Green Cooperative Cognitive Radio: A Multiobjective Optimization Paradigm
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Abstract—In this paper, we apply the cross-entropy optimization (CEO) to the problem of joint multiple relay assignment and source/relay power allocation in green cooperative cognitive radio (GCCR) networks. We use shared-band amplify-and-forward relaying for cooperative communication in this problem. The proposed joint multiple relay assignment and source/relay power allocation jointly performs relay assignment and power allocation in GCCR while optimizing two conflicting objectives: The first one is to maximize the total rate, and the second one is to minimize the greenhouse gas emissions in GCCR networks. This multiobjective optimization problem is a nonconvex combinatorial optimization problem and is NP-hard. We use a Monte–Carlo-based CEO algorithm to solve this nonconvex problem. The CEO has a simplistic model, and its robustness in avoiding local minima/maxima makes it a suitable candidate for solving complex combinatorial optimization problems. We present simulation results that verify the effectiveness of the proposed CEO method for joint multiple relay assignment and source/relay power allocation.

Index Terms—Cooperative cognitive radio (CR), green wireless communication.

NOMENCLATURE

BBOA Branch and bound algorithm with outer approximation.
BS Base station.
CEO Cross-entropy optimization.
CR Cognitive radio.
CSRRP Constraint satisfaction for relay assignment and relay power.
CSSP Constraint satisfaction routine for source power.
GCCR Green cooperative CR.
GHG Greenhouse gas.
ICT Information and communication technology.
LB Lower bound.
MIMO Multiple input multiple output.
MINLP Mixed-integer nonlinear optimization problem.
MOO Multiobjective optimization.
NC-MINLP Nonconvex mixed-integer nonlinear optimization problem.
PU Primary user.
SBAF Shared-band amplify and forward.
SU Secondary user.
UB Upper bound.
WSM Weighted sum method.

I. INTRODUCTION

The ICT sector has seen a rapid increase in wireless communication device usage. This increase has caused this industry to become a significant contributor to GHG emissions [1], [3]. The ICT sector is responsible for approximately 5% of the global electricity demand and GHG emission [4], [5]. In fact, the GHG emission from the ICT sector is close to the airline industry [2], [5]. In the future, this emission is expected to rise, as shown in Fig. 1, where it is shown that mobile devices have a major contribution in GHG emissions. This underlines the importance of looking at ways to mitigate the emissions caused by mobile devices. The purpose of developing green communications is to reduce overall energy consumption, consequently reducing GHG emissions without a significant decrease in throughput. This is unlike traditional communications, where the purpose is to maximize only the throughput. As a result, a very active area of research called green communications has sprung up to reduce the environmental impact to a minimum [6]. In [7] and [8], an energy-efficient management framework of cellular network resources is presented. An energy-efficient green rate-and-power control transmission scheme with quality-of-service and fairness constraints is proposed in [20] for orthogonal frequency division multiple access based multicarrier BSs. In [23], the authors investigated the green wireless localization with non-data-bearing orthogonal frequency division multiplexing transmission. The authors showed that the proposed technique can save 70% energy in wireless positioning system. A green peer-to-peer system with a cloud computing model is proposed in [21].

Cognition has proved to be a useful performance enhancement concept in many fields, e.g., communications [9], [22], radar [10], smart grid [11], wireless sensor networks [12], etc. In wireless communications, CR can help improve the inefficient use of the radio spectrum. A CR system allows the sharing of a spectrum band between unlicensed/secondary users and licensed/primary users, while minimizing the interference caused at the latter’s receiver.
MIMO techniques based on antenna arrays have been shown to significantly reduce the required transmission power for a given throughput requirement due to spatial diversity. The concept of cooperative communication has been proposed to generate a virtual antenna array [13] as it is practically difficult to equip wireless handheld devices with multiple antennas due to size, cost, and equipment constraints. Cooperative communication enables single-antenna mobiles in a multiuser environment to share their antennas and generate a virtual multiple-antenna transmitter. This allows the mobiles to directly transmit [13] to the destination. Cooperative communication, when used for CR, can help in reducing the total transmission power and, consequently, in reducing GHG emissions. Cooperative relaying and coordinated multipoint transmission are two emerging technologies that can be utilized for energy saving since they can also extend coverage and improve throughput [14]. This underlines the importance of the work in the field of GCCR as CR and cooperative sensing/communication can help in reducing energy consumption [15].

One possible means of cooperative communication is the amplify-and-forward relaying [16]. Each relay in this method receives a noisy version of the signal transmitted by the source. The relay amplifies and retransmits this noisy version. The receiver combines the information sent by the source and relay and makes a final decision on the transmitted signal. It has been shown in [17] that relaying techniques enable us to solve the problem of energy efficiency via multihop transmission and extend the battery life. In [18], the authors indicated that two-hop communication consumes less energy than direct communication. However, the energy efficiency of the cooperative communication may degrade with increasing number of cooperators, i.e., more cooperators may not be more energy efficient [19]. Therefore, it is necessary to design energy-efficient cooperative relay assignment strategies. Energy-efficient relay selection for wireless networks has been presented in [25]–[27]. In [25], the energy efficiency of Transmission Control Protocol traffic is analyzed, and relay selection problem is investigated. In [26], an energy allocation scheme for cooperative MIMO communication is presented. In [27], energy efficiency of direct communication and selection of a single relay based on the largest channel gain is compared to show that the latter can improve energy efficiency.

The problem of relay assignment and power allocation for GCCR is an MOO problem. In typical MOO problems, the difficulty arises due to the fact that different objectives can conflict with each other. Optimization with respect to any particular objective can give unacceptable results with respect to the other objectives. For the problem studied in this paper, we have two conflicting objectives: maximize the sum rate and minimize the GHG emissions. Determining the optimal set of decision variables for a single objective, e.g., GHG emission minimization, can result in a nonoptimal set with respect to the other objective, i.e., sum-rate maximization, and vice versa.

In this paper, we apply the CEO [28] to relay assignment problem and show that the performance of the CEO is better than the previously proposed method such as convex optimization. The CEO is a generalized Monte Carlo technique and has been applied to solve combinatorial optimization problems such as the traveling salesman problem, the max-cut problem, etc. The CEO defines a precise mathematical framework for deriving fast updating and learning rules [28]. It has been successfully applied to diverse fields, such as machine learning [28], buffer allocation [29], queuing theory [30], blind signal detection [31], communication networks [32], sensor selection problem [33], etc. The CEO is composed of two phases: In the first phase, the CEO generates a sample of random data according to a specified random mechanism. In the second phase, it uses the random data to update the parameters of the random mechanism. These parameters are typically the parameters of probability distribution to produce a better sample in the next iteration.

The main contribution of this paper is the formulation of an MOO framework for joint power allocation and relay assignment under the constraint of acceptable interference to the primary (licensed) users in GCCR. The proposed multiojective framework jointly performs relay assignment and power allocation in cooperative CR while optimizing two conflicting objectives. The first objective is to maximize the total throughput, and the second objective is to minimize the total transmission power of GCCR. The proposed joint relay selection and power minimization is an NC-MINLP, which is generally NP-hard. The simplicity of the model, ease of implementation, and resistance to being trapped in local minima/maxima make CEO a suitable candidate for solving computationally challenging optimization problems such as the one studied in this paper. To the best of our knowledge, there is no joint multiple relay assignment and power allocation scheme in the literature that deals with the analysis and optimization of energy efficiency in a GCCR system using CEO. The main contributions of this paper are summarized as follows.

1) We propose a multiobjective combinatorial optimization framework that jointly optimizes two conflicting objectives: throughput maximization and GHG emission minimization in a GCCR network.
2) For multiple relay assignment, we use SBAF protocol. The rate expression for SBAF is not a concave function with relay powers. We propose two UBs on the rate expression of SBAF protocol. One UB helps in the normalization, and the second UB converts the nonconca rate expression into concave rate expression.
3) The proposed joint multiple relay assignment and power allocation problem is an NC-MINLP, which is generally NP-hard. We introduce a heuristic algorithm to solve this
problem. The hybrid heuristic algorithm is a combination of CEO algorithm and constraint satisfaction routines to solve multiobjective combinatorial optimization problem. In addition to applying the CEO to the constrained MOO problem, we also introduce adaptive smoothing and premature convergence avoidance mechanisms for CEO.

4) A detailed analysis of the performance of the proposed CEO is presented with simulation results.

We use $A$, $a$, and $a$ to represent a matrix, a vector, and an element of a vector, respectively. When $a_i \geq 0$ for all components $i$ of a vector $a$, we use $a \geq 0$. The rest of this paper is organized as follows. The system model is presented in Section II. In Section IV-A, we present our cross-entropy method and its improved version. Simulation results are presented in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a CR network in TV band comprising of a BS, $K$ SUs, $R$ relays, and $M$ PUs, as shown in Fig. 2. We assume that each SU operates in separate orthogonal frequency band. The channel gains from the BS to the $k$th SU, the $r$th relay, and the $m$th PU in the $k$th SU band are $h_k$, $h_r$, and $g_k^m$, respectively. The channel gains from the $r$th relay to the $k$th SU and the $m$th PU in the $k$th SU band are $h_{r,k}$ and $g_{r,k}^m$, respectively. We denote the $r$th relay’s transmission power by $p_r$. $P_r$ is the maximum power of the $r$th relay, $p_k$ is the source power in the $k$th user band, and $P_s$ is the maximum source power, i.e., $\sum_k p_k \leq P_s$.

The cooperative protocol used in this paper is the half-duplex SBAF protocol. In this protocol, the first time slot is for source transmission, and the second time slot is for the relays. In the second time slot, a user can get the same amplified data from multiple relays simultaneously. The rate of the $k$th user for SBAF is [35]

$$C_k = \frac{1}{2} \log \left( 1 + \frac{p_k}{N_0} \left( \frac{g_k}{\sigma^2} + \frac{\left( \sum_{r=1}^{R} |h_{r,k}| \right)^2}{1 + \sum_{r=1}^{R} \left( \frac{1}{\gamma_{r,k}} \right)^2} \right) \right)$$

where $\gamma_{r,k} = (|h_{r,k}|^{1/p_r}) / (|p_k|h_{r,k}^2 + (N_0/2))$, and $N_0$ is the noise power. The rate formula for SBAF is not a concave function of the relay powers. We define $x_k^r$ as a binary assignment indicator variable, and $x_k^r$ will be 1 if the $r$th relay is assigned to the $k$th user and 0 otherwise. The rate of the $k$th user for SBAF relaying with the binary assignment indicator becomes

$$C_k = \frac{1}{2} \log \left( 1 + \frac{p_k}{N_0} \left( \frac{|h_k|^2}{1 + \sum_{r=1}^{R} \left( \frac{1}{\gamma_{r,k}} \right)^2} \right) \right).$$

Our main goal is multiple relay assignment and source/relay power allocation that will simultaneously maximize the total SUs’ transmission rate and minimize GHG emissions, i.e., the total source/relay transmission power. This paper attempts to achieve a balance between the transmission rate and the total transmission power used. A classical solution will not give us a solution to this problem as data rate maximization and source/relay power minimization are two conflicting objectives. The optimal values of decision variables for one objective can result in a nonoptimal result for the second objective. In our formulation, we use a normalized WSM to combine these conflicting objectives. In a WSM, the weight of each objective is normalized between 0 and 1 and is proportional to its importance in decision making. The range of combined objective function is always within 0 and 1, as a WSM without normalization would result in a biased fitness function [34]. The weights for each objective $w_1$ and $w_2$ can be chosen according to the kind of tradeoffs we are willing to make. The ratio of $w_1$ to $w_2$ will ultimately decide the total transmission power and other variables such as relay assignment. In a time slot with lower rate demand, we may choose a lower $w_1$ and a higher $w_2$. This will give us a power allocation that places more emphasis on power savings. In future, if there is a carbon tax, the exact values of $w_1$ and $w_2$ can be calculated using a cost–revenue analysis. Once we know $w_1$ and $w_2$, the solution of the new problem can be found using the method presented in this paper.

A natural question that arises in green communication is about the basic difference between the classical transmission rate maximization problem and the green rate maximization problem. In the classical transmission rate problem, the main objective is the maximization of throughput subject to total transmission power constraint. It has been shown in [36] that the optimal power allocation that maximizes the total throughput is always on the feasible region boundary of the total transmission power constraint. In the green rate maximization problem, the optimal power allocation that will strike a balance between the throughput/rate and the transmission power may not be at the feasible region boundary of total transmission power constraint.

In our formulation, the first objective is to reduce GHG emissions. The GHG emissions are measured in grams and are directly proportional to the total transmission power. If $P$ is the transmission power used and $X$ is a constant in grams/watts, then $PX$ is the GHG emissions in grams. The value of $X$ depends on the type of fuel used for generation of electricity.
The approximate values of $X$ for natural gas, crude, and diesel oil are 370, 640, and 670 g/W, respectively [4]. We define by $G_r = X P_r$ the GHG emissions due to the $r$th relay and $G_{s,k} = X p_k$ the GHG emissions due to the source transmission power in the $k$th user band. We normalize the GHG emissions objective function with $G_{r, \text{max}} + G_{s, \text{max}}$, where $G_{r, \text{max}} = \sum_r X P_r$, and $G_{s, \text{max}} = X P_s$. The GHG objective can be written as $f_{GHG} = G / G_{\text{max}}$, where $G = \sum_r G_r + \sum_k G_{s,k}$, and $G_{s, \text{max}} = G_{r, \text{max}} + G_{s, \text{max}}$. Our second objective is to maximize the sum-rate capacity $\sum_k C_k$. To normalize it between 0 and 1, we divide it with its decision variable-free UB $\sum_k C_k^{\text{max}}$ given as follows.

**Lemma 1:** The UB of (1) is $C_k^{\text{max}} = 1/2 \log [1 + (P_s/N_0)(|h|^2 + \sum_r |h|^2)]$.

**Proof:** It can be seen that $C_k$ in (1) is an increasing function of the source power. We can set the source power to its maximum transmission power $P_s$. We will get a UB as

$$
C_k = \frac{1}{2} \log \left[ 1 + \frac{p_k}{N_0} \left( |h|^2 + \frac{\left( \sum_r |h|^2 + \sum_r \left| Y_{r,k} \right|^2 \right) \sum_r \left| Y_{r,k} \right|^2}{\left( \sum_r \left| Y_{r,k} \right|^2 \right) \sum_r \left| Y_{r,k} \right|^2} \right) \right].
$$

By Cauchy–Schwart inequality, we will get

$$
\leq \frac{1}{2} \log \left[ 1 + \frac{p_s}{N_0} \left( |h|^2 + \sum_r \left| Y_{r,k} \right|^2 \right) \sum_r \left| Y_{r,k} \right|^2 \right] = C_k^{\text{max}}.
$$

Mathematically, we can write the normalized sum rate as $f_c = \sum_{k=1}^K C_k / \sum_{k=1}^K C_k^{\text{max}}$. The range of the objective function is always within 0 and 1.

The multiobjective problem is to maximize the data rate and minimize the GHG emissions. For MOO, we need to transform both objectives into a joint minimization (or maximization) objective. Since both objectives are normalized and bounded between 0 and 1, we can make the joint minimization objective as $w_1 (1 - c_r) + w_2 f_{GHG}$. The term $(1 - f_c)$ makes the sum-rate maximization a minimization problem. We can formulate the GCCR-MOO problem as

$$
\min \{ X, P_r, p_s \}
$$

subject to

$$
C_1 : \sum_{k=1}^K p_k \leq P_s
$$

$$
C_2 : p_k |g_{m,k}|^2 \leq I_{m,k}^{\text{max}}, \forall (m,k)
$$

$$
C_3 : 0 \leq p_r \leq P_r, \forall r
$$

$$
C_4 : \sum_{r=1}^R x_r p_r |g_{m,k}|^2 \leq I_{m,k}^{\text{max}}, \forall (m,k)
$$

$$
C_5 : \sum_{k=1}^K x_k \leq 1, \forall r
$$

$$
C_6 : x_k \in \{0, 1\}.
$$

In (3), the constraints $C1$ and $C2$ are source sum power and interference constraints, respectively. Constraint $C3$ is the box constraint for relay power, and $C4$ is the relay interference constraint. Constraint $C5$ is the assignment constraint, which ensures that a relay can only be assigned to one SU. The objective function in (3) is bounded below and above by 1 and 0, respectively. This optimization formulation is equally valid for spectrum overlay and underlay paradigms. If the CR network has a capability of spectrum sensing, we can use spectrum overlay; otherwise, we can use spectrum underlay. For spectrum underlay, the value of interference threshold will be less than the value of interference threshold for spectrum overlay system.

The formulation in (3) is a multiobjective nonconvex mixed-integer nonlinear programming problem. To prove the NP-hardness of (3), we can reduce the 0–1 multiple-choice multiple-dimensional knapsack problem to the relay assignment and power allocation problem formulated in (3). Due to NP-complete nature of the problem, we cannot get the optimal solution in polynomial time. A brute force search algorithm for (3) would evaluate all the possible relay assignments, but its complexity exponentially increases with the number of relays and the number of the SUs.

In the next sections, we will present two approaches to solve the problem in (3). The first one uses a branch and bound with outer approximation (BBOA) approach, and the second one applies the CEO method for the proposed MOO problem.

### III. Branch and Bound with Outer Approximation

BBOA is an algorithm used to solve nonlinear mixed-integer problems [24]. The main condition for BBOA is that with fixed binary variables, the objective function and constraints should be a continuously differentiable function. In addition, the constraint qualification (e.g., Slater’s condition) must hold at the solution of every nonlinear programming subproblem resulting from the original nonlinear MINLP [24], [37]. As mentioned in Section II, for a given set of assigned relays, the optimization problem in (3) is not a concave function of the relay powers. Thus, even for a given set of assigned relays, convex optimization techniques cannot be applied to the resulting optimization problem. For comparison, we provide a UB on the rate of SBAF relays. This UB eventually gives the LB of the objective function in (3). The proposed UB is concave for a given realization of integer variables. The concave UB is also derived using the Cauchy–Schwarz inequality without making it a decision variable-free objective. This UB is tighter than the one proposed in Lemma 1. The UB is

$$
C_k = \log \left[ 1 + \frac{p_k}{N_0} \left( |h|^2 + \frac{\left( \sum_{r=1}^R |h|^2 + \sum_{r=1}^R \left| Y_{r,k} \right|^2 \right) \sum_{r=1}^R \left| Y_{r,k} \right|^2}{\left( \sum_{r=1}^R \left| Y_{r,k} \right|^2 \right) \sum_{r=1}^R \left| Y_{r,k} \right|^2} \right) \right].
$$

$$
\leq \log \left[ 1 + \frac{p_s}{N_0} \left( |h|^2 + \sum_r \left| Y_{r,k} \right|^2 \right) \sum_r \left| Y_{r,k} \right|^2 \right] = C_k^{\text{UB}}.
$$
The proposed UB is a concave function of relay powers. Now, we can apply the BBOA method to solve the MINLP optimization problem with concave UB. Mathematically, we can write the normalized sum rate as \( f_c = \frac{\sum_{k=1}^{K} C_k^{UB}}{\sum_{k=1}^{K} C_k^{max}} \). The range of the objective is always within 0 and 1. Again, as done in the previous section, we can write the joint minimization objective as \( w_1 (1 - f_c) + w_2 f_{GHG} \). The term \((1 - f_c)\) makes the sum-rate maximization as a minimization problem. Since \( C_k^{UB} \) is the UB on the capacity objective and we are using the term \((1 - f_c)\) in the joint objective, the implementation of BBOA eventually gives us the LB on the joint multiobjective function.

BBOA is an iterative method that uses divide-and-conquer methodology with the aid of duality theory to solve continuously differentiable mixed-integer programming problems. The basic idea is the generation of UB and LB in each iteration to solve MINLP [24]. The LB results from the solution of the master problem that is derived using primal information. The primal information is the solution of discrete point of the primal and linearization of the nonlinear objective and constraints. This linear approximation is also known as outer approximation of the nonlinear objective and constraints. The LB results from the solution of the problem with fixed discrete variables. These variables are used in the next primal problem. At each iteration, the newly generated UB and LB are nonincreasing and nondecreasing, respectively. It is shown in [24] that these two bounds will converge within finite number of iterations. This convergence means optimal solution.

The main drawback of BBOA is that its worst case complexity is exponential. In the next section, we will present another low-complexity iterative method that uses stochastic search/optimization algorithm, i.e., CEO, for joint relay assignment and power allocation.

IV. CEO FOR RELAY ASSIGNMENT AND POWER ALLOCATION

The cross entropy is a generalized Monte Carlo technique to solve combinatorial optimization problems. In this paper, we propose custom constraint satisfaction routines for relay assignment and power allocation that execute with cross-entropy method to get the suboptimal solution of (3). The CEO algorithm is an iterative probabilistic randomized algorithm that uses Monte Carlo technique and rare event estimation to solve the optimization problem [28]. The CEO algorithm first associates the given optimization problem with a rare-event estimation problem and then solves the problem iteratively in two stages: In the first stage, it generates a sample of random data according to a predefined random mechanism. In the second stage, it updates the parameters of the predefined random mechanism, typically parameters of probability distribution. This update is carried out on the basis of the data to produce a better sample in the next iteration. It has a simplistic model, and its resistance to being trapped in local minima/maxima makes it a suitable candidate for solving complex problems such as the problem presented in (3).

A. CEO

Before presenting CEO for GCCR-MOO, we first provide a brief idea of CEO. Consider a maximization problem

\[
\max_{x \in \chi} F(x).
\]

Let \( \gamma^* \) be the maximum value of function \( F(x) \) that occurs at the point \( x^* \). In general, the iterative probabilistic evolutionary randomized method randomly generates samples (subset of \( \chi \)) in accordance with some probability distribution at each iteration [28]. Subsequently, good candidate solutions are selected from the samples, and the probability distribution is updated on the basis of the selected good candidate solutions. In the next iteration, new samples are generated according to the updated probability distribution. Let us consider an arbitrary probability density function (pdf) \( f(x; u) \), where \( u \) is a parameter that refers to this pdf, and the domain of this pdf is \( \chi \). Suppose a pdf \( f(x; u) \) is used at a stage in iterations. Hypothetically, if the pdf \( g(x; \gamma, u) \) is used as the pdf in the next iteration, then every sample generated from this distribution will be a high-quality candidate, i.e., a candidate whose objective function value is at least \( \gamma \). \( I_{\gamma} \) is an indicator function that takes care of \( F(x) \geq \gamma \). Hypothetically, if \( \gamma = \gamma^* \) were used in (4), then pdf \( g(x; \gamma, u) \) would only generate random samples that are optimal because all probability density is concentrated in the optimal solution(s). However, the optimal value \( \gamma^* \) is unknown to the algorithm. Instead of using a pdf in (4) with \( \gamma = \gamma^* \), the CEO algorithm cautiously increases \( \gamma \) at each new iteration on the basis of samples \( X_i, i = 1, 2, \ldots, \Gamma \) that are randomly generated in accordance with pdf \( f(x; u) \) [28], where \( \Gamma \) is the maximum number of samples per iteration.

Another problem in using (4) is that the pdf in (4) is difficult to compute even for a known \( \gamma \) because computation of \( l(u, \gamma) \equiv \int I_{\gamma} f(x; u) \) could be prohibitive for the case of a large set \( \chi \). The CEO algorithm uses, in place of (4), a pdf that is closer to (4) in terms of Kullback–Leibler (KL) distance (cross entropy) [28], i.e., the pdf \( \nu \) that minimizes

\[
D(g(x; \gamma, u) \| f(x; \nu)) = \int g(x; \gamma, u) \ln \left[ \frac{g(x, \gamma, u)}{f(x, \nu)} \right] = \int g(x; \gamma, u) \ln g(x; \gamma, u) - \int g(x; \gamma, u) \ln f(x; \nu).
\]

Minimizing this KL distance by choosing pdf \( \nu \) is equivalent to maximizing the term \( \int g(x; \gamma, u) \ln f(x; \nu) \). This is also equivalent to maximizing

\[
\int I_{\gamma} f(x; u) \ln f(x; \nu) = E_u [I_{\gamma} \ln f(X; \nu)]
\]

where \( E_u \) denotes the expected value in accordance with pdf \( u \) of random variable \( X \). In order to reduce computational
complexity, the CEO algorithm finds in the family of pdfs a pdf \( \nu \) that results in the largest

\[
\frac{1}{\Gamma} \sum_{i=1}^{\Gamma} I_i \ln f(X; \nu)
\]

(6)

which is the estimate of (5) on the basis of the samples \( X_i, i = 1, 2, \ldots, \Gamma \) [randomly generated in accordance with pdf \( f(x; u) \)]. A typical CEO algorithm is presented in Algorithm 1.

Algorithm 1: Typical CEO Algorithm

1: \( \nu_0 \leftarrow \mu, t \leftarrow 1 \)
2: \( \text{while } t \leq \text{MaxIter} \text{ do} \)
3: \( \text{Generate samples } X_1, \ldots, X_\Gamma \text{ from the density } f(\cdot; \nu^{t-1}) \)
4: \( \text{Evaluate the objective function} \)
5: \( \text{Sort samples as: } F_1 \leq \cdots \leq F_\Gamma \)
6: \( \text{Set the } (1 - \rho) \text{-quantile } \gamma^t \text{ as } \gamma^t \leftarrow F_{\lceil (1-\rho)\Gamma \rceil} \)
7: \( \text{Generate new pdf using (6)} \)
8: \( t \leftarrow t + 1 \)
9: \( \text{end while} \)

B. Cross Entropy for GCCR-MOO

Now, we describe our detailed implementation of the CEO method for the GCCR-MOO problem. First, we define some notations and symbols that will be used in the implementation of the CEO method for the GCCR-MOO problem.

We define the following parameters.

1) \( F = w_1(1 - f_c) + w_2 f_{\text{GHG}} \) is the fitness or objective function.
2) \( \Gamma \) is the maximum number of samples per iteration.
3) \( \Theta, \Sigma \) are the mean and covariance of the best samples, respectively, which are used by the pdf to regenerate the new random samples.
4) \( \rho \) is the rarity parameter. In each iteration, a threshold \( \gamma \) is set to select \( \rho \Gamma \) best samples among \( \Gamma \) samples.
5) \( \alpha \) is the smoothing parameter.

In applying the cross-entropy method to the GCCR-MOO problem, we use the objective function (1) as the fitness function \( F \). We use the multivariate Gaussian distribution \( \mathcal{N}(\Theta, \Sigma) \) to generate the samples. Samples at any iteration \( t \) can be specified by the following \( \Gamma \times (R + K) \) matrix:

\[
\Lambda = \begin{pmatrix}
p_1^1 & p_2^1 & \cdots & p_{\Gamma+K}^1 \\
p_1^2 & p_2^2 & \cdots & p_{\Gamma+K}^2 \\
\vdots & \vdots & \ddots & \vdots \\
p_1^\Gamma & p_2^\Gamma & \cdots & p_{\Gamma+K}^\Gamma 
\end{pmatrix}
\]

(7)

Each row of the matrix \( \Lambda \) represents an individual in the samples, and each individual has \( R + K \) elements. The first \( R \) elements represent relay power, and the next \( K \) elements represent source power. A pseudocode of the cross-entropy method for GCCR-MOO is presented in Algorithm 2. At the start, the algorithm generates samples randomly from the Gaussian distribution using mean \( \Theta^t \) and covariance \( \Sigma^t \). Initial values of \( \Theta^t \) are random, whereas initial \( \Sigma^t \) are set to \( P_r \) for relay and \( P_s \) for source, respectively. After getting the samples, the algorithms run a constraint satisfaction routine that transforms the infeasible samples into feasible samples. A detailed description of these routines is presented in Section IV-D and E. Subsequently, the algorithm evaluates the fitness function and determines \( \Theta^t \) and \( \Sigma^t \) from the best samples. After getting \( \Theta^t \) and \( \Sigma^t \) from the best samples, the algorithm smoothens \( \Theta^t \) and \( \Sigma^t \) with the parameter \( \alpha \). The algorithm continues until some specified termination criterion is satisfied.

Algorithm 2: Cross Entropy for GCCR-MOO

1: \( \text{Initialize: MaxIter, } t, \rho, \alpha, K, R, M \)
2: \( \text{IsAdaptiveUpdate} \leftarrow 0, \text{IsPrematureAvoid} \leftarrow 0 \)
3: \( \text{Initialize } \Theta^t \text{ randomly} \)
4: \( \text{while } t \leq \text{MaxIter} \text{ do} \)
5: \( \text{Generate } \Gamma^t \text{ samples from } \mathcal{N}(\Theta^t, \Sigma^t) \)
6: \( \text{for } i = 1 \text{ to } \Gamma^t \text{ do} \)
7: \( \Gamma^t[i, 1 : R] \to \text{Algorithm 3} (i, \Gamma^t[i, 1 : R], R_k, h_r, h_s, g_{r,k}^m, g_{s,k}^m) \)
8: \( \Gamma^t[i, R + 1 : R + K] \to \text{Algorithm 4} (i, \Gamma^t[i, R + 1 : R + K], R_k, h_r, h_s, g_{r,k}^m, g_{s,k}^m) \)
9: \( \text{end for} \)
10: \( \text{Evaluate fitness function for these } \Gamma^t \text{ samples} \)
11: \( \text{Sort the } \Gamma^t \text{ data samples according to their fitness value} \)
12: \( \Gamma^t \leftarrow F_{(1-\rho)^\Gamma} \)
13: \( \text{Select the samples whose fitness values are greater than } \gamma^t \)
14: \( \text{Select } \rho \Gamma \text{ best samples among } \Gamma \text{ samples to generate } \Theta^{t+1} \)
15: \( \text{if } \text{IsAdaptiveUpdate} \text{ then} \)
16: \( \text{Run Adaptive Alpha Update Routine} \)
17: \( \text{end if} \)
18: \( \Theta^{t+1} \leftarrow (1 - \alpha)\Theta^t + \alpha \Theta^t \)
19: \( \Sigma^{t+1} \leftarrow (1 - \alpha)\Sigma^t + \alpha \Sigma^t \)
20: \( \text{if } \text{IsPrematureAvoid} \text{ then} \)
21: \( \text{Run Premature Avoidance Routine} \)
22: \( \text{end if} \)
23: \( t \leftarrow t + 1 \)
24: \( \text{end while} \)

C. Premature Convergence Avoidance and Adaptive Smoothing of CEO

To avoid premature convergence, the algorithm resets \( \Theta^t \) and \( \Sigma^t \) if their values are larger than some specified threshold \( \lambda \). Furthermore, we use a smoothing parameter \( \alpha \) that has a significant impact on the performance of the cross-entropy method. In this paper, we propose two techniques to adaptively adjust the smoothing parameters. Both methods adjust the smoothing parameters with the number of iterations. We call the methods as uniform adaptation and logarithmic adaptation. Let \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \) be the minimum and maximum values of the smoothing parameters, respectively. Let \( \pi \) be the maximum
number of iterations and $\alpha_{\text{uni}}(t + 1) = \alpha_{\text{log}}(1) = \alpha_{\text{min}}$. We can define uniform adaptation and logarithmic adaptation as

$$\alpha_{\text{uni}}(t + 1) = \alpha_{\text{uni}}(t) + \left( \frac{t}{\pi - 1} \right) (\alpha_{\text{max}} - \alpha_{\text{min}})$$

(8)

$$\alpha_{\text{log}}(t + 1) = \alpha_{\text{log}}(t) + \left( \frac{\log \left( \frac{t}{\pi - 1} \right)}{\pi - 1} \right) (\alpha_{\text{max}} - \alpha_{\text{min}}).$$

(9)

Next, we will explain constraint satisfaction routines that will ensure the feasibility of the solution.

**Algorithm 3: Constraint Satisfaction for Relay Assignment and Relay Power (CSRRP)**

1: **Input**: $i, \Gamma^i[i, 1: R], h_k, h_r, h_r, r, g_m, r, g^m_k$
2: **Initialization**: $\bar{p}_i = p^i_1, S(i) = 0, \forall i, 1, 2, \ldots, R, C_k = 0, \forall k$
3: **Stage 1**: Sort the relays according to the channel
4: $i \leftarrow 1$
5: while $i \leq R$ do
6: $S(i) \leftarrow \arg \max_{k \in \{1, 2, \ldots, K\}} |h_i| |h_{i,k}| / \max(|g^1_{i,k}|^2, |g^2_{i,k}|^2, \ldots, |g^M_{i,k}|^2)$
7: $i \leftarrow i + 1$
8: end while
9: **Stage 2**: Perform assignment and relay power repair
10: $\bar{p}_i \leftarrow \min(p_i, \Gamma^i_{\text{max}} / |g^1_{i,k}|^2, \ldots, \Gamma^i_{\text{max}} / |g^M_{i,k}|^2)$, $\forall i, 1, 2, \ldots, R$
11: $k \leftarrow 1$
12: while $k \leq K$ do
13: $\Psi_k \rightarrow \{i | S(i) = k\}$
14: if $\Psi_k \neq \emptyset$ then
15: $flag \leftarrow 0$
16: while $flag \leftarrow 0$ do
17: if $I_{\Psi_k} \leq \Gamma^i_{\text{max}}/\gamma_{\text{min}}$ then
18: $flag \leftarrow 1$
19: $C_k \rightarrow $ Get capacity of the $k$th user with $\Psi_k$
20: $x_r \in \Psi_k \leftarrow 1$
21: else
22: $i \leftarrow \arg \max_{i \in \Psi_k} I_{\Psi_k}$
23: $\Psi_k \leftarrow \Psi_k \setminus \{i\}$
24: end if
25: end while
26: end if
27: $k \leftarrow k + 1$
28: end while
29: $\Gamma^i[i, 1 : R] = \bar{p}_i, \forall i, 1, 2, \ldots, R$
30: **Output**: $\Gamma^i[i, 1 : R]$

**D. CSRRP**

Here, we will present iterative CSRRP allocation. This routine will be executed on each cross-entropy individual in the samples. A pseudocode of the routine is presented in Algorithm 3. The proposed algorithm has two stages: In the first stage, based on channel conditions, relays are assigned to the SUs without satisfying the interference constraint. In the second stage, the algorithm performs final assignment under the constraint that interference to the PUs is satisfied.

For developing this routine, we can view the product of the channel from the $i$th relay to the $k$th SU and the channel from the source to the $i$th relay as profit taken from investing one unit transmission power to relay $i$. We also view the channel from the $i$th relay to its PUs as a loss. In particular, our algorithm views $\max(|g^1_{i,k}|^2, |g^2_{i,k}|^2, \ldots, |g^M_{i,k}|^2)$ as a loss incurred from investing unit transmission power to relay $i$.

**Stage 1**: In this stage, the algorithm assigns each relay to the SU that gives the maximum profit-to-loss ratio. The profit is the SU’s channel gain, i.e., $|h_i| |h_{i,k}|$, and the loss is the maximum channel gain from the SU to the PUs, i.e., $\max(|g^1_{i,k}|^2, |g^2_{i,k}|^2, \ldots, |g^M_{i,k}|^2)$. Mathematically, for each relay $i$, the algorithm temporarily assigns SU $i$

$$S(i) = \arg \max_{k \in \{1, 2, \ldots, K\}} \frac{|h_i| |h_{i,k}|}{\max(|g^1_{i,k}|^2, |g^2_{i,k}|^2, \ldots, |g^M_{i,k}|^2)}$$

where $S$ is an $R$-dimensional vector that stores this assignment. At the end of stage 1, relays are assigned to the SUs with the power $\bar{p}_i, i = 1, \ldots, R$.

Note that the relays’ power levels randomly generated by the cross-entropy algorithm can violate the constraint of limited interference to the PUs. In the next stage, the algorithm refines the relay assignment carried out in stage 1 and adjusts the power level of each relay so that interference to the PUs is satisfied.

**Stage 2**: At the start of the second stage, the algorithm starts adjusting the relays’ power levels if there is a violation of the PU’s interference constraints. First, the algorithm examines for each relay $i$ whether its transmission power would violate any interference constraint even if all the other relays’ power levels were set to zero. If $\bar{p}_i$ violates any of the interference constraints, $I^\text{max}_{m,k}$, even under the assumption that other relays’ transmission power levels are all set to 0, the algorithm first makes the following adjustment:

$$\tilde{p}_i = \min (\bar{p}_i, \frac{I^\text{max}_{m,k}}{|g^1_{i,k}|^2, |g^2_{i,k}|^2, \ldots, |g^M_{i,k}|^2})$$

With this power adjustment, each user individually guarantees constraint satisfaction of the PU’s interference constraint. After the power adjustment, the algorithm iterates over the SUs and completes the assignment of relays. At the $k$th iteration, the algorithm determines the set of relays $\Psi_k$ that are assigned to the $k$th SU in stage 1. Then, the algorithm checks whether the relays in the set $\Psi_k$ satisfy the interference constraint at all the PUs. If the relays in the set $\Psi_k$ violate the interference constraint at any PU, the algorithm iteratively removes the relay from the set $\Psi_k$ that causes maximum interference to the PUs. Mathematically, this determines the relay with the highest interference from the set $\Psi_k$ as

$$i = \arg \max_{i \in \Psi_k} I_{\Psi_k}.$$ 

This relay removal process continues until the relays in the set $\Psi_k$ satisfy the interference constraint. When the algorithm
has a set of relays that satisfy the interference constraint, the algorithm determines the capacity of the \( k \)th user and sets \( x_{rk} = 1 \). The algorithm executes until all the SUs get their assigned relays. In the next subsection, we will present CSSP.

### Algorithm 4: Constraint Satisfaction for Source Power (CSSP)

1. **Input:** \( i, \Gamma^i, |i, R + 1 : R + K|, h_k, h_r, h_{r,k}, g_k^m, g_{r,k}^m \)
2. **Initialization:**
3. \( \xi_k = p_k^i, \forall i = R + 1, R + 2, \ldots, R + K, \xi_s(i) \leftarrow 0, \forall i \leftarrow 1, \ldots, K, \xi_k \rightarrow \{1, 2, \ldots, K\} \)
4. **Stage 1: Sort the source to relay channel**
5. while \( \xi_k \neq \emptyset \) do
6. \( \xi_s(k) \leftarrow \arg \max_{k \in \xi_k} |h_{i,k}|/|h_{i,k}| / \max\{|g_{1,k}^1|, |g_{1,k}^2|, \ldots, |g_{M,k}^M|\} \)
7. \( \xi_k \leftarrow \xi_k \setminus \{k\} \)
8. end while
9. **Stage 2: Perform source power repair**
10. \( i \leftarrow 1 \)
11. \( \tilde{p}_k \leftarrow \min\{\tilde{p}_k, \max_{k \rightarrow 1} |g_{s,1}|^2, \max_{k \rightarrow 2} |g_{s,2}|^2, \ldots, \max_{k \rightarrow M} |g_{s,M}|^2\} \)
12. while (1) do
13. \( \text{if } \sum_k \tilde{p}_k \leq P \text{ then} \)
14. \( \text{break} \)
15. end if
16. \( \tilde{p}_{s,k,i} \leftarrow \tilde{p}_{s,k,i} / \delta \)
17. \( i \leftarrow i + 1 \)
18. \( \text{if } i > K \text{ then} \)
19. \( i \leftarrow 1 \)
20. end if
21. end while
22. \( \Gamma^i[i, R + 1 : R + K] = \tilde{p}_k, \forall k \)
23. **Output:** \( \Gamma^i[i, R + 1 : R + K] \)

### CSSP

A pseudocode of the routine is presented in Algorithm 4. We denote by \( \tilde{p}_k = p_k^i, \forall i = R + 1, R + 2, \ldots, R + K \), the source power level at the \( k \)th user band in the \( j \)th sample drawn by the CEO. We denote by \( \xi_s \) a vector that will be used for user indices, \( \xi_k \) a set of users, and \( \delta \) a power control factor. The power control factor is used for source power adjustment. This adjustment will be done iteratively until all the interference constraints relating to the source power are satisfied. The proposed CSSP is also a two-stage algorithm.

**Stage 1:** In the first stage, users are ranked according to their channel conditions. Similar to the CSRRP, maximum profit-to-loss ratio criterion is used to rank the users. The profit is the SUs' channel gain, i.e., \( |h_i||h_{i,k}| \), and the loss is the maximum channel gain from the SU to the PUs, i.e., \( \max\{|g_{1,k}^1|^2, |g_{1,k}^2|^2, \ldots, |g_{M,k}^M|^2\} \). Mathematically

\[
\xi_s(i) = \arg \max_{k \in \xi_k} \frac{|h_i||h_{i,k}|}{\max\{|g_{1,k}^1|^2, |g_{1,k}^2|^2, \ldots, |g_{M,k}^M|^2\}}.
\]

In the next stage, the power of each user will be adjusted according to their ranks.

**Stage 2:** At the start of the second stage, the algorithm starts adjusting the source power levels if there is a violation of the PU's interference constraint using the expression

\[
\tilde{p}_k = \min\{\tilde{p}_k, \max_{k \rightarrow 1} |g_{s,1}|^2, \max_{k \rightarrow 2} |g_{s,2}|^2, \ldots, \max_{k \rightarrow M} |g_{s,M}|^2\} \quad \forall k.
\]

After the source power adjustment with the interference constraint, the algorithm verifies the power constraint. If the source power constraint is not satisfied, then the algorithm adjusts the source power using power control factor \( \delta \) until the constraint is satisfied. The user with the worst channel condition is reduced first using the power control factor \( \delta \). This process will be executed for all users until we get a feasible solution. At the end of the algorithm, we shall have a feasible solution. As we are using half-duplex amplify-and-forward protocol, both CSRRP (Algorithm 3) and CSSP (Algorithm 4) will execute independently.

### V. Numerical Results

Here, we present simulation results to demonstrate the performance of the proposed CEO algorithm. The impact of network parameters (e.g., number of SUs and number of relays) is also investigated.

**A. Simulation Setup and Results**

In all simulations, the channels between sources, relays, and destinations have independent complex Gaussian distribution. The channel gain \( h \) is modeled as \( h = \Phi K_o (d_o/d)^{\beta} \), where \( K_o \) is a constant that depends on the antenna characteristic and average channel attenuation, \( d_o \) is the reference distance for the antenna far field, \( d \) is the distance between transmitter and receiver, \( \beta \) is the path loss constant, and \( \Phi \) is the Rayleigh random variable [36]. Since this formula is not valid in the near field, we assume that \( d \) is greater than \( d_o \) in all the simulation results. In addition, \( d_o = 20 \) m, \( K_o = 50 \), and \( \beta = 3 \) in all the results. The PU’s protected distance \( R_m \) is set to 10 m. The SUs and PUs are uniformly distributed. All simulations are performed using Monte Carlo runs. For each PU, there is a PU protection area wherein the strength of the CR signals must be constrained. Unless otherwise specified, we use the following common simulation values: \( P_s = 1 \) W, \( P_s = 10 \) W, \( \alpha = 0.3 \), \( \Gamma = 100 \), and \( \rho = 0.5 \), \( w_1 = w_2 = 0.5 \).

We compare the proposed cross-entropy method with a combination of BBOA. Figs. 3–5 present the fitness (objective) function versus iterations plots for different numbers of relays, PUs, and SUs. The fitness function is \( w_1 (1 - f_s) + w_2 f_{CHG} \). For Figs. 3–5, the parameters are set to \( (R, M, K, f_{max}) = (\{8, 12\}, 1, 6, 1 W), (8, 1, 5), 8, 100 \) mW, and \( 12, 1, 8, 10 \), 1 W, respectively. In the plots, the term LB is the lower bound obtained using BBOA, as mentioned in Section III, and CEO is the cross-entropy method. From the results, we can observe that the performance of cross entropy is very close to the LB obtained by BBOA. We can also note that the fitness values with greater number of relays are better than the fitness values with lesser relays for a fixed number of SUs. This is because greater number of relays gives more degree of freedom for assignment.
In the simulation results, we also observe that the objective function is better at low number of PUs. The reason is that the relay assignment needs to satisfy more interference constraints as the number of PUs increases.

Fig. 6 focuses on the method of applying thresholds on cross entropy to avoid premature convergence. The parameters for simulations are set to $(R, M, K, I_{\text{max}}) = (10, 6, 5, 100 \text{ mW})$. The values of threshold are $\lambda = \{0, 0.05, 0.1, 0.3\}$. Note that setting $\lambda = 0$ means not applying any threshold. This means that, after some iterations, the algorithm generates the samples from an almost identical distribution in each iteration, i.e., the algorithm does not take advantage of the randomization. A proper setting of $\lambda$ can facilitate the algorithm for early convergence. From Fig. 6, we can observe that $\lambda$ equal to 0.05 and 0.1 give better results than $\lambda$ equal to 0.3 and 0. A higher value of $\lambda$ means more randomization that eventually decreases the performance of CEO. Fig. 7 presents the results of adaptation on the parameter $\alpha$. The results show that adaptation increases the convergence of the cross-entropy algorithm and that the logarithmic adaptation performs better than the uniform adaptation.
VI. CONCLUSION

In this paper, we have presented a multiobjective GCCR network that uses multiple relay assignment and source/relay power allocation in SABF relaying. The proposed problem is a mixed-integer nonlinear nonconvex optimization problem. We presented CEO with constraint satisfaction algorithms to solve the MOO problem. We also proposed schemes to avoid premature convergence and adaptation in the cross-entropy method. We compared the results with LB that uses BBOA approach. The results show that the proposed cross-entropy method is close to the LB. Simple underlying concept and ease of implementation of the proposed cross-entropy algorithm make it a suitable candidate for GCCR network.

REFERENCES

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