TABLE I
MEASURED AND SIMULATED RADIATION PATTERN PARAMETERS OF THE ARRAY WHEN HORN 1 IS EXCITED

Freq (MHz)	Gain (dBi)		SLL (E-plane) (dB)		HPBW (H-plane) (°)	
	Meas	Sim	Meas	Sim	Meas	Sim
900	8.4	8.8	-11.8	-10.9	84	86
1750	9.2	9.6	-10.7	-10.2	76	80
2170	9.8	10.1	-13.6	-11.9	73	82
2400	9.8	10.1	-15.1	-15.0	75	78
2490	12.0	11.1	-10.9	-17.6	72	70

V. CONCLUSION

A low-profile four-element array has been designed and fabricated for wideband and wide-coverage applications. Particularly, it is suitable for wide spectrum monitoring systems on moving platforms. Each H-plane horn fed by a thick-cylinder-loaded probe is chosen as the array element, owing to its attractive impedance matching feature. The measured frequency bandwidth for VSWR < 2 is about 100%. The method of inserting two pins in each horn reduces the weights of higher order modes and suppresses the E-field in the edge of the horn at high frequency. Thus, the H-plane radiation pattern improves, and the measured beamwidth is stable from 860 to 2500 MHz. The measured results prove the effectiveness of the proposed design guidelines.

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Hierarchical Decision-Making With Information Asymmetry for Spectrum Sharing Systems

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***Abstract*—In this paper, observing the information asymmetry phenomenon among multiple secondary users (SUs) spectrum sharing, we analyze the hierarchical decision-making and the strategic interaction of information-poor and information-rich SUs. A Stackelberg capacity-maximization game is formulated with leaders and followers, and closed-form solutions are mathematically derived for the optimal Stackelberg equilibrium solution. Moreover, the existence and uniqueness of equilibrium solutions are investigated via the quasi-variational inequality method. Finally, the distributed algorithm with partial asymmetric information awareness is designed to reach the solution. Numerical results demonstrate that the proposed algorithm achieves improved individual and system performance with mild condition on the ratio of information-poor leaders and information-rich followers.**

***Index Terms*—Cognitive radio, hierarchical decision making, spectrum sharing, Stackelberg game.**

I. INTRODUCTION

It has been recognized that the scarcity of the radio spectrum is mainly due to the inefficiency of traditionally static spectrum allocation policies [1]. Using the idea of the cognitive radio (CR), many advanced and autonomous spectrum management schemes have gotten attention from both academia and industry. From both technical and economic perspectives, if the operator or transmitter can attain the spectrum utilization or feedback information, then this side information can be exploited for capacity gain. For instance, Goldsmith and Varaiya [2]

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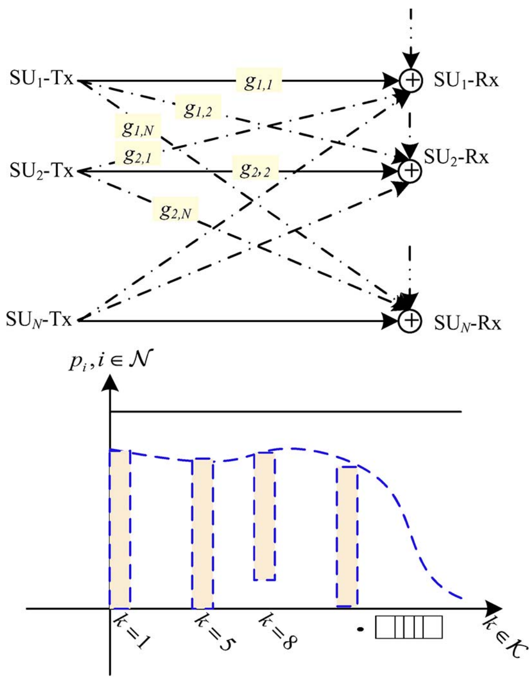

Fig. 1. (a) System model. (b) Power selection coupled with channel allocation.

obtained the Shannon capacity of a fading channel with channel side information at the transmitter and receiver. The optimal power adaptation in the former case is water pouring in time. The difference between this paper and the current literature is that we investigate the hierarchical decision-making and the strategic interaction of information-poor and information-rich secondary users (SUs), e.g., power control among SUs with asymmetric side information in CR networks.

One should pay attention to strategic behaviors and dynamic interactions among intelligent SUs in the multiuser spectrum sharing games. Multiple SUs coexist to sense and share the same spectrum holes in the CR networks. They may perceive different spectrum utilization status and mutual interference state information due to their specific geographical locations and cognition capabilities. This naturally causes the information asymmetry among multiple SUs, further leading to a short-sighted or foresighted spectrum sharing decision. Different SUs may select different available channels in which to pour power resources with their perceived and useful context-aware information. Such an asymmetric case deserves to be characterized and investigated in CR networks. In this paper, we concentrate on the analysis of hierarchical strategic decision-making behaviors of multiple SUs, with the aim of exploring and exploiting the diversity of information asymmetry to improve the performance of both information-rich and information-poor SUs.

II. System Model, Problem Formulation, and Initial Analysis

We consider a CR network where N SUs coexist and opportunistically share K channels with each other; here, we assume that $N \geq K$. Different capabilities of exploring and exploiting mutual interference information are assumed between information-rich and information-poor SU pairs. Each pair of secondary transmitter and receiver is defined as one SU pair, and we use SU and SU pair interchangeably in this paper. The spectrum sharing system model is illustrated in Fig. 1(a), where N SUs coexist with and are interfered with by a primary unit (PU) I_{PU} on K channels. Secondary users who share the same channel will interfere with each other. If SU_1–Tx and SU_2–Tx share the same channel, then $g_{1,1}$ and $g_{2,2}$ are communication channel gains, and $g_{1,2}$ and $g_{2,1}$ are mutual interference channel gains. Meanwhile, each SU–Rx has the background noise power σ^2 and the interference power I_{PU} from the PU system.

Different SU–Tx pairs will receive the relevant noise/interference information; however, leaders and followers make different use of this received information. For instance, in Fig. 1(b), we assume that SU_1–Tx is a leader and that SU_2–Tx is a follower. First, SU_1–Tx pours high power resources into the channel $k = 1, 5, 8$, only treating mutual interference as noise power, but followers can extract more useful information, which can assist in decision-making. SU_2–Tx will explore and exploit the mutual interference introduced by SU_1–Tx and then determine the power on its selected channels, e.g., $k = 5, 8$ not on channel $k = 1$ due to the fact that high power has been poured for SU_1–Tx. Therefore, the advantage of categorizing users into leaders and followers in the following formulated game is that followers can make more intelligent power control policy by exploring the side information of the perceived interference. This will help improve the network throughput and mitigate the critical interference in the network.

A. Utility Function

We study the capacity-maximization game among multiple SUs that share common channels, and we choose the utility function $u_l(\cdot)$ for $\text{SU}_l, l \in \mathcal{N} = \{1, 2, \ldots, l, \ldots N\}$ as the aggregate capacity over all of its selected channels [3]. Mathematically, $u_l(\cdot)$ represents

$$u_l(\mathbf{p}_l; \mathbf{p}_{-l}) = \sum_{k=1}^{K} \log_2 (1 + \gamma_l(k)) \quad \forall\, l \in \mathcal{N} \tag{1}$$

where $\mathbf{p}_l$ is the power vector of SU_l on all of its selected channels, and $\mathbf{p}_{-l}$ is the power vector of its opponents $-l = \{1, 2, \ldots l-1, l+1, \ldots N\}$. Moreover

$$\gamma_l(k) = \frac{p_l(k) g_{l,l}(k)}{\xi_l(k)} \tag{2}$$

is the received signal-to-interference-plus-noise ratio (SINR) of SU_l on its selected channel k, the value of which is determined by its own power $p_l(k)$, the communication channel gain between l's secondary transmitter to its receiver $g_{l,l}(k)$, and the aggregate interference plus noise $\xi_l(k)$. $\xi_l(k)$ captures all the interference from other SUs' power $\mathbf{p}_{-l}$, as well as the noise variance $\sigma_l^2(k)$. In the SINR $\gamma_l(k)$, the aggregate noise $\xi_l(k)$ in the denominator can be further expressed by

$$\xi_l(k) = \mathcal{I}_l^{\text{SU}}(k) + \mathcal{I}_l^{\text{PU}}(k) + \sigma_l^2(k) \tag{3}$$

where $\mathcal{I}_l^{\text{SU}}(k)$ represents the total interference from its opponents, and $\mathcal{I}_l^{\text{PU}}(k)$ is the total interference caused by all the primary users that is assumed fixed in the following analysis. Specifically, our analysis can be easily extended to the other forms of utility function (e.g., an energy-efficiency function).

B. Stackelberg Capacity-Maximization Game

Definition 1: The Stackelberg capacity-maximization game is defined as $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$, which has the following elements.

- Player set: $\mathcal{N} = \{1, 2, \ldots, N\}$ is divided into leader set $\mathcal{L}$ and the follower set $\mathcal{F}$, containing multiple leader SUs and multiple follower SUs, respectively. We set short-sighted SUs as leaders, where we assume that leaders are with some blindness when they make decisions. That is, leaders make quick decisions once obtaining the channel state information. However, foresighted SUs as followers will first analyze and then obtain the useful context information, which will be conducive to arrive at more intelligent decisions.

- Strategy set: $\mathcal{S} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_K$, which is the Cartesian product space, where $\mathcal{S}_l = \{p_l(k), k \in \mathcal{K}\}$ is the feasible strategy set of SU_l formed by non-negativity constraint and spectral mask constraints of primary users, where $\mathcal{K} = \{1, 2, \ldots k, \ldots, K\}$ is the feasible channel set.
- Utility function set: $\mathcal{U} = \{u_1, \ldots, u_l, \ldots u_N\}\}$, where the individual utility function $u_l(\mathbf{p}_l; \mathbf{p}_{-l})$ is given by (1).

The Stackelberg capacity-maximization game is carried out in two stages.

- First, leader $i \in \mathcal{L}$ begins to play a game and decides the individually optimal power $\mathbf{p}_i^\star$ by treating aggregated interference plus noise $\xi_i(k)$ from other players as noise.
- Then, follower $j \in \mathcal{F}$ reacts to the leader's power selections $p_i^\star(k)$ and selects its own more intelligent power $p_j^\star(k)$.

C. Initial Analysis: Strategic Design for Leaders

During the first stage of the Stackelberg capacity-maximization game, the power control game among leaders can be formulated as a noncooperative capacity-maximization game, i.e.,

$$\max_{\mathbf{p}_i} \quad u_i(\mathbf{p}_i; \mathbf{p}_{-i})$$

$$\text{subject to} \quad \sum_{k=1}^{K} p_i(k) \leq P_i^{\max} \tag{4a}$$

$$p_i(k) \in \mathcal{S}_i, \quad k \in \mathcal{K} \tag{4b}$$

where each leader SU_i, $i \in \mathcal{L}$ tries to maximize the capacity-like utility function (1) by implementing the feasible power $\mathbf{p}_i = \{p_i(k), k \in \mathcal{K}\}$. Here, $P_i^{\max}$ in (4a) limits the total transmission power of SU_i. $\mathcal{S}_i$ in (4b) is the feasible strategy set of SU_i formed by a nonnegativity constraint and spectral mask constraints of primary users. We assume that $\mathcal{S}_i$ is convex and compact in this paper.

Since leaders make decisions in the absence of any knowledge of the power strategies of followers or other leaders, each leader is self-interested and employs an optimal power strategy. The Nash equilibrium solution of the optimal power selection strategy $\mathbf{p}_i^\star = \{p_i^\star(k)\}_{k\in\mathcal{K}}$ exists and is unique for the rational leader SU_i, $i \in \mathcal{L}$—derivation is similar to water-filling [2]. It takes a closed form and is given by

$$p_i^\star(k) = \left[\frac{1}{\lambda_i} - \frac{\xi_i(k)}{g_{i,i}(k)}\right]^+$$

$$\triangleq \mathcal{F}_i\left(\lambda_i\left(\mathbf{p}_{-i}^\star(k)\right)\right) \tag{5}$$

where $\xi_i(k)$ is the perceived interference plus noise of the leader i. Lagrange multiplier λ_i is introduced by constraint (4a). Furthermore, we conclude that the optimal power of leader i is influenced by all the power values $\mathbf{p}_{-i}(k) := \{p_{i'}(k)\}_{i'\in\mathcal{L}\setminus i}$ of other leaders via λ_i, where the multiplier λ_i satisfies

$$0 \leq \lambda_i \perp \left(\sum_{k=1}^{K} p_i(k) - P_i^{\max}\right) \geq 0 \tag{6}$$

where the compact notion $\perp$ is shown as $0 \leq a \perp b \geq 0$, meaning that $a \cdot b = 0$, $a \geq 0$, and $b \geq 0$.

III. Asymmetric Observations, Analysis, and Strategy Design of Followers

At the second stage of the Stackelberg capacity-maximization game, foresighted followers form and solve another capacity-maximization game among themselves, based on the power selections $\{\mathbf{p}_i^\star\}_{i\in\mathcal{L}}$ of all leaders in advance. These first power allocations of short-sighted leaders will reembody in the form of interference introduced to foresighted followers.

For each user $j \in \mathcal{F}$, the set of interfering users $\mathcal{N} \setminus j$ can be partitioned into two subsets, i.e., $\mathcal{N} \setminus j = \mathcal{L} \cup (\mathcal{F} \setminus j)$, which correspond to two sets of power selections $\mathbf{p}^{\mathcal{L}} := \{\mathbf{p}_i\}_{i\in\mathcal{L}}$ of all leaders and $\mathbf{p}_{-j}^{\mathcal{F}} := \{\mathbf{p}_{j'}\}_{j'\in\mathcal{F}\setminus j}$ of all other followers, respectively. Apparently, $\mathbf{p}_{-j} = \{\mathbf{p}_{-j}^{\mathcal{F}}, \mathbf{p}^{\mathcal{L}}\}$. The following spectrum sharing problem is formulated for deciding $\mathbf{p}_j$:

$$\max_{\mathbf{p}_j} \quad u_j\left(\mathbf{p}_j; \mathbf{p}_{-j}^{\mathcal{F}}, \mathbf{p}^{\mathcal{L}}\right)$$

$$\text{subject to} \quad \sum_{k=1}^{K} p_j(k) \leq P_j^{\max} \tag{7a}$$

$$p_j(k) \in \mathcal{S}_j, \quad k \in \mathcal{K} \tag{7b}$$

where the utility function has been defined in (1).

A. Stackelberg Equilibrium

For the Stackelberg capacity-maximization game defined by the two power control models in (4) and (7) for the follower $i \in \mathcal{L}$ and leader $j \in \mathcal{F}$, respectively, where $\mathcal{N} = \mathcal{L} \cup \mathcal{F}$, the equilibrium of the strategy selection is defined as follows.

Definition 2: The Stackelberg equilibrium solution $\mathbf{p}^\star$ of the Stackelberg capacity-maximization game $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$ is composed of two parts: the Nash equilibrium solution $\mathbf{p}_i^\star : \{p_i^\star(k) = \mathcal{F}_i(\lambda_i(\mathbf{p}_{-i}^\star(k)))\}_{k\in\mathcal{K}}$ of the leader players i as in (5) $\forall\, i \in \mathcal{L}$, and $\mathbf{p}_j^\star$ will be given by solving (7), $\forall\, j \in \mathcal{F}$. Therefore, the strategy profile $\mathbf{p}^\star$ is a final Stackelberg equilibrium solution if only if

$$u_j\left(\mathbf{p}_j^\star, \{\mathbf{p}_i^\star\}_{i\in\mathcal{L}}\right) \geq u_j\left(\mathbf{p}_j, \{\mathbf{p}_i^\star\}_{i\in\mathcal{L}}\right) \quad \forall\, \mathbf{p}_j \in \mathcal{S}_j \tag{8}$$

where $\mathbf{p}_i^\star = \mathcal{F}_i(\lambda_i(\mathbf{p}_{-i}))$ denoted in (5).

B. Dynamic Behavior Caused by Asymmetric Information

We note that the total interference $\xi_j(k)$ in the SINR definition in (3) can be written as

$$\xi_j(k) = \overbrace{I_i^{\mathrm{SU}}(k) + I_{-j}^{\mathrm{SU}}(k)}^{I_j^{\mathrm{SU}}(k)} + I_j^{\mathrm{PU}}(k) + \sigma_j^2(k)$$

$$= \sum_{i\in\mathcal{L}} p_i(k) g_{i,j}(k) + I_{-j}^{\mathrm{SU}}(k) + I_j^{\mathrm{PU}}(k) + \sigma_j^2(k) \tag{9}$$

where $I_{-j}^{\mathrm{SU}}(k) = \sum_{j'\in\mathcal{F}\setminus j} p_{j'}(k) g_{j',j}(k)$. Here, (9) explicitly expresses the role of leaders' power $p_i(k)$ in the total interference to a follower j because followers can perceive $\{p_i(k)\}_i$ in the hierarchical structure. Further, we know from (5) that the optimal power selection of leader $i \in \mathcal{L}$ is

$$p_i^\star(k) = \left[\frac{1}{\lambda_i^\star} - \frac{\xi_i^\star(k)}{g_{i,i}(k)}\right]^+. \tag{10}$$

To show the complex interference relationship among secondary leaders and followers, we rewrite $\xi_i(k)$ in (10) as

$$\begin{aligned}\xi_i(k) &= \overbrace{I_j^{\mathrm{SU}}(k) + I_{-i}^{\mathrm{SU}}(k)}^{I_i^{\mathrm{SU}}(k)} + I_i^{\mathrm{PU}}(k) + \sigma_i^2(k) \\ &= \sum_{j\in\mathcal{F}} p_j(k) g_{j,i}(k) + I_{-i}^{\mathrm{SU}}(k) + I_i^{\mathrm{PU}}(k) + \sigma_i^2(k) \qquad (11)\end{aligned}$$

where $I_{-i}^{\mathrm{SU}}(k) = \sum_{i'\in\mathcal{L}\setminus i} p_{i'}(k) g_{i',i}(k)$. Because a follower j can observe the power selections $p_i^\star(k)$ of the leaders, we substitute (9) into (11) to compute $p_j(k)$. By combining (5), (9), and (11), it can be shown that $\xi_j(k)$ is an explicit function of both $p_j(k)$ and $\{p_i(k)\}_{i\in\mathcal{L}}$, leading to

$$\frac{\partial \xi_j(k)}{\partial p_j(k)} = \sum_{i\in L} \frac{\partial \xi_j(k)}{\partial p_i(k)} \cdot \frac{\partial p_i(k)}{\partial \xi_i(k)} \cdot \frac{\partial \xi_i(k)}{\partial p_j(k)}. \qquad (12)$$

To compute $\partial\xi_j(k)/\partial p_j(k)$ in the form of (12), apparently, we should acquire complete information of channel states, strategy selections, and interference power values, which cause heavy overhead and implementation complexity. In addition, it is hard to develop the distributed algorithm. To reduce the implementation complexity, we approximate the summation on the right-hand side of (12) by its dominating summand indexed by i_j. That is

$$\frac{\partial \xi_j(k)}{\partial p_j(k)} := \frac{\partial \xi_j(k)}{\partial p_{i_j}(k)} \cdot \frac{\partial p_{i_j}(k)}{\partial \xi_{i_j}(k)} \cdot \frac{\partial \xi_{i_j}(k)}{\partial p_j(k)}. \qquad (13)$$

C. Closed-Form Power Control Policy of Followers

Theorem 1: The optimal power selection of the follower $j \in \mathcal{F}$ can be shown to be

$$p_j^\star(k) = \frac{\lambda_j^\star g_{i,i}(k)\left(\xi_j^\star(k)\right)^2 - g_{i,i}(k) g_{j,j}(k)\xi_j^\star(k)}{g_{i,j}(k) g_{j,i}(k) g_{j,j}(k) - \lambda_j g_{i,i}(k) g_{j,j}(k)\xi_j^\star(k)} \qquad (14)$$

where the multiplier $\lambda_j^\star$ satisfies

$$0 \le \lambda_j^\star \perp \left(\sum_{k=1}^{K} p_j^\star(k) - P_j^{\max}\right) \ge 0. \qquad (15)$$

D. Closed-Form Stackelberg Equilibrium Solution

Combining the optimal power of the leaders given in (5), we finally reach the equilibrium solution of the Stackelberg capacity-maximization game, summarized as follows in Theorem 2.

Theorem 2: The Stackelberg equilibrium solution optimal power selection of the proposed Stackelberg capacity-maximization game model is $\mathbf{p}^\star = \mathbf{p}_j^\star \cup \mathbf{p}_i^\star$, where $\mathbf{p}_j^\star = \{p_j^\star(k), k\in\mathcal{K}\}$ of the follower $j\in\mathcal{F}$ can be given in closed form as

$$p_j^\star(k) = \frac{\lambda_j^\star \xi_j^\star(k)^2 - g_{j,j}(k)\xi_j^\star(k)}{\nu_j(k) g_{j,j}(k) - \lambda_j^\star g_{j,j}(k)\xi_j^\star(k)} \qquad (16)$$

where $\nu_j(k) = g_{i,j}(k) g_{j,i}(k)/g_{i,i}(k)$ is the only decision-making information needed from the leader players, and the multiplier $\lambda_j^\star$ satisfies (15). Meanwhile, $\mathbf{p}_i^\star = \{p_i^\star(k), k\in\mathcal{K}\}$ of the leader player $i\in\mathcal{L}$ implements the optimal power selection

$$p_i^\star(k) = \left[\frac{1}{\lambda_i^\star} - \frac{\xi_i^\star(k)}{g_{i,i}(k)}\right]^+ \qquad (17)$$

where $\lambda_i^\star$ satisfies

$$0 \le \lambda_i^\star \perp \left(\sum_{k=1}^{K} p_i^\star(k) - P_i^{\max}\right) \ge 0. \qquad (18)$$

Proof for Theorems 1 and 2 are omitted due to space limitation.

IV. Stackelberg Equilibrium Solution: Existence and Uniqueness

The Stackelberg capacity-maximization game model denoted above with the asymmetric strategy information can well be captured by the variational inequality model [4]. That is, if $p_j^\star(k)\in\mathcal{S}_j$ is the equilibrium solution of the followers, it should satisfy $(p_j(k) - p_j^\star(k))^T \mathcal{F}(p_j^\star(k)) \ge 0$ for $p_j(k)\in\mathcal{S}_j$, where $\mathcal{S}_j$ is the potential function of $p_j^\star(k)$, termed as $\mathcal{S}_j(p_j^\star(k))$. For simplicity, we still use $\mathcal{S}_j$ in the following. According to the definition of quasi-variational inequality and its applications in CR networks [5], the primal problem of the above spectrum sharing can be denoted the quasi-variational inequality model $\mathcal{QVI} = \{\mathcal{K},\mathcal{F}\}$, where $\mathcal{K} = \mathcal{S}_1\times\cdots\times\mathcal{S}_F$, and

$$\begin{aligned}\mathcal{F} = \Big\{ -\nabla_{p_1(k)}\mathcal{L}_1\left(p_1(k), \mathbf{p}_{-1}(k)\right), \ldots \\ - \nabla_{p_F(k)}\mathcal{L}_F\left(p_F(k), \mathbf{p}_{-F}(k)\right)\Big\} \qquad (19)\end{aligned}$$

where $F = |\mathcal{F}|$, and $\mathbf{p}_{-j} = \{\mathbf{p}_{-j}^{\mathcal{F}}, \mathbf{p}^{\mathcal{L}}\}$. Meanwhile, $\mathcal{S}_j$ is the feasible strategy set, and $\nabla_{p_j(k)}\mathcal{L}_j(\mathbf{p}_j,\mathbf{p}_{-j})$ is the partial differential function of $\mathcal{L}_j(\mathbf{p}_j,\mathbf{p}_{-j})$ with respect to $p_j(k)$. Computing the first-order differential function, we obtain

$$\begin{aligned}&\nabla_{p_j(k)}\mathcal{L}_j\left(p_j(k), \mathbf{p}_{-j}(k)\right) \\ &\quad = \frac{g_{i,i}(k) g_{j,j}(k)\xi_j(k) + p_j(k) g_{i,j}(k) g_{j,i}(k) g_{j,j}(k)}{g_{i,i}(k)\xi_j(k)\left[\xi_j(k) + p_j(k) g_{j,j}(k)\right]} - \lambda_j. \qquad (20)\end{aligned}$$

Then, according to the definition of the quasi-variational inequality model in (19), we easily get the following as its most important element:

$$\begin{aligned}&\mathcal{F}_j\left(p_j^\star(k), \mathbf{p}_{-j}^\star(k)\right) \\ &\quad = \lambda_j - \frac{g_{i,i}(k) g_{j,j}(k)\xi_j(k) + p_j(k) g_{i,j}(k) g_{j,i}(k) g_{j,j}(k)}{g_{i,i}(k) g_{j,j}(k)\left(\xi_j(k) + p_j(k) g_{j,j}(k)\right)}. \qquad (21)\end{aligned}$$

Theorem 3: Given the Stackelberg capacity-maximization game model $\mathcal{G} = \{\mathcal{N},\mathcal{S},\mathcal{U}\}$ denoted as above, and the quasi-variational inequality model $\mathcal{QVI} = \{\mathcal{K},\mathcal{F}\}$, we conclude that the game $\mathcal{G}$ is equivalent to the $\mathcal{QVI}$ with $\mathcal{K}=\mathcal{S}$ and $\mathcal{F}$ denoted as (21), if and only if we have the following:

- The feasible set $\mathcal{S}_j$ is convex and compact.
- The utility function $u_j(\mathbf{p}_j,\mathbf{p}_{-j})$ is continuously differentiable in $\mathcal{S}_j$ and convex in $\mathcal{S}_j$ for every fixed $\forall\,\mathbf{p}_{-j}(k)\in\mathcal{S}_{-j}$.

Lemma 1: The Stackelberg equilibrium solution $\mathbf{p}^\star = \mathbf{p}_j^\star \cup \mathbf{p}_i^\star$ of $\mathcal{G} = \{\mathcal{N},\mathcal{S},\mathcal{U}\}$ model is equivalent to that of the quasi-variational inequality model: $\mathcal{QVI} = \{\mathcal{K},\mathcal{F}\}$, with $\mathcal{K}=\mathcal{S}$ and $\mathcal{F}$ denoted as (21).

That is, if $p_j^\star$ is the Stackelberg equilibrium solution of player j, then it is also the solution of the $\mathcal{QVI} = \{\mathcal{K},\mathcal{F}\}$. It means

$(\mathbf{p}_j - \mathbf{p}_j^\star)\mathcal{F}(\mathbf{p}_j^\star, \mathbf{p}_{-j}^\star) \geq 0,\ \mathbf{p}_j \in \mathcal{S}_j$, where

$$\mathcal{F}_j\left(p_j^\star(k), \mathbf{p}_{-j}^\star(k)\right) = -\nabla_{p_j(k)}\mathcal{L}_j\left(p_j(k), \mathbf{p}_{-j}(k)\right). \tag{22}$$

A. Existence

To guarantee the existence of Stackelberg equilibrium solution of $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$, according to the variational inequality theory [4], the $\mathcal{QVI} = \{\mathcal{K}, \mathcal{F}\}$ model is solvable if $\mathcal{K}$ is a nonempty, convex, and compact subset of a finite dimensional Euclidean space $\mathcal{R}^n$. Meanwhile, the mapping $\mathcal{F}$ denoted in (21) is a continuous mapping function.

Theorem 4: There always exists ν_i to guarantee the existence of the Stackelberg equilibrium solution $p_j^\star = \{p_j^\star(k), k \in \mathcal{K}\}, j \in \mathcal{F}$ of the $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$ model, where $\nu_j = \| \max_{\hat{p}_j(k) \in \mathcal{S}_j} \{g_{j,j}(k)/(\xi_j^\star(k) + \hat{p}_j(k) g_{j,j}(k))\}\|^2$.

B. Uniqueness

Here, we further investigate the uniqueness of the equilibrium solution. We know that there always exists a Nash equilibrium solution $\mathbf{p}_j^\star = \{p_j^\star(k), k \in \mathcal{K}\},\ j \in \mathcal{F}$ of the $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$ model. Moreover, if $\mathcal{F}$ is strongly monotone on $\mathcal{S}$, the $\mathcal{QVI}(\mathcal{K}, \mathcal{F})$ admits a unique solution.

Theorem 5: There always exists λ_j to guarantee the uniqueness of the Nash equilibrium solution $\mathbf{p}_j^\star = \{p_j^\star(k), k \in \mathcal{K}\},\ j \in \mathcal{F}$ of $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$ model, where $\lambda_j = \| \min_{\hat{p}_j(k) \in \mathcal{S}_j} \{g_{j,j}(k)/(\xi_j^\star(k) + \hat{p}_j(k) g_{j,j}(k))\}\|^2$.

Proofs for Theorems 3 and 4 are omitted due to space limitations.

C. Distributed Algorithm Design

Based on the given analysis, we illustrate the implementation overflow of the proposed distributed algorithm. There are three crucial steps. First, who will be the leader or follower? As we stated above, the most important assumption is the super cognitive capabilities of the SUs. If the SU can observe the information asymmetry phenomenon and become an information-rich SUs, then it will be the foresight player in the formulated Stackelberg capacity-maximization game; otherwise, it is the leader player.

The following two steps concentrate on the distributed algorithm for the final Stackelberg equilibrium solution, which is different from the conventional iterative water-filling algorithm (IWFA) [3]. We further divide it into two substeps of *Algorithms 1* and *2* designated for a leader and a follower, respectively. *Algorithm 1* searches for the power solution for the leader $i \in \mathcal{L}$. Here, *Algorithm 1* approximates the Nash equilibrium solution in the traditional Nash game for leaders only.

Algorithm 1: Distributed Algorithm for Leaders

Step 1: Set $\lambda_i^{(0)}$ and $p_i^{(0)}(k)$ for $\forall i \in \mathcal{L},\ \forall k \in \mathcal{K}$;
Step 2: Observe the necessary information, e.g., $g_{i,i}(k)$ and compute the total interference power $\xi_i^{(n)}(k)$;
Step 3: Get the power level during the next step for $\forall i \in \mathcal{L},\ \forall k \in \mathcal{K}$:

$$p_i^{(n+1)}(k) = \frac{1}{\lambda_i^{(n)}} - \frac{\xi_i^{(n)}(k)}{g_{i,i}(k)};$$

Step 4: Choose $\alpha_i^{(n)} > 0$ and update

$$\lambda_i^{(n+1)} = \lambda_i^{(n)} + \alpha_i^{(n)}\left\{\sum_k p_i^{(n)}(k) - p_i^{\max}\right\};$$

Step 5: Set $n \leftarrow n+1$, go to **step 2**, and repeat above steps until convergence.

Algorithm 2: Distributed Algorithm for Followers

Step 1: for $j \in \mathcal{F}, \forall k \in \mathcal{K}$:
1.1. Set $\lambda_j^{(0)}$, and $p_j^{(0)}(k)$, for $\forall j \in \mathcal{F},\ \forall k \in \mathcal{K}$;
1.2. Observe the necessary information, e.g., $g_{j,j}(k)$ and compute the total interference power $\xi_j^{(n)}(k)$;
1.3. Get the power level during the next step for $\forall j \in \mathcal{F},\ \forall k \in \mathcal{K}$:

$$p_j^{(n+1)}(k) = \frac{\lambda_j^{(n)}\left(\xi_j^{(n)}(k)\right)^2 - g_{j,j}(k)\xi_j^{(n)}(k)}{\nu_j(k)g_{j,j}(k) - \lambda_j^{(n)}g_{j,j}(k)\xi_j^{(n)}(k)};$$

Step 2: Choose $\beta_j^{(n)} > 0$, and update

$$\lambda_j^{(n+1)} = \lambda_j^{(n)} + \beta_j^{(n)}\left\{\sum_k p_j^{(n)}(k) - p_j^{\max}\right\};$$

Step 3: Set $n \leftarrow n+1$, go to **step 1**, and repeat above steps until convergence.

In *Algorithm 1*, $\alpha_i^{(n)} > 0$ is the iteration step, which is a function of iteration number n, i.e., we select $\alpha_i^{(n)} = 1/n$. If $\sum_k p_i^{(n)}(k) - p_i^{\max} > 0$, which means that the power selection of SU i on all selected channels is larger than the power constraint of itself, then the water-filling level $\lambda_i^{(n)}$ during the next iteration will increase according to $\lambda_i^{(n+1)} = \lambda_i^{(n)} + \alpha_i^{(n)}\{\sum_k p_i^{(n)}(k) - p_i^{\max}\}$ so that the optimal power will decrease according to $p_i^{(n+1)}(k) = [(1/\lambda_i^{(n)}) - (\xi_i^{(n)}(k)/g_{i,i}(k))]^+$. Hence, the convergence condition is denoted as $\|\lambda_i^{(n+1)} - \lambda_i^{(n)}\| \leq \epsilon$, where ϵ is a small constant value. For the last part of power selection of followers in *Algorithm 2*, we have the similar description.

V. Simulation and Numerical Results

We consider a cognitive Wi-Fi 2.0 scenario, where multiple cognitive access points (CogAPs) are randomly distributed in a square area of 100 m $\times$ 100 m to share multiple channels [6]. We assume each CogAP has only one cognitive device, which is termed as one pair of SUs in the simulation. Multiple SUs are divided into leaders and followers. We use the channel fading model as $\bar{P}(d)[\mathrm{dB}] = \bar{P}(d_0) + 10N\log(d/d_0)$, where $\bar{P}(d_0) = 20\log(4\pi d_0/\tau)$, where $\tau = 2.4$ GHz is the carrier frequency, and without loss of generality, we choose $d_0 = 1$.

A. Specific Cognitive Wi-Fi 2.0 Networks Scenario

We investigate system performance (e.g., using the SINR as the measurement criteria) with variation of the CogAP pair ratio of CogAP–leader to CogAP–follower shown in Fig. 2. We randomize a specific fading channel model where the mean channel fading factor is -185 dB.

From the figure, we can conclude the following.

- Fig. 2 describes the average system SINR of each CogAP portfolio, where the CogAP pair ratio changes from $20:0 \Rightarrow 0:20$. Meanwhile, $20:0$ means there are 20 leader CogAPs and 0 follower, and the average system SINR is computed as the mean SINR of 20 CogAPs of each variation of the CogAP pair ratio.
- From the figure, we can see the basic trend of the average system SINR achieves more improvement with the CogAP pair ratio becoming less, which means the number of followers becomes larger. However, when the number of more foresighted follower

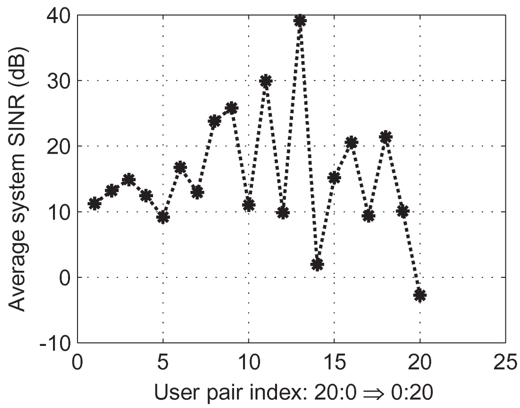

Fig. 2. Average SINR system performance.

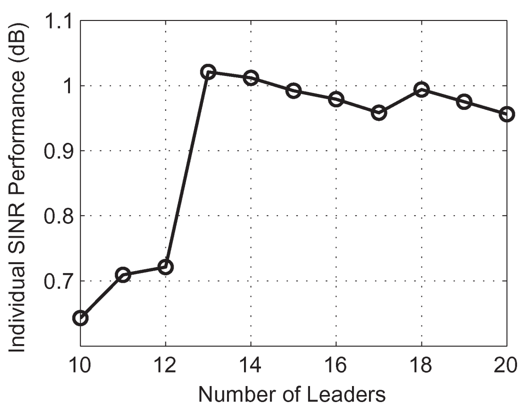

Fig. 3. Individual QoS measured by individual SINR performance.

reaches a certain level, e.g., the leader number is 13, it achieves the optimal system performance. Further, when there are more followers, system performance declines.

- Finally, we conclude that when the CogAP pair ratio is 13 : 7, which means there are 13 leaders and seven followers, it achieves the optimal system performance in the selected scenario.

The main reason for these results can be summarized as follows.

- When the CogAP pair ratio gets smaller, the number of more foresighted followers becomes larger. Therefore, there are more CogAPs who can select the more rational and intelligent power strategy, which leads to the improvement system SINR performance.
- However, when the CogAP pair ratio becomes further smaller, which also means that the number of leaders who will be selected by more foresighted follower gets smaller, the competition case among the followers will increase; meanwhile, mutual interference among more followers will deteriorate during the process. Both of these effects will affect the final performance.

B. Optimal Performance Tradeoff

Further, we simulate the scenario of the total number of leaders changing from 10 to 20, and the number of follower is fixed as 7. The individual performance of the proposed algorithm is shown in Fig. 3. A similar result was observed for overall performance as well.

Fig. 3 shows that the achieved individual SINR reaches the optimal value of 1 dB when the number of CogAP-Leader is 13; meanwhile, we also can see the individual SINR varies from 0.6 to 1 dB when the channel fading factor change from -195 to -181 dB.

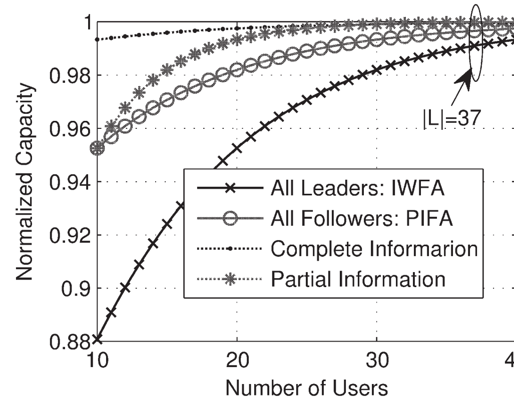

Fig. 4. Capacity performance competition with typical IWFA and PIFA measured by normalized capacity.

C. Normalized Capacity Performance

Finally, we investigate the capacity performance, comparing it with the algorithms of conventional IWFA in [3] and a newly proposed price-based iterative water-filling algorithm (PIFA) in [3], which are, respectively, activated in the cases of all leaders and all followers. Here, the two cases of all leaders and all followers in conventional IWFA and PIFA can be seen as the symmetric information case. PIFA assumes that the SUs always achieve the pricing information from the specific center compared to IWFA, which only regards the interference as noise. Meanwhile, we consider the complete information case of the centralized algorithm, and the distributed algorithm in the partially information-dependent case, which is shown in Fig. 4.

VI. Conclusion

In this paper, we have investigated a multileader and multifollower case, where foresighted SUs are followers with more rationality, and myopic SUs are leaders using an IWFA-like power allocation strategy. We derived a closed form of power Stackelberg equilibrium solution for followers, and based on this, we developed distributed algorithms for approximating the optimal Stackelberg equilibrium solution.

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