Energy-Efficient Frequency and Power Allocation for Cognitive Radios in Television Systems

Kandasamy Illanko, Muhammad Naeem, Alagan Anpalagan, and Dimitrios Androutsos

***Abstract*—Energy-efficient resource allocation for cognitive radios operating in television systems (TV white spaces) presents a unique challenge compared with other cognitive radios because the interference constraint is for the whole frequency band rather than per carrier. This paper presents a subchannel and power allocation protocol that maximizes the energy efficiency (EE) of transmissions from a cognitive base station operating in the TV white spaces. The system model conforms to the IEEE 802.22 standard, and the proposed two-step solution to the EE maximization problem satisfies users' minimum rate requirements and keeps the interference to the primary users in the neighboring areas below a specified threshold. The first step of the protocol is a near-optimal but low-complexity subchannel assignment. This is followed by an optimal power allocation procedure that is obtained by analyzing the Karush–Kuhn–Tucker conditions. The computational complexity of the resulting resource allocation protocol is the same as that of the least complex resource allocation protocol for orthogonal frequency division multiple access (OFDMA) downlinks in the literature. Simulation results show that our protocol achieves higher EE compared with a modified and improved version of the OFDMA protocol from the literature.**

***Index Terms*—Channel assignment, Charnes–Cooper transformation, cognitive radio, energy efficiency, green communications, optimization, power allocation, resource allocation, TV white spaces.**

I. Introduction

According to a report [1], powering cellular base stations around the world would have cost approximately \$36 billion in 2012. In large base stations, the power amplifier typically takes 67% of the power, with another 11% for air conditioning, for a total of 78% of electricity consumption [1]. If not for economical, environmental reasons make it crucial that we improve the energy efficiency (EE) of wireless communication systems [2], [3].

Cognitive radio [4] has been proposed as a general approach for higher EE in wireless communication systems for two reasons. First, the EE-related functionalities can be embedded into the cognitive operational cycle. Second, from the green perspective, spectrum is a natural resource, which should not be wasted on idle licensed channels but be shared [5].

TV white spaces are vacant frequencies made available for unlicensed use at locations where spectrum is not being used by licensed services, such as television broadcasting. This spectrum is located in the very high frequency (54–216 MHz) and ultrahigh frequency (470–698 MHz) bands and has characteristics that make it highly desirable for wireless communications [6]. A cognitive radio standard, IEEE 802.22 has been released as early as 2009 to guide the design of devices operating in TV white spaces [7]. The focus of this paper is the frequency and power allocation that maximizes the bit/joule/hertz EE of orthogonal frequency division multiple access (OFDMA)-based transmissions from a cognitive base station operating in the TV white spaces.

Energy consumption of an OFDM-based cognitive radio network is indirectly addressed in [8], by maximizing the expected capacity. Transmission duration and power in a cognitive radio network are the variables in [9], which maximize the number of bits transmitted per unit of energy in a frame. Parameterized convex programming is used in [10] to minimize the energy spent per bit of reliably transmitted information. Conserving the energy was the target of [11], where adaptive and distributed beamforming was used to direct the main transmission beam toward cognitive users while creating nulls to licensed users.

To the best of our knowledge, there are no studies that directly address the bit/joule/hertz EE of transmissions from a cognitive base station in the TV white space. Lack of articles dealing with cognitive radio EE in general is understandable, because cognitive radio EE maximization problem adds only one more constraint to the OFDMA EE maximization problem—a per-carrier interference constraint. However, as we explain in detail in the next section, maximizing the EE of cognitive radios in the TV white spaces presents a whole new problem because the interference constraint is for the whole vacant frequency band. In the following, we give an overview of the research that is closest to that undertaken in this paper, which is the EE of OFDMA-based transmissions.

A number of studies optimizing the EE of OFDMA-based transmissions consider only the power allocation. Since EE is not concave in the powers, a search method is used in [12] to find the transmission rates that maximize the EE. The computational complexity of the method in [12] is at least polynomial in the ratio of the number of subchannels to the resolution of the search method. Reference [13] uses Charnes–Cooper transformation to convert the optimization problem into a concave one and then uses a fixed-point algorithm to find the optimal power

Manuscript received November 29, 2013; revised October 20, 2014; accepted December 3, 2014. Date of publication February 17, 2015; date of current version February 23, 2016. This work was supported in part by the National Sciences and Engineering Research Council of Canada through Discovery Grants.

K. Illanko, A. Anpalagan, and D. Androutsos are with the Department of Electrical and Computer Engineering, Ryerson University, Toronto, ON M5B 2K3, Canada (e-mail: killanko@rnet.ryerson.ca; alagan@ee.ryerson.ca; dimitri@ee.ryerson.ca).

M. Naeem is with the Wireless Networks and Communications Research Laboratory, Ryerson University, Toronto, ON M5B 2K3, Canada, and also with the Department of Electrical Engineering, COMSATS Institute of Information Technology (IT), Islamabad 33597, Pakistan (e-mail: muhammadnaeem@gmail.com).

Digital Object Identifier 10.1109/JSYST.2015.2393834

allocation. Their algorithm has linear complexity in the number of subchannels.

Xiong *et al.* [14] considered both subchannel and power allocation that maximize the EE of OFDMA-based transmissions. They started by proposing an optimal power allocation procedure that maximizes the EE for a given subchannel assignment. They then commented that this procedure has too high a complexity to be practical and offered a near-optimal power allocation procedure, which is based on optimizing an upper bound on the EE. The subchannel assignment proposed by Xiong *et al.* is heuristic and uses the same upper bound on the EE to rank the users to facilitate the channel assignment. It should be noted that, of the work that consider both subchannel and power allocation, Xiong *et al.* [14] had the solution with the lowest computational complexity. Reference [15] uses the same algorithm as in [12] for power allocation, but their subchannel assignment algorithm has a complexity that is at least a few orders of magnitude greater than the algorithm of Xiong *et al.* [14].

This paper proposes a subchannel and power allocation protocol that maximizes the EE of transmissions from a cognitive base station operating in the TV white spaces. The proposed protocol consists of a two-step approach where a near-optimal but low-complexity subchannel assignment is followed by an optimal power allocation. A low-complexity subchannel assignment is used because an optimal one would be prohibitively computationally expensive. The EE is maximized subject to the constraints, namely, minimum acceptable user rates, a total power constraint, and an interference constraint to protect the primaries or other devices operating in nearby areas. We compare the complexity of our protocol with that of modified and improved versions of existing resource allocation protocols that maximize EE of OFDMA downlinks. We provide support for the analysis using simulated numerical results and demonstrate that our protocol achieves higher EE when compared with improved and modified versions of existing protocols.

Contribution: The following list summarizes the unique contribution of this paper.

1) Unlike the popular method of using iterative algorithms to solve the Lagrangian dual problem, this paper analyzes the Karush–Kuhn–Tucker (KKT) conditions to find the optimal power allocation strategy.
2) It is shown that, for a given subchannel assignment, the power allocation that maximizes the EE, when it occurs inside the cognitive interference constraint, can be obtained by solving a *single* nonlinear equation regardless of the number of users or subchannels.
3) The proposed power allocation procedure that maximizes the EE (for a given subchannel assignment) has constant time complexity.
4) For the case where the maximum EE occurs on the power constraint plane, a closed-form solution is provided.
5) A power and frequency allocation protocol for maximizing the EE of cognitive radio is proposed. It has the same complexity as the protocol of Xiong *et al.* [14], which is only for OFDMA downlink and not for cognitive radios. Furthermore, unlike the protocol of Xiong *et al.*, which only works with near-optimal EE, our algorithm finds the optimal EE.

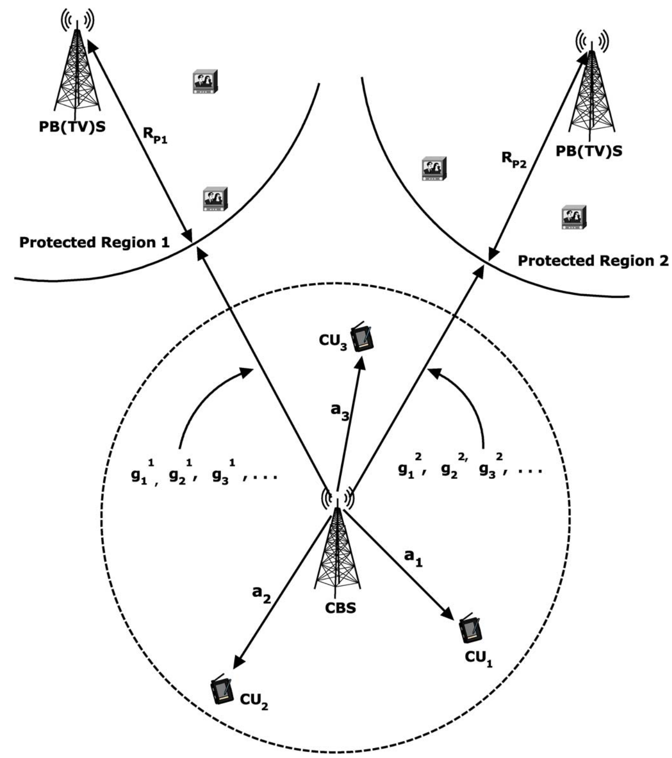

Fig. 1. Cognitive cell.

6) We modify the protocol of Xiong *et al.* [14] to suit cognitive radios and improve their algorithm by making it work with our optimal EE instead of near-optimal EE and compare it with our protocol using simulations. We demonstrate that our protocol produces better results than the modified and improved protocol of Xiong *et al.*

The rest of this paper is organized as follows. Section II describes the system model and forms the optimization problem. The analysis of the problem and the solution in terms of the subchannel and power allocation protocol are presented in Section III. Simulation results that support the analysis, demonstrate the resource allocation protocol in action, and compare it with existing protocols are given in Section IV. This paper concludes in Section V.

II. System Model

The system model considered in this paper conforms to the IEEE 802.22 standard [7], and as much as possible, we will use the terminology of the same standard in describing our model. Consider the OFDM-based transmissions on the downlink of a cognitive cell shown in Fig. 1. Transmissions from the cognitive base station (CBS) to the N cognitive users (CU) are to take place over a TV frequency band that is licensed to a primary user (PU) but is not in use at this time in the area of the cognitive cell. The conclusion that this frequency band is not in use is arrived at either by sensing or by accessing a database, exactly as suggested in the IEEE 802.22 standard [7], but is beyond the scope of this paper. The IEEE 802.22 standard divides the frequency band into 1680 subcarriers, but states that the elementary unit for resource allocation is the subchannel, which consists of 28 subcarriers. Therefore, the elementary unit for frequency allocation in this paper will be a subchannel (and there will be 60 of these). Since the frequency bandwidth of the TV channels varies from continent to continent, the standard allows the bandwidth of the subchannels to be scaled accordingly. Therefore, instead of working with subchannels of a particular bandwidth, this paper will work with quantities that are normalized with respect to the bandwidth of the subchannel. In other words, all the results obtained in this paper will be for per hertz of the bandwidth of each subchannel.

We will denote the total number of subchannels by K. These K subchannels are distributed among N users, using a yet to be determined channel assignment protocol, to be used in the downlink transmissions from the CBS to the N cognitive users. While many subchannels may be allocated to one user, any particular subchannel is allocated to only one user. Suppose a total of K_1 subchannels—subchannel 1 through subchannel k_1—is assigned to User 1. A total of K_2 subchannels—subchannel k_1+1 through subchannel k_2—is assigned to User 2 and so on. A total of K_n subchannels—subchannel $k_{n-1}+1$ through subchannel k_n—is assigned to User n.

Let a_k denote the gain on subchannel k from the CBS to the CU to whom this subchannel is assigned and $\aleph_k$ denote the background noise plus interference power in watts per hertz. Since the primary users are TV stations, the transmit powers from these stations are known, and any interference from the primary users to the cognitive users can be dumped with the background noise to form $\aleph_k$. If $h_k = a_k/\aleph_k$ then User n's transmission rate r_n, in bits per second per hertz, can be written as

$$r_n = \sum_{k=k_{n-1}+1}^{k_n} \log_2(1 + h_k p_k) \tag{1}$$

where p_k is the transmission power used on subchannel k. If R_n is the minimum rate acceptable to user n, then we must have

$$r_n - R_n \geq 0. \tag{2}$$

On the outskirts of this area, there are other primary users (TV stations) that might be using the same frequency band. It is also possible that, in the outskirts, there may be other cognitive devices that use this frequency band. The power levels of the downlink transmissions from our CBS must be controlled to protect these users [8]. In order to do this, each primary base station in the outskirts is designated as a protected area [8], and the cognitive transmissions are required to limit their interference at the edge of the protected areas.

We consider an interference threshold I on the entire primary frequency band. This was motivated by the fact that, in the IEEE 802.22 standard [7], the primary base stations represent the TV stations, and they would only specify a tolerable interference on the entire TV band. Furthermore, if another cognitive device is operating in the outskirts, it may decide to divide the frequency band in a different way or not divide the frequency band at all. Note that dividing I by the number of subchannels and placing individual interference constraints on each subchannel would place too much restriction on the transmission rates of the users. Since green communications philosophy advocates squeezing every bit of performance from every joule of energy, such conservative per subchannel interference constraints are not suitable.

Referring to Fig. 1, let g_k^i denote the gain on subchannel k from the cognitive base station CBS to the edge of the protected area i. If the power used on subchannel k is p_k, then the total interference measured at the edge of the protected area i will be

$$I_i = \sum_k g_k^i p_k. \tag{3}$$

If the interference that can be tolerated by an incumbent user in any of the protected areas is I, then we would like

$$\max_i \{I_i\} \leq I. \tag{4}$$

Using (3)

$$\max_i \left\{ \sum_k g_k^i p_k \right\} \leq I. \tag{5}$$

However

$$\max_i \left\{ \sum_k g_k^i p_k \right\} \leq \sum_k \max_i \{g_k^i\} p_k. \tag{6}$$

Letting

$$g_k = \max_i \left\{ g_k^i \right\} \tag{7}$$

$$\max_i \left\{ \sum_k g_k^i p_k \right\} \leq \sum_k g_k p_k. \tag{8}$$

Suppose we make sure that (this is the condition that we later impose on the system model)

$$\sum_k g_k p_k \leq I. \tag{9}$$

Then from (8), we would have

$$\max_i \left\{ \sum_k g_k^i p_k \right\} \leq \sum_k g_k p_k \leq I \tag{10}$$

and this is the same as

$$\max_i \{I_i\} = \max_i \left\{ \sum_k g_k^i p_k \right\} \leq \sum_k g_k p_k \leq I. \tag{11}$$

The preceding argument shows that

$$\sum_k g_k p_k \leq I \Longrightarrow \max_i \{I_i\} \leq I. \tag{12}$$

In other words, we would like $\max_i \{I_i\} \leq I$ to be true, but making sure that it happens directly is very difficult. However, imposing $\sum_k g_k p_k \leq I$ is easy, and that makes sure that $\max_i \{I_i\} \leq I$ is true.

In practice, g_k is easily obtained from (7). It is the maximum among all the gains on subchannel k from the cognitive base station to the different edges of the protected areas.

If P_T is the maximum power allowed on the OFDM transmissions on this frequency band, then we have

$$\sum_{k=1}^{K} p_k \leq P_T. \tag{13}$$

The purpose of this paper is to present a two-step resource management strategy that maximizes the EE of the downlink transmissions subject to the constraints in (2), (9), and (13). The EE of the transmissions in bits per joule per hertz can be written as

$$\frac{\sum_k \log_2(1 + h_k p_k)}{p_c + \psi \sum_k p_k} \tag{14}$$

where p_c is the circuit power [12], [14], and ψ is the reciprocal of the efficiency of the power amplifier.

Suppose a suitable channel assignment strategy has been used to assign the subchannels to the users. Then the power allocation among the K subchannels that maximizes the EE can be determined by solving the following optimization problem:

$$\begin{aligned} &\max_{\mathbf{p}} \frac{\sum_k \log_2(1 + h_k p_k)}{p_c + \psi \sum_k p_k} \\ &\text{subject to} \\ &\quad \mathrm{C1}: I - \sum_k g_k p_k \geq 0 \\ &\quad \mathrm{C2}: P_T - \sum_k p_k \geq 0 \\ &\quad \mathrm{C3}: r_n - R_n \geq 0, \qquad \text{for } n = 1, 2, \ldots, N. \end{aligned} \tag{15}$$

III. Analysis and Solution

Here, we develop a resource allocation protocol to maximize the EE of the downlink transmissions from a cognitive base station operating in the TV white space. As mentioned before, an optimal channel assignment protocol that satisfies the rate requirements would have too high a complexity to be useful in practice. Because of this, we first present a low-complexity channel assignment protocol. Then, we analyze the power allocation problem in (15) and propose a power allocation procedure that has constant complexity. We finish this section by comparing the complexity of our resource allocation protocol with a particular one in the literature that maximizes the EE of OFDMA transmissions from a base station.

A. Channel Assignment

To increase the system EE, one should assign each subchannel to the user with the largest gain on that subchannel. On the other hand, from a fairness point of view, one might want to assign each user the subchannel for which this user has the greatest channel gain. We take an approach that balances these two perspectives.

The channel assignment protocol we propose starts by reordering or reindexing the users in the descending order of their minimum rate requirements. After that, it alternates between the following two rounds until all subchannels are assigned. On the first round, each user is assigned to the subchannel on which it has the largest gain. Note that the reordering of the users enables the users with the higher rate requirements to pick the best subchannels in this first round. On the second round, for each user, it is checked if that user happens to be the user with the largest gain on any particular subchannel; if so, that subchannel is assigned to that user. We now give a more formal description of the proposed protocol, which we call Ratefair.

The Ratefair Channel Assignment Protocol

1. Relabel the users in the descending order of their minimum rate demands.
2. while (there are subchannels left to assign)
3.   for $n = 1$ to N
4.   while (there are subchannels left to assign)
5. $\tilde{k} := \arg\max_k \{a_{kn}\}$
6.   assign subchannel $\tilde{k}$ to user n.
7.   remove subchannel $\tilde{k}$ from available subchannels.
8.   end while
9.   end for
10.   for $i = 1$ to N
11.   while (there are subchannels left to assign)
12.   for every k in available subchannels
13. $\tilde{n} : \arg\max_n \{a_{kn}\}$
14.   if $i = \tilde{n}$
15.   assign subchannel k to user i.
16.   remove subchannel k from available subchannels.
17.   end if
18.   end for
19.   end while
20.   end for
21. end while

After the subchannel assignments, we can determine the power allocation that maximizes the EE by solving the optimization problem (15) given in the previous section. Before we do that, however, it is important to check if a solution is feasible at all.

B. Feasibility and Admission Control

The constraints C1 through C3 in our optimization problem (15) immediately pose a feasibility problem. If one or more of the minimum rate requirements or the associated interfering channel gains g_k are too large or if the direct channel gains a_k are too small, we will not be able to meet these constraints simultaneously. As a consequence, we may not have any solution to the optimization problem. In order to avoid this, we add the following admission control protocol as a prerequisite to our power allocation strategy. This protocol is based on calculating the power levels p_k that minimize the sum interference $\sum_k g_k P_k$ subject to the constraints imposed by the rate requirements of the users and checking if the interference threshold is exceeded or not. The simple optimization problem that finds the required p_k values is solved in Appendix A. The protocol then repeats this for the total power constraint.

1) Obtain the rate requirements R_n from all the users, compute the p_k^I values using (42) in Appendix A, and check if letting $p_k = p_k^I$ for all k satisfies the inequality in constraint C1. That is, check if $I - \sum_k g_k p_k^I \geq 0$. Similarly,

compute the p_k^T values using (43) in Appendix A and check if letting $p_k = p_k^T$ for all k satisfies the inequality in constraint C2. That is, check if $P_T - \sum_k p_k^T \geq 0$.

2) If either C1 or C2 is not satisfied, then do the following until both are satisfied.

Remove the user with the largest $R_n \sum_k (g_k/h_k)$ value.

This admission control protocol has the effect of removing the "worst offenders," that is, the users who are experiencing the worst possible channel conditions and yet expecting a relatively higher quality of service.

C. Power Allocation

The obstacle one faces in solving the optimization problem in (15) comes from the fact that the objective function is not concave in the powers. At this point, we take the same approach as in [13] to overcome this difficulty. However, at a crucial juncture soon, we take a completely different path than what is in [13]. It is this different path that enables us to reduce the computational complexity of the solution considerably.

Since the numerator of the objective function is positive and concave and the denominator is positive and convex (affine), (15) belongs to a class of optimization problems called *concave fractional programs* [19]. Furthermore, a concave fractional program with an affine denominator can be transformed into a concave program using a transformation proposed by Charnes and Cooper [19].

Throughout this paper, we use bold letters to represent vector variables. Consider an optimization problem where we want to maximize the quotient $N(\mathbf{x})/D(\mathbf{x})$ subject to the constraints $M_i(\mathbf{x}) \geq 0, i = 1, 2, \ldots, k$. Here, $N(\mathbf{x}), D(\mathbf{x})$, and each $M_i(\mathbf{x})$ are scalar functions of vector variable $\mathbf{x}$.

Charnes–Cooper Transformation: The concave fractional program $\max\{N(\mathbf{x})/D(\mathbf{x}) | M_i(\mathbf{x}) \geq 0, i = 1, 2, \ldots, k\}$ can be reduced to the concave program $\max\{tN(\mathbf{y}/t) | tM_i(\mathbf{y}/t) \geq 0, i = 1, 2, \ldots, k, tD(\mathbf{y}/t) = 1, t > 0\}$, by using the transformation $t = 1/D(\mathbf{x}), \mathbf{x} = \mathbf{y}/t$.

Proof: Please see [19, Ch. 7, p. 216].

Applying the Charnes–Cooper transformation to our optimization problem (15), by substituting $p_k = y_k/t$ for all k, $t = 1/(p_c + \psi \sum_k p_k)$, and rewriting (15), we obtain the standard concave maximization problem

$$\max_{\mathbf{y}t} \quad f(\mathbf{y}, t) = t \sum_k \log_2 (1 + h_k y_k / t)$$

subject to

$$\begin{aligned} &It - \sum_k g_k y_k \geq 0 \quad P_T t - \sum_k y_k \geq 0 \\ &t(r_n - R_n) \geq 0, t > 0 \quad \psi \sum_k y_k + t p_c - 1 = 0. \end{aligned} \tag{16}$$

Henceforth, as the first line above indicates, we will refer to the objective function by f. The Lagrangian is formed as

$$\begin{aligned} L(\mathbf{y}, t, \lambda, \boldsymbol{\mu}, \nu) &= f + \lambda \left(\psi \sum_k y_k + t p_c - 1 \right) \\ &+ \sum_n \mu_n t(r_n - R_n) + \eta t + \nu(It - \sum_k g_k y_k) + \gamma(P_T t - \sum_k y_k) \end{aligned}$$

where λ, μ, η, ν, and γ are the dual variables.

Since (16) is a standard concave problem, the KKT conditions are necessary and sufficient for optimality. They are

$$\frac{\partial f}{\partial y_k} + \psi\lambda + \sum_n \mu_n \frac{\partial (t r_n)}{\partial y_k} - \nu g_k - \gamma = 0 \quad \text{for all } k \tag{17a}$$

$$\frac{\partial f}{\partial t} + \lambda p_c + \sum_n \mu_n \frac{\partial (t r_n)}{\partial t} + \eta + \nu I + \gamma P_T = 0 \tag{17b}$$

$$\mu_n t(r_n - R_n) = 0 \quad \text{for all } n \tag{17c}$$

$$\eta t = 0 \tag{17d}$$

$$\nu(It - \sum_k g_k y_k) = 0 \tag{17e}$$

$$\gamma(P_T t - \sum_k y_k) = 0 \tag{17f}$$

$$\sum_k y_k + t p_c - 1 = 0 \tag{17g}$$

$$It - \sum_k g_k y_k \geq 0 \tag{17h}$$

$$P_T t - \sum_k y_k \geq 0 \tag{17i}$$

$$\sum_n t(r_n - R_n) \geq 0 \quad \text{for all } n \tag{17j}$$

$$t > 0 \tag{17k}$$

$$\eta \geq 0, \nu \geq 0, \gamma \geq 0, \mu_n \geq 0 \quad \text{for all } n. \tag{17l}$$

Note that (17d) and (17k) together implies $\eta = 0$. Equation (17e) presents us with two cases: either $\nu = 0$ or $It - \sum_k g_k y_k = 0$. Similarly, (17f) presents us with two cases: either $\gamma = 0$ or $P_T t - \sum_k y_k = 0$. Equation (17c), however, leads to $2N$ number of cases because each of the μ_n creates two cases. It appears there are a large number of cases. However, a careful look reveals that we can group all these possibilities into three cases. We start with Case I, where S represents the set of all N users in the system.

Case I: $\nu = \gamma = 0$ and $\mu_n = 0$ for all $n \in S$: We have the following three equations and three inequalities:

$$\frac{\partial f}{\partial y_k} + \lambda\psi = 0 \qquad \text{for all } k \in S \tag{18a}$$

$$\frac{\partial f}{\partial t} + \lambda p_c = 0 \tag{18b}$$

$$\sum_k y_k + t p_c - 1 = 0 \tag{18c}$$

$$It - \sum_k g_k y_k \geq 0 \tag{18d}$$

$$P_T t - \sum_k y_k \geq 0 \tag{18e}$$

$$r_n - R_n \geq 0 \qquad \text{for all } n \in S. \tag{18f}$$

Eliminating λ from (18a) and (18b)

$$\frac{\partial f}{\partial y_1} = \frac{\partial f}{\partial y_2} = \cdots \frac{\partial f}{\partial y_K} = \frac{\psi}{p_c} \frac{\partial f}{\partial t}. \tag{19}$$

From the K equalities on the left, and recalling that $p_k = y_k/t$

$$\frac{1 + h_1 p_1}{h_1} = \frac{1 + h_2 p_2}{h_2} = \frac{1 + h_3 p_3}{h_3} = \cdots = \frac{1 + h_K p_K}{h_K}.$$

If we let

$$w = \frac{1 + h_1 p_1}{h_1} = \frac{1 + h_2 p_2}{h_2} = \frac{1 + h_3 p_3}{h_3} = \cdots = \frac{1 + h_K p_K}{h_K}$$

we have

$$p_k = w - \frac{1}{h_k}. \tag{20}$$

This shows that the optimum power levels, in this case, can be obtained by a water-filling-like solution. Using (20) on the last equality in (19) and performing some simplifications give us the following equation in w, i.e.,

$$F(w) = K\log_2(w) - A - \frac{B}{w} = 0 \tag{21}$$

where $A = K - \log_2(\prod_k h_k)$, $B = p_c/\psi - \sum_k(1/h_k)$,, and K is the number of subchannels. $F(w)$ is a notation we introduce here to facilitate quick later reference to this equation. Since the optimization problem is concave, we expect only one solution to (21). In Appendix B, we prove that this equation indeed has a unique solution.

Suppose the solution of (21) is w^*. Then from (20)

$$p_k^* = w^* - \frac{1}{h_k}. \tag{22}$$

We now check if the solution obtained satisfy the last three inequalities. It can be shown that these p_k values will satisfy the inequality (18d), only if

$$w^* \le \frac{I + \sum_k \frac{g_k}{h_k}}{\sum_k g_k}. \tag{23}$$

Similarly, the inequality (18e) will be satisfied, only if

$$w^* \le P_T + \sum_k \frac{1}{h_k}. \tag{24}$$

From (43) in Appendix A, we can deduce that the last inequality (18f) will be satisfied only if

$$w^* \ge \left[\frac{2^{R_n}}{\prod_{k=k_{n-1}+1}^{k_n} h_k}\right]^{1/K_n} \qquad \text{for all } n \in S. \tag{25}$$

In other words, if w^* satisfies (23)–(25), then (22) is the solution. Suppose w^* satisfies (23) and (24), but (25) is satisfied only for some n, for example, for $n \in S_1$. This leads to Case II. If either (23) or (24) is not satisfied, then that leads to Case III.

Case II: $\nu = \gamma = 0 \mu_n = 0$ for $n \in S_1$, $\mu_n > 0$ for $n \in S_2$: Note that, for those $n \in S_2$, the only way to satisfy the KKT condition (17c) is to let

$$r_n = \sum_{k=k_{n-1}+1}^{k_n} \log_2(1 + h_k p_k) = R_n, \qquad \text{for } n \in S_2. \tag{26}$$

From (25), we know that power levels dictated by the water level

$$w = \left[\frac{2^{R_n}}{\prod_{k=k_{n-1}+1}^{k_n} h_k}\right]^{1/K_n} \qquad \text{for all } n \in S \tag{27}$$

will produce the minimum rates in (26). However, these are not the only power levels that produce those rates. Among the sets of power levels that can produce the rates in (26), we must find the one that maximizes the EE of users in S_2. That is, we must solve the optimization problem

$$\max_{\mathbf{p}} \frac{\sum_{n\in S_2} \log_2(1 + h_k p_k)}{p_c + \psi \sum_{k\in S_2} p_k}$$
$$\text{subject to} \sum_{k=k_{n-1}+1}^{k_n} \log_2(1 + h_k p_k) = R_n, \qquad \text{for } n \in S_2 \tag{28}$$

where $k \in S_2$ means subchannels assigned to users in S_2. At first, it appears that we have to repeat the whole optimization process for the users in S_2. However, there is a clever way out. Notice that this optimization problem is the same as

$$\max_{\mathbf{p}} \frac{\sum_{n\in S_2} R_n}{p_c + \psi \sum_{k\in S_2} p_k}$$
$$\text{subject to} \sum_{k=k_{n-1}+1}^{k_n} \log_2(1 + h_k p_k) = R_n, \qquad \text{for } n \in S_2. \tag{29}$$

Furthermore, the numerator of the objective function and p_c are constants. Therefore, this optimization problem is equivalent to

$$\min_{\mathbf{p}} \sum_{k\in S_2} p_k$$
$$\text{subject to} \sum_{k=k_{n-1}+1}^{k_n} \log_2(1 + h_k p_k) = R_n, \qquad \text{for } n \in S_2. \tag{30}$$

Fortunately, we have already solved this problem in Appendix A. From there, we learn that, for every $n \in S_2$, we must allocate

$$p_k = \left[\frac{2^{R_n}}{\prod_{k=k_{n-1}+1}^{k_n} h_k}\right]^{1/K_n} - \frac{1}{h_k},$$
$$\text{for } k = k_{n-1}+1, k_{n-1}+2, \ldots, k_n. \tag{31}$$

Coincidently, these are the same power levels given by the water level in (27). However, the analysis would have been incomplete without the forgoing.

After assigning these powers to users who fall in S_2, we can remove users in S_2 from our analysis and go to Case I with $S := S_1, P_T := P_T - \sum_{k\in S_2} P_k$ and $I := I - \sum_{k\in S_2} g_k P_k$, where $k \in S_2$ means subchannels assigned to users in S_2. We solve (21) for the reduced set of users in S_1, find a new w^*, and proceed to check the inequalities again.

Case III: $It \sum_k g_k y_k = 0$ or $P_T t - \sum_k y_k = 0$ and $\mu_n = 0$ for all $n \in S$: We have four equations and one inequality, i.e.,

$$\frac{\partial f}{\partial y_k} + \lambda\psi + \nu g_k = 0 \qquad \text{for all } k \in S \tag{32a}$$
$$\frac{\partial f}{\partial t} + \lambda p_c + \nu I = 0 \tag{32b}$$
$$It - \sum_k g_k y_k = 0 \quad \text{or} \quad P_T t - \sum_k y_k = 0 \tag{32c}$$
$$\sum_k y_k + t p_c - 1 = 0 \tag{32d}$$
$$r_n - R_n \ge 0 \qquad \text{for all } n \in S. \tag{32e}$$

From (32a)

$$p_k = \frac{1}{-\lambda\psi - \nu g_k} - \frac{1}{h_k}. \tag{33}$$

This shows that there is no common water level in this case. To obtain the optimum power levels, it appears that we have to solve K nonlinear equations.

However, there is a clever way out of this, and in fact, we can obtain the optimal solution in closed form. Note that this is the case where the solution occurs on one of the planes in (32c). Suppose that the solution occurs in

$$P_T t - \sum_k y_k = 0 \text{ or equivalently on } \sum_k p_k = P_T. \tag{34}$$

The situation we are facing is the same as maximizing the EE subject to the preceding total power constraint. In other words, we are trying to solve the following simple optimization problem:

$$\max_{\mathbf{p}} \frac{\sum_k \log_2(1 + h_k p_k)}{p_c + \psi P_T}$$

$$\text{subject to} \sum_k p_k = P_T.$$

Since the denominator of the objective function is a constant, the preceding problem is the same as the following:

$$\max_{\mathbf{p}} \sum_k \log_2(1 + h_k p_k)$$

$$\text{subject to} \sum_k p_k = P_T.$$

This is the familiar OFDMA throughput maximization problem, for which we know the solution

$$p_k = p_k^{**} = \left[w^{**} - \frac{1}{h_k}\right]^+ \tag{35}$$

where

$$w^{**} = \frac{P_T}{K} + \frac{1}{K}\sum_k \frac{1}{h_k}.$$

Suppose the solution lies on the second of the planes in (32c). That is, on

$$\sum_k g_k p_k = I.$$

There is no clever way out of this case. Instead of solving the K nonlinear equations, we offer a suboptimal but fast solution. Note that (33) shows that p_k increases with g_k in a nonlinear fashion. The suboptimal solution we offer assumes that this relationship is linear. In other words, we let

$$p_k = \rho g_k - \frac{1}{h_k} \tag{36}$$

where ρ is a constant yet to be determined. We can use the fact that our solution lies on the plane $\sum_k g_k p_k = I$ to determine ρ, and thus p_k. This way, we obtain

$$p_k = p_k^{***} = \frac{\left(I + \sum_k \frac{g_k}{h_k}\right) g_k}{\sum_k g_k^2} - \frac{1}{h_k}. \tag{37}$$

To determine which one of p^{**} or p^{***} is the actual solution, we take a look at the original optimization problem in (15). We notice that the solution must lie inside both planes. This forces us to choose the minimum of the two solutions in (35) and (37), i.e.,

$$p_k = \text{Min}\,\{p_k^{**}, p_k^{***}\}. \tag{38}$$

We still have not checked if this solution satisfies inequality (32e). We handle this situation exactly like the way we did in the previous case. Let S_3 represent the set of users for whom the p_k values given by (38) do not produce their minimum rate. For these users, we give the powers dictated by (31), make the following modification to I and P_T:

$$I := I - \sum_{k\in S_3} g_k p_k \text{ and } P_T := P_T - \sum_{k\in S_3} p_k$$

and recalculate p_k^{**} and p_k^{***} from (35) and (37), this time performing the summations on the reduced set of users $S := S - S_3$.

We now incorporate the results of the analysis into our power allocation procedure.

The Optimal Power Allocation Procedure

1) Solve $F(w) = 0$ in (21) and obtain the common water level w^*.
2) If $w^* \geq [2^{R_n}/\prod_{k=k_{n-1}+1}^{k_n} h_k]^{1/K_n}$, for all $n \in S$ and $w^* \leq (I + \sum_{k\in S}(g_k/h_k)/\sum_{k\in S} g_k)$ and $w^* \leq P_T + \sum_{k\in S}(1/h_k)$ then $p_k := w^* - (1/h_k)$ for all $k \in S$.
3) If $w^* \leq (I+\sum_{k\in S}(g_k/h_k)/\sum_{k\in S} g_k)$ and $w^* \leq P_T + \sum_{k\in S}(1/h_k)$ and $w^* \geq [(2^{R_n}/\prod_{k=k_{n-1}+1}^{k_n} h_k)]^{1/K_n}$, for all $n \in S_1$ and $w^* < [(2^{R_n}/\prod_{k=k_{n-1}+1}^{k_n} h_k)]^{1/K_n}$, for all $n \in S_2$ then $p_k = [(2^{R_n}/\prod_{k=k_{n-1}+1}^{k_n} h_k)]^{1/K_n} - (1/h_k)$ for $k = k_{n-1}+1, k_{n-1}+2, \ldots k_n$, and for all $n \in S_2$, and go to (1) with $S := S_1$.
4) If $w^* > (I + \sum_k(g_k/h_k)/\sum_k g_k)$ or $w^* > P_T + \sum_k(1/h_k)$ then Calculate p_k^{**} and p_k^{***} from (35) and (37). Calculate the associated water levels w^{**} and w^{***} by adding $1/h_k$.
   a) If Min $\{w^{**}, w^{***}\} \geq [(2^{R_n}/\prod_{k=k_{n-1}+1}^{k_n} h_k)]^{1/K_n}$ for all $n \in S$ then $p_k := \text{Min}\{p_k^{**}, p_k^{***}\}$ for all k.
   b) If Min $\{w^{**}, w^{***}\} < [(2^{R_n}/\prod_{k=k_{n-1}+1}^{k_n} h_k)]^{1/K_n}$ for $n \in S_3$ then $p_k = [(2^{R_n}/\prod_{k=k_{n-1}+1}^{k_n} h_k)]^{1/K_n} - (1/h_k)$ for $k = k_{n-1}+1, k_{n-1}+2, \ldots k_n$ for all $n \in S_3$, and go to 4) with $I := I - \sum_{k\in S_3} g_k p_k$, $P_T := P_T - \sum_{k\in S_3} p_k$ and $S := S - S_3$.

D. Complexity Comparison

Note that the most computationally expensive part of the Ratefair channel assignment protocol, or any channel assignment protocol for that matter, is the sorting of the channel gains. We will use the well-known result that sorting an array of n elements takes, at worst, a time on the order of $n\log_2(n)$. Ratefair should sort the users according to their rate requirements and then sort each row and column of the channel matrix according to the gain. This would take a time on the order of $N\log_2(N) + NK\log_2(K) + KN\log_2(N)$. As mentioned before, an updated list can be used afterward every time *argmax* is mentioned in the description of Ratefair. We cannot be sure of the number of subchannels that will be assigned in each iteration of the outer loop of the algorithm in steps 9 through 19. However, we are sure exactly N subchannels will be assigned in each run of steps 2 through 8. This means, in the worst case scenario, Ratefair would need K/N iterations of the outer loop to assign all subchannels. Since sorting is performed only once at the beginning, Ratefair has a complexity on the order of $NK\log_2(K) + (K+1)N\log_2(N) + K/N$.

The most computationally expensive part of our power allocation procedure is solving a single nonlinear equation. The time it takes to accomplish this does not depend on the number of users or subchannels. In other words, our power allocation procedure has constant time complexity. Thus, the whole resource management protocol has a time complexity of $NK\log_2(K)+(K+1)N\log_2(N)+K/N+C$, where C is a constant. Note that the dominant term here is $NK\log_2(K)$. Hence, the complexity of our resource management protocol is $O(NK\log_2(K))$.

There are no other EE maximizing resource management protocols for TV white spaces or cognitive radios in the literature for us to compare the complexity with. Of the works dealing with OFDMA downlinks that use a channel assignment protocol along with power allocation, the protocol of Xiong *et al.* [14] has the lowest complexity. Their channel assignment protocol starts by calculating a parameter necessary for the algorithm using binary search over two layers that takes a time on the order of at least KN. Then, the protocol assigns the worst subchannel to each user. After this, the protocol repeats the following two steps until all subchannels are assigned. Approximate the EE of each user by calculating a lower bound on EE. Assign its best subchannel to the user with the worst approximate EE, if the change in EE is greater than a certain predetermined number; if not, try the next user with the worst EE. A binary search is used to approximate the EE, and it takes at least a time of N. Note that, in each iteration of the last step, sorting has to be done anew on the list of user's EE, and in the worst case, one has to go through all N users before assigning a subchannel. Thus, the time complexity of channel assignment amounts to $NK+NK\log_2(K)+(N+N^2\log_2(N))(K-N)/N$. With power allocation, the resource management protocol of Xiong for OFDMA downlink has the time complexity of $NK+NK\log_2(K)+(N+N^2\log_2(N))(K-N)/N+N$.

For large K, we can assume $K \gg N$ and $K-N \simeq K$. Then, the complexity of Xiong reduces to $NK+NK\log_2(K)+NK\log_2(N)+K+N$. The dominant term here, too, is $NK\log_2(K)$. In other words, the protocol of Xiong has the same complexity as ours, namely, $O(NK\log_2(K))$.

Channel gains are obtained by superimposing Raleigh fading and log-normal shadowing on path loss calculated with a coefficient of 4.

Distance to the boundary of the protected regions from the cognitive base station are chosen randomly from the interval [1300, 1500] meters.

SCENARIO - 1: Cognitive cell radius = 1000 meters

Number of users **N** = 6 Number of subchannels **K** = 60
Interference threshold **I** = 10^{-13} Watts/Hz
6 users are placed at randomly chosen distances in the range of [50, 1000] meters from the cognitive base station.

Minimum rate requirements: 55, 50, 45, 40, 35, 30 bits/s/Hz

SCENARIO - 2: **N** = 6 **K** =60 **I** = 10^{-13}Watts/Hz

Cognitive cell radius R is varied from 300 to 1400 meters.
For each R,
6 users are placed at randomly chosen distances in the range of [50, R] meters from the cognitive base station.

SCENARIO - 3: Cognitive cell radius R = 1000 meters

Number of users **N** varied from 10 to 18

Number of subchannels **K** varied from 96 to192

Interference threshold **I** varied from 10^{-13} to 10^{-18} Watts/Hz

Fig. 2. Simulation details.

IV. Simulation Results

A detailed description of the simulation parameters used in this paper is given in Fig. 2. The subchannel gains a_k are calculated using the formula

$$a_k = (d_0/d_k)^n\, R_R R_{\mathrm{LN}} \tag{39}$$

where d_k is the transmitter-to-receiver distance in meters, d_0 is the reference distance in meters, n is the path-loss coefficient, R_R is the Rayleigh random variable, and R_{LN} is the log-normal random variable. n is assumed to be 4, and the reference distance d_0 is taken to be 1 m. It is customary to assume that the losses up to the reference distance are negligible. Since the base station antennas are typically about 1 m in dimension, this assumption makes sense. Please refer to [22] on how to generate the Rayleigh and log-normal random variables R_R and R_{LN}, respectively. For a particular channel realization from Scenario 1, the bar chart on the left in Fig. 3 illustrates the nature of the solution at the optimum water level, after channel assignment by Ratefair. The power levels obtained for the 60 channels turned out to be within the total power constraint P_T and inside the cognitive interference constraint. However, the minimum rate requests of User 2 and User 5 were not met. Step 3 of our power allocation procedure would have taken care of this situation and increased the power levels for the subchannels allocated to these two users until their rate demand were met and then go to step 1. The bar chart on the right in Fig. 3 shows the resulting transmission rates. However, as expected, we notice that this was achieved at a slight reduction in EE.

As mentioned before, the protocol of Xiong *et al.* is designed only for OFDMA downlink and uses an approximation to the optimum EE during the channel allocation part and later for the power allocation. We extended the protocol of Xiong *et al.*

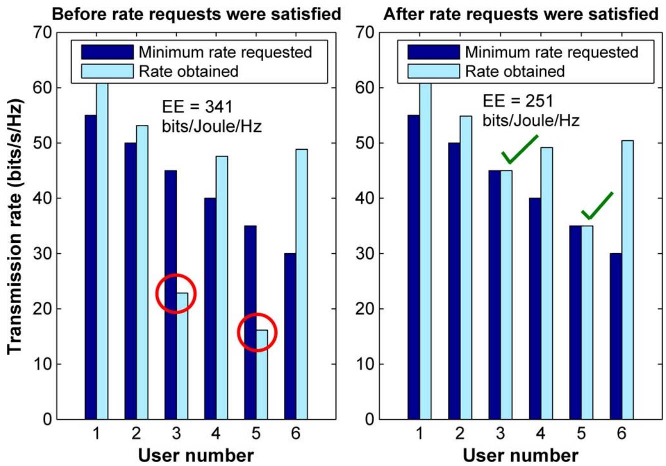

Fig. 3. Nature of the solution for Scenario 1.

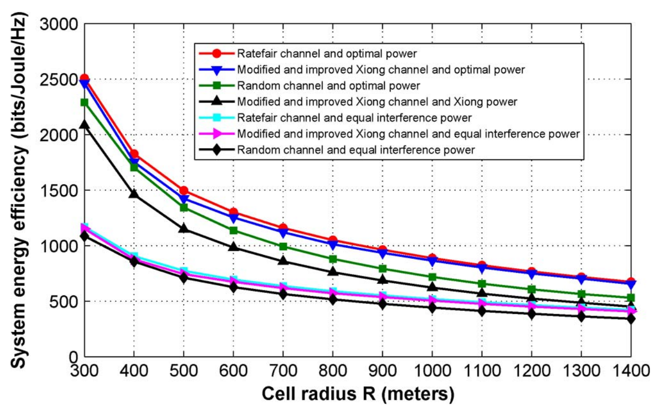

Fig. 4. Scenario 2: Comparing various resource management protocols: Variation of EE with cell radius.

to work with cognitive radios first. Then, we improved their protocol by making it work with the optimum EE.

To compare the various combinations of channel and power allocation protocols, we used Scenario 2 but ignored the rate demands. The graph in Fig. 4 shows the EE obtained by various resource allocation strategies at various cell sizes. For each cell radios, the EE values shown are the averages from 50 different placements of the users within the cell and the corresponding channel realizations. Equal interference power means that $p_k = I/(Kg_k)$. We notice that Ratefair channel assignment and optimal power allocation produce the highest EE. The extended and improved version of Xiong *et al.* comes a very close second. The difference between our protocol and the protocol of Xiong *et al.* extended for cognitive radios, but before the improvement, is striking.

As the cell size approaches 1400 m, we see the difference between the EE values achieved by the optimum power protocols and the equal interference power protocols decreasing. This is because a larger and larger proportion of the optimal solution is now occurring on the cognitive interference plane. Close to the cell size of 300 m, the difference between the best protocols is decreasing. This is particularly noticeable among the top three protocols. This is because in small cell sizes, there is not enough variation in the subchannel gains to bring out the strength of the best protocol. When the subchannel gains are almost the same, intelligent channel assignment cannot result

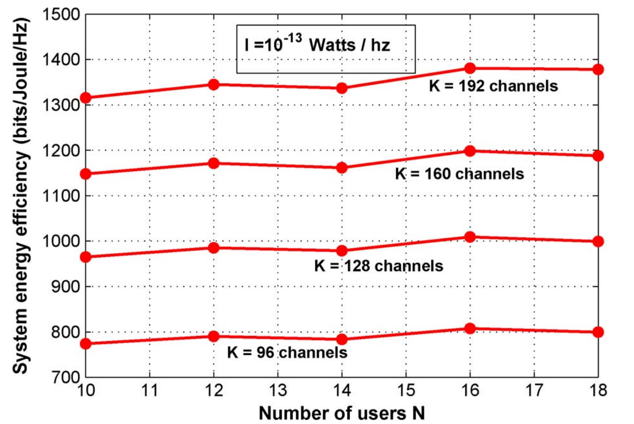

Fig. 5. Scenario 3: Variation of EE with number of users and channels for $I = 10^{-13}$ W/Hz.

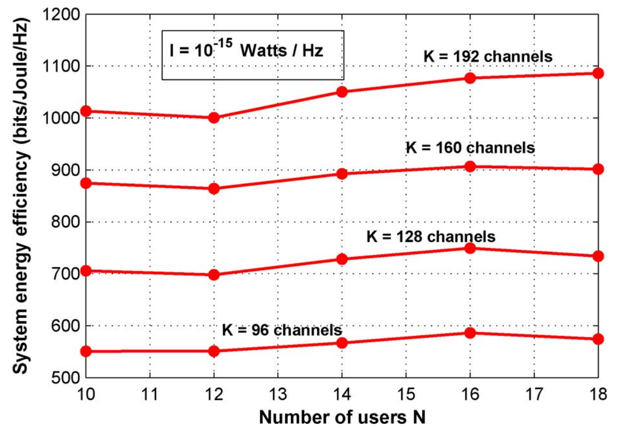

Fig. 6. Scenario 3: Variation of EE with number of users and channels for $I = 10^{-15}$ W/Hz.

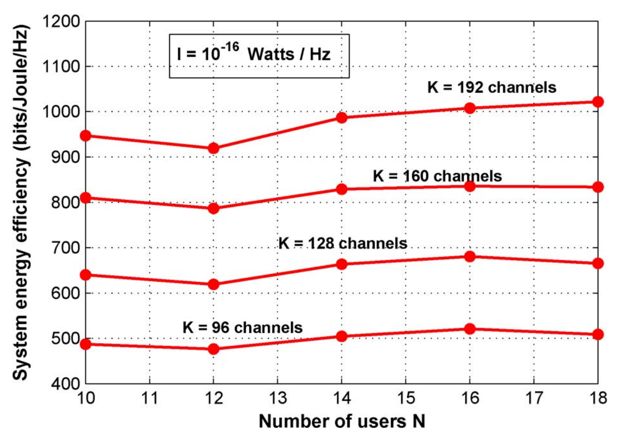

Fig. 7. Scenario 3: Variation of EE with number of users and channels for $I = 10^{-16}$ W/Hz.

in performance that is very different from random subchannel assignment. In addition, when the subchannel gains are almost the same, optimum power levels cannot be that different from equal power levels.

Figs. 5–7, show the variation of EE with number of users for fixed number of subchannels. In all three figures, we see the EE increasing with the number of channels K. This can be explained as follows. Consider the EE defined in the following:

$$\frac{\sum_{k=1}^{K} \log_2(1 + h_k p_k)}{p_c + \psi \sum_{k=1}^{K} p_k}. \tag{40}$$

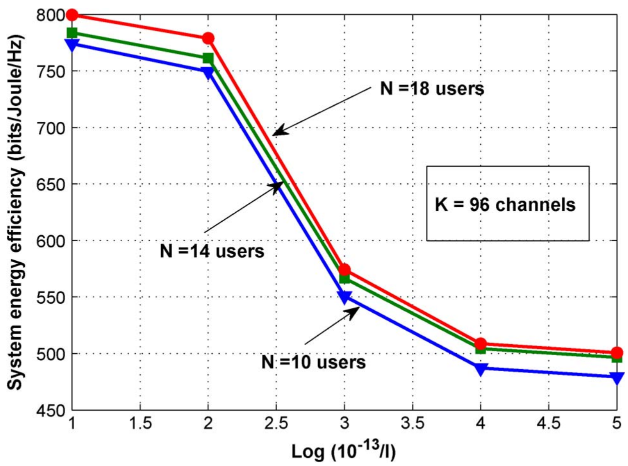

Fig. 8. Scenario 3: Variation of EE with interference threshold I.

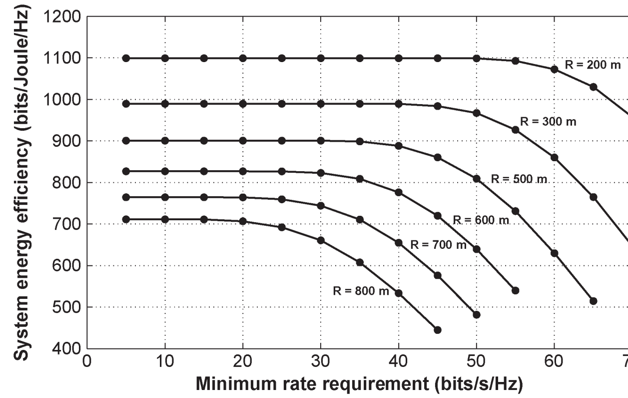

Fig. 9. Variation of EE with minimum rate requirement. $N = 6$, $K = 60$. R varied from 200 to 800 m. All users have the same minimum rate requirement. This requirement is varied from 5 to 80 bit/s/Hz.

As K increases, the sums in both the numerator and the denominator increase. At first, it looks like it is difficult to predict what will happen to the fraction. Assume for a moment that the channel gains are the same for all the channels. Then the optimal power allocation will be *equal* power allocation, that is, all the h_k are equal and all the p_k are equal. Suppose we now add one more channel. The numerator will increase by a fraction of $1/K$. The $\psi \sum_k p_k$ term in the denominator will also increase by the same fraction. However, because of the presence of the term p_c, the fractional increase in the denominator will be less than $1/K$. Hence, the EE, which is the fraction, will increase. This argument shows that, in average, the optimal EE increases with the number of channels.

In each of Figs. 5–7, we see very little increase in EE with the number of users N, when the number of channels K is fixed. This is understandable, because, regardless of the number of users, they are all using the same old channels.

When we go from Figs. 5–7, the interference threshold I becomes stricter. We see a corresponding drop in the EE values. The next plot in Fig. 8 shows this variation of EE with interference threshold I in a clear manner. In Fig. 8, we see a sudden drop in EE between the numbers 2 and 4 in the interference (horizontal) axis. This is the region where the interference threshold I begins to affect the maximum EE.

Suppose the power allocation that maximizes the EE without the interference constraint is p_k^*. If this p_k^* falls inside the interference constraint plane $\sum_k g_k p_k = I$, then p_k^* will be also the solution to the maximization problem with the interference constraint $\sum_k g_k p_k \leq I$. If p_k^* falls outside the plane $\sum_k g_k p_k = I$, then the power allocation that maximizes the EE with the interference constraint will be different from p_k^* and will be a p_k^{**} that lies on the plane $\sum_k g_k p_k = I$ (Case III in Section III or step 4 in the power allocation algorithm). The maximum EE with the interference constraint will be definitely smaller than the maximum EE without the interference constraint.

In Fig. 8, as $\text{Log}(10^{-13}/I)$ goes from 2 to 4, I decreases, and the plane $\sum_k g_k p_k = I$ moves closer to the origin. For more and more simulated placements of the users, p_k^* begins to fall outside the plane $\sum_k g_k p_k = I$, and the optimum point moves from p_k^* to p_k^{**}. The region between the numbers 2 and 4 in the interference (horizontal) axis is the transition zone where we go from more p_k^* to more p_k^{**}.

The last plot in Fig. 9 shows the effect of the minimum rate demand on EE. At all cell radii, the EE begins to drop when the minimum rate demand exceeds a certain value. A greater rate demand forces the users with bad channel conditions to use higher power—a power level higher than the unconstrained optimal power p^*. The EE at the new power level is lower than the one at p^*. Fig. 9 also shows that the onset of this drop in EE occurs at a lower rate demand for larger cell sizes. The larger the cell size, the more users with lower channel gains, and this forces more and more users to use a higher power than p^*, at a lower rate demand.

V. Conclusion

A frequency and power allocation protocol that maximizes the EE of a cognitive base station operating in the TV white spaces has been presented. The protocol satisfies users' minimum rate requirements, adheres to a total power constraint, and keeps the interference to the primary users in the neighboring areas below a specified threshold. After a low-complexity subchannel assignment, Charnes–Cooper transformation was applied to the power allocation problem to obtain an optimal solution. Simulation results showing our protocol achieving higher EE compared with a modified and improved version of the protocol from the literature were provided.

Appendix A
Minimum Interference Power That Satisfies the Rate Requests

We wish to minimize

$$\sum_{k \in S_2} g_k p_k$$

subject to the constraints

$$\sum_{k=k_{n-1}+1}^{k_n} \log_2(1 + h_k p_k) - R_n = 0, \qquad \text{for } n \in S_2. \quad (41)$$

Notice that this is the opposite of the OFDMA power allocation problem that maximizes the throughput subject to an interference constraint. Therefore, here, too, the solution is obtained using the Lagrange multipliers

$$p_k = p_k^I = \frac{1}{g_k}\left[\frac{2^{R_n}}{\prod_{k=k_{n-1}+1}^{k_n}(h_k/g_k)}\right]^{1/K_n} - \frac{1}{h_k}, \quad \text{for } k = k_{n-1}+1, k_{n-1}+2, \ldots, k_n. \tag{42}$$

There is also the total power constraint. The power levels that minimize the total power subject to the rate constraints can be obtained by letting $g_k = 1$ for all k in (42), i.e.,

$$p_k = p_k^T = \left[\frac{2^{R_n}}{\prod_{k=k_{n-1}+1}^{k_n} h_k}\right]^{1/K_n} - \frac{1}{h_k}, \quad \text{for } k = k_{n-1}+1, k_{n-1}+2, \ldots, k_n. \tag{43}$$

Appendix B
Proof: $F(w) = 0$ Has a Unique Solution

Recall (21), i.e.,

$$F(w) = K\log_2(w) - A - \frac{B}{w} = 0$$

where $A = K - \log_2(\prod_k h_k)$, and $B = p_c/\psi - \sum_k(1/h_k)$. We start by showing that $B > 0$ for any practical system

$$B = \frac{p_c}{\psi} - \sum_{k=1}^{K}\frac{1}{h_k} > \frac{p_c}{\psi} - \frac{K}{[h_k]_{\mathrm{Min}}}. \tag{44}$$

Now, $[h_k]_{\mathrm{Min}}$ will occur at the boundary of the cell. Since path loss is the dominant contributor to the channel gain, we can write

$$[h_k]_{\mathrm{Min}} = \frac{R^{-4}}{\aleph} \tag{45}$$

where R is the cell radius, and $\aleph$ is the cumulative background noise per hertz. Hence

$$\frac{K}{[h_k]_{\mathrm{Min}}} = KR^4\aleph. \tag{46}$$

Suppose the circuit power p_c is τ times the total transmission power. If the average transmission power per subchannel is p, then

$$p_c = \tau K p. \tag{47}$$

Suppose the received signal-to-noise ratio at the boundary of the cell is γ. Then

$$\gamma = [h_k]_{\mathrm{Min}}p.$$

Using (45) and (47)

$$p_c = \tau K p = \tau K R^4 \gamma \aleph. \tag{48}$$

Using (48) and (46) on (44)

$$B > \frac{KR^4\tau\gamma\aleph}{\psi} - KR^4\aleph = KR^4\aleph\left(\frac{\tau\gamma}{\psi} - 1\right).$$

This shows that $B > 0$ if $\gamma > \psi/\tau$. Typically, base station power amplifiers have efficiency of 50%. This makes $\psi = 2$. If we assume that circuit power is at least 20% of the total transmit power, then $\tau = 0.2$. The required condition becomes $\gamma > 10$. However, the recommended γ at the boundary of cells is 20 dB, which is a ratio of 100.

Notice that $F(w)$ is continuous when $w > 0$. Furthermore, since $B > 0$, $\lim_{w\to\infty} F(w) = +\infty$, and $\lim_{w\to 0^+} F(w) = -\infty$. Hence, by intermediate value theorem, $F(w) = 0$ has at least one positive solution.

Suppose that $F(w) = 0$ has two positive solutions. Since $F(w)$ is differentiable for $w > 0$, by Rolle's theorem, $F'(w) = 0$ for some w. However

$$F'(w) = \frac{K}{\ln 2}\frac{1}{w} + \frac{B}{w^2} > 0$$

which is a contradiction.

References

[1] D. Talbot, *Efficiency Breakthrough Promises Smartphones That Use Half the Power*. Cambridge, MA, USA: MIT Press, Oct. 31, 2012.

[2] A. Fehske, G. Fettweis, J. Malmodin, and G. Biczók, "The global footprint of mobile communications: The ecological and economic perspective," *IEEE Commun. Mag.*, vol. 49, no. 8, pp. 55–62, Aug. 2011.

[3] J. Quittek, K. Christensen, and B. Nordman, "Energy-efficient networks," *IEEE Netw. Mag.*, vol. 25, no. 2, pp. 4–5, Mar. 2011.

[4] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, Feb. 2005.

[5] G. Gur and F. Alagoz, "Green wireless communications via cognitive dimension: An overview," *IEEE Netw. Mag.*, vol. 25, no. 2, pp. 50–56, Mar./Apr. 2011.

[6] Spectrum Bridge, Inc., White Space Overview. [Online]. Available: http://spectrumbridge.com/ProductsServices/WhiteSpacesSolutions/WhiteSpaceOverview.aspx

[7] C. R. Stevenson *et al.*, "IEEE 802.22: The first cognitive radio wireless regional area network standard," *IEEE Commun. Mag.*, vol. 47, no. 1, pp. 130–138, Jan. 2009.

[8] Z. Hasan, G. Bansal, E. Hossain, and V. K. Bhargava, "Energy-efficient power allocation in OFDM-based cognitive radio systems: A risk–return model," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 6078–6088, Dec. 2009.

[9] L. Li *et al.*, "Energy-efficient transmission in cognitive radio networks," in *Proc. IEEE Consum. Commun. Netw. Conf.*, 2010, pp. 1–5.

[10] R. S. Prabhu and B. Daneshrad, "An energy-efficient water-filling algorithm for OFDM systems," in *Proc. IEEE Int. Conf. Commun.*, Cape Town, South Africa, May 23–27, 2010, pp. 1–5.

[11] X. Lian, H. Nikookar, and L. P. Ligthart, "Green communications and positioning by integration of adaptive and beam-forming technologies in cognitive radio systems," in *Proc. Int. Conf. Wireless Commun., Veh. Technol., Inf. Theory Aerosp. Electron. Syst. Technol.*, 2011, pp. 1–6.

[12] G. Miao, N. Himayat, and G. Y. Li, "Energy-efficient link adaptation in frequency-selective channels," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 545–554, Feb. 2010.

[13] C. Isheden and G. P. Fettweis, "Energy-efficient multi-carrier link adaptation with sum rate-dependent circuit power," in *Proc. IEEE Global Commun. Conf.*, Miami, FL, USA, Dec. 6–10, 2010, pp. 1–6.

[14] C. Xiong, G. Y. Li, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient resource allocation in OFDMA networks," in *Proc. IEEE Global Commun. Conf.*, Houston, TX, USA, Dec. 2011, pp. 3767–3778.

[15] Y. Hu, Y. Huang, L. Yang, and J. Zhou, "Energy-efficient resource allocation in multi-user OFDMA systems," in *Proc. Int. Conf. WCSP*, 2011, pp. 1–5.

[16] A. Akbari, R. Hoshyar, and R. Tafazolli, "Energy-efficient resource allocation in wireless OFDMA systems," in *Proc. IEEE Int. Symp. Pers., Indoor Mobile Radio Commun.*, Sep. 2010, pp. 1731–1735.

[17] S. Schaible, "Fractional programming," *Zeitschrift für Oper. Res.*, vol. 27, pp. 39–54, Oct. 1982.

[18] W. Dinkelbach, "On Nonlinear Fractional Programming," *Manage. Sci.*, vol. 13, no. 7, pp. 492–498, Mar. 1967.

[19] M. Avriel, W. E. Diewert, S. Schaible, and I. Zhang, *Generalized Concavity*. Philadelphia, PA, USA: SIAM, 2010.
[20] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, Mar. 2004.
[21] K. Illanko, A. Anpalagan, and D. Androutsos, "Dual methods for power allocation for radios coexisting in unlicensed spectra," in *Proc. IEEE Global Commun. Conf.*, Miami, FL, USA, Dec. 6–10, 2010, pp. 1–5.
[22] J. G. Proakis, M. Selahi, and G. Bauch, *Contemporary Communication Systems using Matlab*. Stanford, CA, USA: Cengage Learning, 2012.

Kandasamy Illanko received the B.Sc.(Hons.) degree in electrical and electronic engineering from the University of Peradeniya, Peradeniya, Sri Lanka, in 1985, the M.A.Sc. degree in computer engineering from the University of Toronto, Toronto, ON, Canada, in 1991, and the Ph.D. degree in electrical and computer engineering from Ryerson University, Toronto, ON, Canada in 2014.

After completing the B.Sc. degree, he became an Assistant Lecturer with the Department of Electrical and Electronic Engineering, University of Peradeniya. In 1987, he was awarded a commonwealth scholarship to pursue graduate studies at the University of Toronto. After completing the M.A.Sc. degree, he worked in the industry for many years, mainly in system-level programming in the C language/Unix environment. He is currently with the radio resource management and radio access and networking research group in the Wireless Networks and Communications Research Lab, Ryerson University. His research interests include nonconvex optimization, evolutionary algorithms, and game theory applications to resource allocation in cognitive radios and energy-efficient transmissions in wireless communications.

Muhammad Naeem received the B.S. and M.S. degrees in electrical engineering from the University of Engineering and Technology, Taxila, Pakistan, in 2000 and 2005, respectively, and the Ph.D. degree from Simon Fraser University, Burnaby, BC, Canada, in 2011.

From 2000 to 2005, he was a Senior Design Engineer with Comcept (Pvt) Ltd., where he participated in the design and development of smart-card-based GSM and CDMA pay phones at the design department. From 2012 to 2013, he was a Postdoctoral Research Associate with the Wireless Networks and Communications Research Lab (WINCORE Lab), Ryerson University, Toronto, ON, Canada. Since August 2013, he has been an Assistant Professor with the Department of Electrical Engineering, COMSATS Institute of Information Technology, Islamabad, Pakistan, and a Research Associate with WINCORE Lab. His research interests include optimization of wireless communication systems, nonconvex optimization, resource allocation in cognitive radio networks, and approximation algorithms for mixed-integer programming in communication systems.

Dr. Naeem has been a recipient of an NSERC Canada Graduate Scholarship. He is a Microsoft Certified Solution Developer.

Alagan Anpalagan received the B.A.Sc., M.A.Sc., and Ph.D. degrees from the University of Toronto, Toronto, ON, Canada, all in electrical engineering.

In 2001, he joined the Department of Electrical and Computer Engineering, Ryerson University, Toronto, ON, Canada, where he served the department as the Graduate Program Director in 2004–2009, the Interim Electrical Engineering Program Director in 2009–2010, and became a Full Professor in 2010. During his sabbatical (2010–2011), he was a Visiting Professor with Asian Institute of Technology, Bangkok, Thailand, and a Visiting Researcher with Kyoto University, Kyoto, Japan. He is currently with the WINCORE Lab, Ryerson University, where he directs a research group working on radio resource management and radio access and networking areas. His current research interests include cognitive radio resource allocation and management, wireless cross-layer design and optimization, collaborative communication, green communications technologies, machine-to-machine communication, and small-cell networks. He has been a coauthor/editor of *Small Cell Networks* (Cambridge Press, 2014), *Handbook of Green Information and Communication Systems* (Academic Press, 2012), and *Routing in Opportunistic Networks* (Springer-Verlag, 2013). He has authored or coauthored over 200 technical papers in his research areas.

Dr. Anpalagan held various positions within IEEE Canada, including ComSoc Toronto Chapter Chair (2004–2005), IEEE Toronto Section Chair (2006–2007), IEEE Canada Professional Activities Committee Chair (2009–2011), and IEEE Central Canada Chair (2013). He served as an Associate Editor of Springer Wireless Personal Communications (2009), the IEEE COMMUNICATIONS LETTERS (2010–2013), and the IEEE Communications Surveys and Tutorials (2012). He was an Editor of EURASIP Journal of Wireless Communications and Networking (2004–2009). He also served as a Guest Editor for special issues in EURASIP Radio Resource Management in 3G+ Systems (2006), EURASIP Fairness in Radio Resource Management for Wireless Networks (2008), and ACM/Springer MONET Green Cognitive and Cooperative Communication and Networking (2012). He was the recipient of the Faculty Scholastic, Research and Creativity Award (2010); the Faculty Service Award (2010); and the Dean's Teaching Award (2011) at Ryerson University. He is a Registered Professional Engineer in the Province of Ontario. He also completed a course on Project Management for Scientist and Engineers at the University of Oxford Continuing Professional Development Center.

Dimitrios Androutsos received the B.A.Sc., M.A.Sc., and Ph.D. degrees from the University of Toronto, Toronto, ON, Canada, in 1992, 1994, and 1999, respectively, all in electrical and computer engineering.

He has held numerous industry positions in both the U.S. and Canada. He is currently a Full Professor with the Department of Electrical and Computer Engineering, Ryerson University, Toronto, ON, Canada, where he served as an Industrial Internship Coordinator (2005–2012), a Program Director of Electrical Engineering (2006–1012), and an Interim Department Chair (2009–2010). He also held the NSERC Chair in Design Engineering (2010–2012). He has coauthored over 100 technical research papers in numerous research areas. His recent active research interests are in the areas of multimedia processing and transmission, stereoscopy, 3-D digital cinema, and computation image processing.

Prof. Androutsos is an Associate Editor of the Journal of Multimedia. He was an Associate Editor of the *IEEE Signal Processing Letters* and was a Guest Editor of the IEEE Signal Processing Magazine.