}_n} \hat{s}_n^{(i)}\left(a_n, a_{\mathcal{B}_n^{(i)}}^*, \omega[t]\right).\end{aligned} \tag{13}$$

- According to (1), players in $\mathcal{B}_n^{(0)}$ cause co-channel interference to player n, which implies that

$$\sum_{a_n \in \mathcal{A}_n} \hat{s}_n^{(0)}\left(a_n, a_{\mathcal{B}_n^{(0)}}^*, \omega[t]\right) = \sum_{k \in \mathcal{B}_n^{(0)}} c_k^t. \tag{14}$$

- Players in $\mathcal{B}_n^{(i)}$, $i = 1, 2, 3, 4$, not only cause co-channel interference to player n, but also cause partially overlapping channel interference to player n. Thus,

$$\sum_{a_n \in \mathcal{A}_n} \hat{s}_n^{(i)} \left(a_n, a^*_{\mathcal{B}_n^{(i)}}, \omega[t]\right) \leq (2i+1) \sum_{k \in \mathcal{B}_n^{(i)}} c_k^t. \quad (15)$$

Applying (14) and (15), the following inequality holds:

$$\sum_{i=0}^{4} \sum_{a_n \in \mathcal{A}_n} \hat{s}_n^{(i)} \left(a_n, a^*_{\mathcal{B}_n^{(i)}}, \omega[t]\right) \leq \sum_{i=0}^{4} (2i+1) \sum_{k \in \mathcal{B}_n^{(i)}} c_k^t. \quad (16)$$

Substituting (16) into (13), we can derive

$$\sum_{a_n \in \mathcal{A}_n} s_n \left(a_n, a^*_{\mathcal{B}_n}\right) \leq \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=0}^{4} (2i+1) \sum_{k \in \mathcal{B}_n^{(i)}} c_k^t$$
$$= \sum_{i=0}^{4} (2i+1) \sum_{k \in \mathcal{B}_n^{(i)}} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} c_k^t = \sum_{i=0}^{4} (2i+1) \sum_{k \in \mathcal{B}_n^{(i)}} \theta_k, \quad (17)$$

which along with (12) yields

$$s_n \left(a^*_n, a^*_{\mathcal{B}_n}\right) \leq \frac{\sum_{i=0}^{4} (2i+1) \sum_{k \in \mathcal{B}_n^{(i)}} \theta_k}{|\mathcal{A}_n|}, \quad \forall n \in \mathcal{N}. \quad (18)$$

Accordingly, it follows that

$$I(a_{NE}) = \sum_{n \in \mathcal{N}} s_n \left(a^*_n, a^*_{\mathcal{B}_n}\right) \leq \frac{\sum_{n \in \mathcal{N}} \sum_{i=0}^{4} (2i+1) \sum_{k \in \mathcal{B}_n^{(i)}} \theta_k}{|\mathcal{A}_n|}, \quad (19)$$

where $|\mathcal{A}_n| \equiv M$. This concludes the proof.

Theorem 2 provides a generic analysis of the interference bound for any network topology, which is implicitly reflected by the set of neighboring nodes $\mathcal{B}_n^{(i)}$. Besides, Theorem 2 shows that to achieve lower aggregate interference, larger number of available channels is preferable, which coincides with the idea of employing POCs. In addition, lower active probabilities of nodes can achieve lower aggregate interference. It is like the time-division multiplexing that each node shares a small portion of time slots for channel access.

IV. Stochastic Learning in Dynamic and Distributed Network Topology

In this section, we design a stochastic learning automata (SLA) [12] based distributed algorithm to obtain NE in the dynamic environment where active players dynamically vary.

Algorithm 1: SLA Based Dynamic Channel Selection

Initialization: At time $t = 0$, initialize each player's channel selection probability vector as $\mathbf{p}_n^t = (p_{n1}^t, \ldots, p_{nM}^t) = \left(\frac{1}{M}, \ldots, \frac{1}{M}\right)$, where p_{nm}^t denotes the probability for n to select channel m at time t.

Loop for $t = 0, 1, 2, \ldots$

1) **Updating channel selection strategy**: At time t, each active player, say n, stochastically selects a channel a_n^t according to its current channel selection probability vector $\mathbf{p}_n^t$. The non-active players keeps their channel selection strategies unchanged.

2) **Estimating received payoff**: The game is played once by the active players, and then all the active players estimate the received utilities $\hat{u}_n^t$ using (2) and (5).

3) **Updating channel selection probability**: All the active players update their channel selection probabilities according to the following rules:

$$p_{nm}^{t+1} = p_{nm}^t + b\, r_n^t \left(1 - p_{nm}^t\right), \quad m = a_n^t$$
$$p_{nm}^{t+1} = p_{nm}^t - b\, r_n^t p_{nm}^t, \quad m \neq a_n^t, \quad (20)$$

where $0 < b < 1$ is the learning step size, r_n^t is the normalized received payoff defined as $r_n^t = \hat{u}_n^t / L$. The non-active players keeps their channel selection probabilities unchanged.

End loop until the variation of the aggregate network interference during a period is trival.

The proposed learning algorithm is fully distributed, since the updating rule specified by (20) only depends on the individual experienced action-reward, which can be computed by using the similar estimation approach in [8], [9]. Besides, as shown in [9], the proposed algorithm runs in a low complexity with $T_{it}(\mathcal{O}(C_1) + \mathcal{O}(2M+2))$, where T_{it} denotes the number of iterations needed for the algorithm convergence, and C_1 is a constant determined by the duration of the estimation period and the used estimation approaches.

Following the similar proof in [12], [13], we achieve the following theorem by utilizing the ordinary differential equation (ODE) and stochastic approximation theory. Due to the space limitation, the detailed process of proof is omitted.

Theorem 3: With a sufficiently small step size b, the proposed SLA based dynamic channel selection algorithm converges to a pure NE point of $\mathcal{G}$.

V. Simulation Results and Analysis

In simulations, we consider a canonical network where communication nodes are randomly scattered in a 1000 m-by-1000 m square area. The IEEE 802.11b standard with 2Mb/s data rate is applied. The interference range of co-channel communications is set to 200 m [6], and the interference ranges of different channel separations are given by $R_I(1) = 112.5$ m, $R_I(2) = 75$ m, $R_I(3) = 37.5$ m, $R_I(1) = 12.5$ m [8]. Without loss of generality, we assume that the active probabilities of all the nodes are equal to 0.6. In addition, the positive constant in (5) is set to be $L = 2$, and the step size of the proposed algorithm is set to be $b = 0.1$.

For convergence analysis of the proposed stochastic learning algorithm, we simulate a random network topology involving 60 nodes. The convergence behavior of one arbitrarily selected node is presented in Fig. 1. At the initial time, it randomly selects the channels with equal probabilities. As the algorithm iterates, its channel selection probabilities evolve with the time and converge in about 450 iterations. We can see P_{10} converges to 1 while other channel selection probabilities converge to 0, which indicates that the node finally choose channel 10 for communication.

Figs. 2 and 3 plot performance comparison for different solutions in terms of the expected aggregate interference and

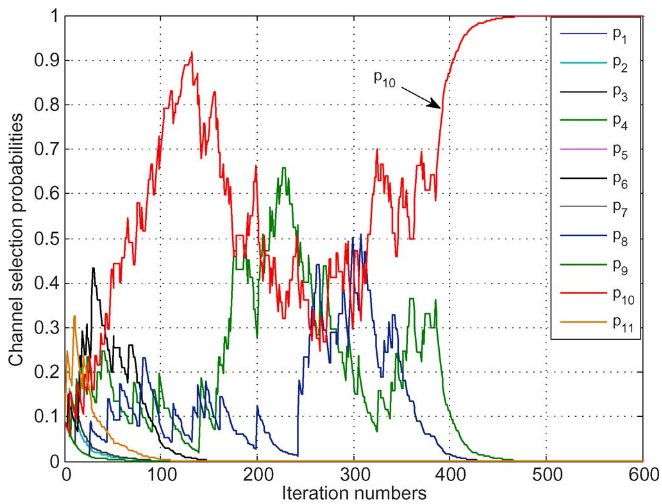

Fig. 1. Convergence behavior.

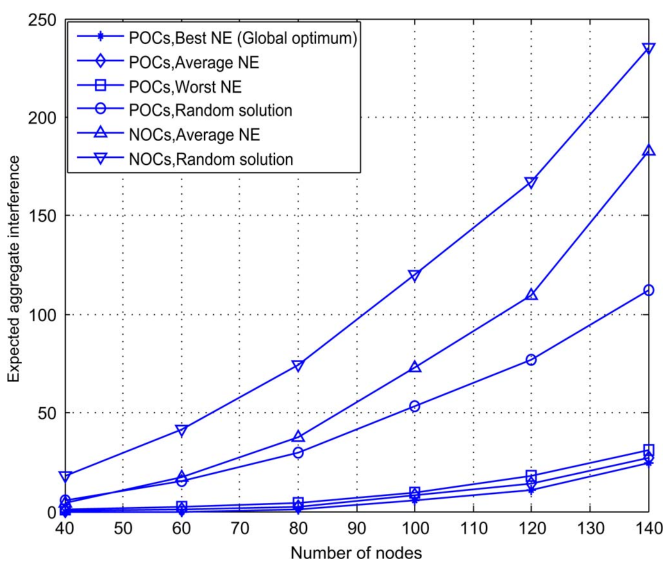

Fig. 2. Performance comparison of the expected aggregate interference.

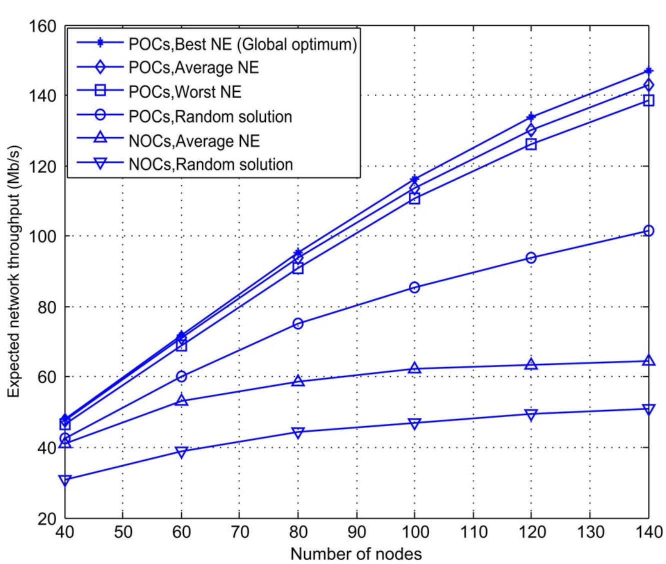

Fig. 3. Performance comparison of the expected network throughput.

expected network throughput, respectively. For the random solution, each node randomly chooses a channel in each time slot. Besides, the globally optimal solution is obtained by running the proposed algorithm for multiple times and taking the best NE solution, since Theorem 1 demonstrates the best NE is the global optimum. The presented results are obtained by simulating 1000 independent trials. Firstly, as the number of nodes increase, the expected aggregate interference obtained by all the solutions becomes more severe, and the network throughput decreases accordingly. Secondly, the solutions for POCs are much better than for NOCs, which proves the advantage of employing POCs. Thirdly, the NE achieved by our proposed algorithm outperforms the random solution. Moreover, the gap between the best NE (i.e., the global optimum) and the worst NE is small, as shown in both figures.

VI. Conclusion

In this letter, we investigated the distributed channel allocation in a canonical communication network with partially overlapping channels, in which the active users dynamically vary. This problem was formulated as a dynamic game, which was proved to be an exact potential game and the performance bounds of the NE solutions were derived and analyzed. Moreover, we designed a distributed stochastic learning algorithm to converge to the NE. Simulation results demonstrated the effectiveness of our proposed algorithm.

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