This is because the proposed scheme requires only a small buffer size to achieve the optimal diversity gain $2K$ and approach the selection bound, which yields a short average delay.

VI. Conclusion

This paper proposed a max-weight relay selection scheme, in which each link is assigned with a weight related to the buffer status, then the qualified link with the largest weight is selected. The closed-form expression for the outage probability and the diversity gain was derived by introducing several MCs to model the evolution of the buffer status. In contrast to the existing max-link relay selection scheme, the proposed scheme can achieve the optimal diversity gain for a small buffer size, i.e., $L \geq 3$, which implies that it efficiently exploits relay buffers without suffering from long packet delay. Future work of interest is to extend the proposed max-weight scheme to other more complicated networks, such as cognitive relay networks and multiple-access relay networks.

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Load-Aware Dynamic Spectrum Access for Small-Cell Networks: A Graphical Game Approach

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***Abstract*—In this paper, we investigate the problem of dynamic spectrum access for small-cell networks using a graphical game approach. Compared with existing studies, we take the features of different cell loads and local interference relationship into account. It is proven that the formulated spectrum access game is an exact potential game (EPG) with the aggregate interference level as the potential function, and the Nash equilibrium (NE) of the game corresponds to the global or local optima of the original optimization problem. A lower bound of the achievable aggregate interference level is rigorously derived. Finally, we propose an autonomous best response (BR) learning algorithm to converge toward its NE. It is shown that the proposed game-theoretic solution converges rapidly, and its achievable performance is close to the optimum solution.**

***Index Terms*—Dynamic spectrum access, fifth-generation (5G) networks, potential game, small-cell networks.**

I. Introduction

Small cell is an enabling technology for fifth-generation (5G) networks since it has been regarded as the most promising approach for providing a thousand-fold mobile traffic over the next decade [1]. Technically, the use of very dense and low-power small cells exploits the following two fundamental effects [2]: 1) The decreasing distance between the base station and the user leads to higher transmission rates, and 2) the spectrum is more efficiently exploited due to the

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improved spectrum spatial reuse ratio. As the network becomes denser, temporal–spatial variations of mobile traffic can be observed, and the small cells are usually deployed randomly and dynamically [3]. As a result, traditional centralized optimization approaches, e.g., the graph coloring algorithm [4], cannot be applied in practice. To overcome this shortage, there are some distributed spectrum access approaches using, e.g., the sensing-based access approach [5], utility-based learning approach [6], reinforcement-learning-based self-organizing scheme [7], coalitional-game-based scheme [8], evolutionary-game-based scheme [9], and hierarchical dynamic game approach [10].

However, there are two limitations in existing distributed approaches: 1) The fact that the small cells have different loads was not addressed, i.e., most existing work assumed that there is only one mobile user (MU) in each small cell; and 2) the feature of local interference due to the low transmission power, e.g., the transmission of a small cell only directly affects its nearby cells, was not considered. In this paper, we consider load-aware dynamic spectrum access for small-cell networks, taking into account different cell loads and a local interference relationship.

In this paper, we consider a sensing-based autonomous spectrum access mechanism, i.e., a small cell transmits on the channels which are detected idle [5]. In such scenarios, it is desirable to decrease the number of neighboring cells choosing the same channel [4]. We first define a new optimization metric to capture the interference among the small cells. Then, we formulate the spectrum access problem as a graphical game and propose a self-organizing distributed spectrum access algorithm. To summarize, the contributions of this paper are as follows.

1) We formulate the spectrum access problem for the small cells as a graphical game, taking the inherent features of different cell loads and local interference relationship into account. It is proved that it is an exact potential game (EPG) with the aggregate interference level as the potential function; furthermore, the Nash equilibrium (NE) of the game corresponds to the global or local optima of the original problem. Moreover, a lower bound of the aggregate interference level is rigorously derived.
2) We propose an autonomous best response (BR) algorithm to converge toward the NE of the game. Compared with the standard BR algorithm, the proposed algorithm converges rapidly and is scalable when the number of small cells becomes large. Simulation results show that its performance is very close to the global optimum.

Note that game-based spectrum access approaches for small-cell networks have been extensively studied [6]–[12]. In methodology, the differences in this paper are as follows. First, in the existing work, it is assumed that there is only one serving MU in each cell, and the task is to choose an operational channel. When different cell loads are considered, each cell generally needs multiple channels rather than one channel. However, existing game-theoretic optimization with singleton action selection cannot be applied. Second, we consider a graphical game model, i.e., the direct interaction only exists between neighboring users and hence is significantly different from previous global interactive game models, i.e., the interaction emerges among all users. Finally, we define a new metric to capture the interference relationship among neighboring small cells.

The remainder of this paper is organized as follows. In Section II, the system model and problem formulation are presented. In Section III, the graphical game model is formulated and analyzed, and an autonomous BR learning algorithm is proposed to achieve its NE. Finally, simulation results and discussion are presented in Section IV, and conclusion is drawn in Section V.

II. System Model and Problem Formulation

Consider a small-cell network consisting of N small-cell access points (SAPs), where each SAP serves several MUs. It is assumed that the small cells and the macrocell operate on orthogonal channels; hence, the main optimization objective is to eliminate mutual interference among the small cells. Note that this assumption has been extensively used in previous work [4], [6], [13]–[15]. Moreover, it is in line with the Third-Generation Partnership Project (3GPP) [16] and particularly represents the scenarios in the LTE-U network [17], which is an active research topic.

There are M channels available for the SAPs. Denote the SAP set as $\mathcal{N}$, i.e., $\mathcal{N} = \{1, \ldots, N\}$, and the available channel set as $\mathcal{M}$, i.e., $\mathcal{M} = \{1, \ldots, M\}$. It is shown that, as the small cells become denser in 5G networks, the more spatial load fluctuation is observed by each SAP [2]. To capture such a fluctuation, it is assumed that each SAP chooses K_n channels for data transmission of the MUs. The number K_n can be regarded as the load of each SAP, which is jointly determined by the number of active MUs and their traffic demands.[1] Similar to previous work [6], [11], [12], we focus on the spectrum access problem and do not consider the problem of optimizing the required number of channels of each SAP. In practice, some simple but efficient approaches, e.g., the one proposed in [13], can be applied to estimate the cell load.

Due to the spatial distribution and lower transmission power of SAPs, the transmission of a small cell only directly affects the neighboring small cells [4], [13], [14], [18]. To characterize the interference relationship among the small cells, the following interference graph is introduced. Specifically, if the distance d_{ij} between SAP i and j is lower than a threshold d_0, then they interfere with each other when transmitting on the same channel. Therefore, the potential interference relationship can be captured by an interference graph $\mathcal{G} = \{V, E\}$, where V is the vertex set (the SAP set) and E is the edge set, i.e., $V = \{1, \ldots, N\}$ and $E = \{(i, j) | i \in \mathcal{N}, j \in \mathcal{N}, d_{ij} < d_0\}$. For presentation, denote the neighboring SAP set of SAP n as $\mathcal{J}_n$, i.e., $\mathcal{J}_n = \{j \in \mathcal{N} : d_{nj} < d_0\}$.

If two or more neighboring small cells choose the same channel, mutual interference may occur. Thus, to mitigate interference among the small cells, it is desirable to allocate nonoverlapping channels for them as soon as possible. Denote the choice of channels by SAP n as $a_n = \{c_1, c_2, \ldots, c_{K_n}\}, c_i \in \mathcal{M}\ \forall 1 \leq i \leq K_n$. Note that a_n is a K_n-combination of $\mathcal{M}$ and the number of all possible chosen channel profiles of player n is $C_M^{K_n} = (M(M-1)\ldots(M-K_n+1))/(K_n(K_n-1)\ldots 1)$. Motivated by the graph coloring for spectrum allocation problems [4], we define the experienced interference level as follows:

$$s_n = \sum_{j \in \mathcal{J}_n} \sum_{e \in a_n} \sum_{f \in a_j} \delta(e, f) \tag{1}$$

where $\delta(e, f)$ is the following indicator function:

$$\delta(e, f) = \begin{cases} 1, & e = f \\ 0, & e \neq f. \end{cases} \tag{2}$$

That is, if two selected channels e and f are the same, then the indication function takes one; otherwise, it takes zero.

The rationale behind the experienced interference level is briefly explained as follows: In autonomous small-cell networks, a small cell

[1]Since the users in the small cells are always random and dynamic, it is hard to allocate spectrum resources based on the instantaneous network state; instead, it is preferable to allocate spectrum resources according to their loads in a relatively longer decision period.

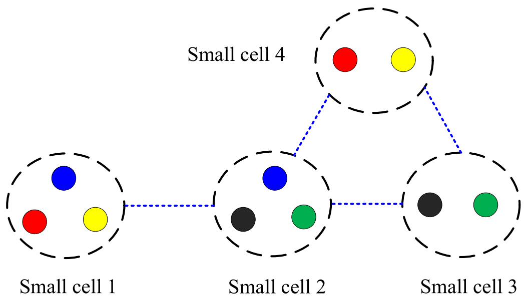

Fig. 1. Considered interference model, in which different colors represent different channels. To reduce the interference in the network, 1) for intracell spectrum access, it is mandatory to allocate different channels for the users in the same cell; and 2) for intercell spectrum access, the number of overlapping channels should be minimized. According to (1), the interference levels of the cells are $s_1 = 1$, $s_2 = 3$, $s_3 = 2$, and $s_4 = 0$.

transmits only when the received energy on the dedicated channel is below a threshold. This is similar to the carrier sense multiple access and has been regarded as a proposing approach for cognitive small-cell networks [5] and LTE-U small cells [17]. Therefore, decreasing the number of interfering cells would increase the achievable throughput.

Note that s_n is the number of channels also chosen by the neighboring SAPs. For an individual SAP n, the interference level s_n should be minimized. From a network-centric perspective, the aggregate interference level of all the SAPs, i.e., $\sum_{n\in\mathcal{N}} s_n$ should be minimized. The considered interference model is illustratively shown in Fig. 1. Thus, we formulate the problem of load-aware spectrum access for small-cell networks as follows:

$$\mathrm{P1}: \quad \min \sum_{n\in\mathcal{N}} s_n. \tag{3}$$

It is noted that the definition of the interference model is different from that of traditional PHY-layer interference. Here, the interference level is used to characterize the mutual influence among neighboring SAPs from a higher level view. Such an interference model has also been applied for single channel selection in opportunistic spectrum access networks [19]–[22]. In comparison, this paper extends previous single channel selection to load-aware multiple channel access. With the allocated channels, the small cell can perform power control to further reduce the mutual interference among different cells. However, this problem is beyond the scope of this paper.

III. Graphical Game Model and Distributed Learning Algorithm

To implement self-organizing and distributed spectrum access, we formulate a graphical game model to address the local interference relationship among the cells.

A. Graphical Game for Dynamic Spectrum Access

Formally, the spectrum access game is denoted as $\mathcal{F} = [\mathcal{N}, \mathcal{G}, \{\mathcal{A}_n\}_{n\in\mathcal{N}}, \{u_n\}_{n\in\mathcal{N}}]$, where $\mathcal{N} = \{1,\dots,N\}$ is a set of players (small cells), $\mathcal{G}$ is the potential interference graph among the players, $\mathcal{A}_n = \{1,\dots,M\}$ is a set of the available actions (channels) for each player n, and u_n is the utility function of player n. Due to the limited interference range, the utility function can be expressed as $u_n(a_n, a_{\mathcal{J}_n})$, where a_n is the action of player n, and $a_{\mathcal{J}_n}$ is the action profile of the neighboring players of n. Thus, the formulated spectrum access game belongs to the *graphical game* [20], [21]. As discussed before, each small cell prefers a lower interference level, which motivates us to define the utility function as follows:

$$u_n(a_n, a_{\mathcal{J}_n}) = -s_n \tag{4}$$

where s_n is the experienced interference level of player n, as characterized by (1). The players in the game are selfish and rational to maximize their individual utilities, i.e.,

$$(\mathcal{F}): \qquad \max_{a_n\in A_n} u_n(a_n, a_{\mathcal{J}_n}) \quad \forall n\in\mathcal{N}. \tag{5}$$

To analyze the properties of the formulated spectrum access game, we first present the following definitions.

Definition 1 (Nash Equilibrium [23]): An action profile $a^* = (a_1^*,\dots,a_N^*)$ is a pure-strategy NE if and only if no player can improve its utility by deviating unilaterally, i.e.,

$$u_n(a_n^*, a_{\mathcal{J}_n}^*) \ge u_n(a_n, a_{\mathcal{J}_n}^*) \ \forall n\in\mathcal{N},\ \forall a_n\in\mathcal{A}_n, a_n \ne a_n^*. \tag{6}$$

Definition 2 (Exact Potential Game [23]): A game is an EPG if there exists a potential function $\phi : A_1 \times \cdots \times A_N \to R$ such that for all $n\in\mathcal{N}$, all $a_n\in\mathcal{A}_n$, and $a_n'\in\mathcal{A}_n$, the following holds:

$$u_n(a_n, a_{\mathcal{J}_n}) - u_n(a_n', a_{\mathcal{J}_n}) = \phi(a_n, a_{\mathcal{J}_n}) - \phi(a_n', a_{\mathcal{J}_n}). \tag{7}$$

That is, when an arbitrary player unilaterally changes its selection, the change in the utility function is the same with that in the potential function. EPG admits the following two promising features [23]: 1) Every EPG has at least one pure-strategy NE, and 2) an action profile that maximizes the potential function is also an NE.

Theorem 1: The formulated spectrum access game $\mathcal{F}$ is an EPG, which has at least one pure-strategy NE. In addition, the global optima of problem $P1$ are pure-strategy NEs of $\mathcal{F}$.

Proof: To prove this theorem, we first construct the following potential function:

$$\Phi(a_n, a_{-n}) = -\frac{1}{2}\sum_{n\in\mathcal{N}} s_n(a_1,\dots,a_N) \tag{8}$$

where s_n is characterized by (1).

Recalling that the chosen channels of player n is denoted $a_n = \{c_1, c_2, \dots, c_{K_n}\}$, define $\mathcal{I}_n(c_i, a_{J_n})$ as the set of neighboring players choosing a channel c_i, $1\le i\le K_n$, i.e.,

$$\mathcal{I}_n(c_i, a_{J_n}) = \{j\in\mathcal{J}_n : c_i\in a_j\} \tag{9}$$

where $\mathcal{J}_n$ is the neighbor set of player n. Then, we denote

$$h_n(c_i, a_{J_n}) = |\mathcal{I}_n(c_i, a_{J_n})| \tag{10}$$

as the experienced interference level on channel c_i, where $|A|$ is the cardinality of set A, i.e., the number of elements in $|A|$. Accordingly, the aggregate experienced interference level of player n is also given by

$$s_n(a_n, a_{J_n}) = \sum_{e\in a_n} h_n(e, a_{J_n}). \tag{11}$$

Now, suppose that an arbitrary player n unilaterally changes its channel selection from $a_n = \{c_1, c_2, \dots, c_{K_n}\}$ to $a_n^* = \{c_1^*, c_2^*, \dots, c_{K_n}^*\}$. For presentation, we classify the channels into the following three sets.

- $\mathcal{C}_0 = a_n \cap a_n^*$. That is, the channels in set $\mathcal{C}_0$ are chosen by player n both before and after its unilateral action change. Note that $\mathcal{C}_0$ may be a null set.

- $\mathcal{C}_1 = a_n \setminus \{a_n \cap a_n^*\}$, where $A \setminus B$ means that B is excluded from A. That is, the channels in $\mathcal{C}_1$ are only chosen by player n before its unilateral action change.
- $\mathcal{C}_2 = a_n^* \setminus \{a_n \cap a_n^*\}$. That is, the channels in $\mathcal{C}_2$ are only chosen by player n after its unilateral action change.

Then, the change in utility function of player n caused by its action unilateral action change is given by

$$u_n(a_n^*, a_{J_n}) - u_n(a_n, a_{J_n}) = \sum_{e \in \mathcal{C}_1} h_n(e, a_{J_n}) - \sum_{e \in \mathcal{C}_2} h_n(e, a_{J_n}). \tag{12}$$

Moreover, the change in the potential function caused by the unilateral change of player n is as follows:

$$\begin{aligned}\Phi\left(a_n^*, a_{-n}\right) &- \Phi(a_n, a_{-n}) \\ = \frac{1}{2}\Bigg\{ & u_n\left(a_n^*, a_{J_n}\right) - u_n(a_n, a_{J_n}) \\ &+ \sum_{k \in \mathcal{D}_1} \left\{u_k\left(a_k, a_{J_k}^*\right) - u_k(a_k, a_{J_k})\right\} \\ &+ \sum_{k \in \mathcal{D}_2} \left\{u_k\left(a_k, a_{J_k}^*\right) - u_k(a_k, a_{J_k})\right\} \\ &+ \sum_{k \in \mathcal{D}_3, k \neq n} \left\{u_k\left(a_k, a_{J_k}^*\right) - u_k(a_k, a_{J_k})\right\}\Bigg\} \end{aligned} \tag{13}$$

where $\mathcal{D}_1 = \cup_{e \in \mathcal{C}_1} \mathcal{I}_n(e, a_{J_n})$, $\mathcal{D}_2 = \cup_{e \in \mathcal{C}_2} \mathcal{I}_n(e, a_{J_n})$, $\mathcal{D}_3 = \mathcal{N} \setminus \{\mathcal{D}_1 \cup \mathcal{D}_2\}$, and $u_k(a_k, a_{J_k}^*)$ is the utility function of player k after n's unilateral action change. Note that player n belongs to the neighboring player set of player k, i.e., $n \in \mathcal{J}_k$. Since the action of player n only affects its neighboring players, the following equations hold:

$$u_n\left(a_k, a_{J_k}^*\right) - u_n(a_k, a_{J_k}) = 1 \quad \forall k \in \mathcal{D}_1 \tag{14}$$

$$u_n\left(a_k, a_{J_k}^*\right) - u_n\left(a_k, a_{J_k}\right) = -1 \quad \forall k \in \mathcal{D}_2 \tag{15}$$

$$u_n\left(a_k, a_{J_k}^*\right) - u_n(a_k, a_{J_k}) = 0 \quad \forall k \in \mathcal{D}_3, k \neq n. \tag{16}$$

Based on (14) and (15), we have

$$\sum_{k \in \mathcal{D}_1} \left\{u_k\left(a_k, a_{J_k}^*\right) - u_k(a_k, a_{J_k})\right\} = |\mathcal{D}_1| = \sum_{e \in \mathcal{C}_1} h_n(e, a_{J_n}) \tag{17}$$

$$\sum_{k \in \mathcal{D}_2} \left\{u_k\left(a_k, a_{J_k}^*\right) - u_k(a_k, a_{J_k})\right\} = -|\mathcal{D}_2| = -\sum_{e \in \mathcal{C}_2} h_n(e, a_{J_n}). \tag{18}$$

Now, combining (12)–(18) yields the following equation:

$$\Phi\left(a_n^*, a_{-n}\right) - \Phi(a_n, a_{-n}) = u_n\left(a_n^*, a_{-n}\right) - u_n(a_n, a_{-n}) \tag{19}$$

which satisfies the definition of EPG, as characterized by (7). Thus, the formulated spectrum access game $\mathcal{F}$ is an EPG, which has at least one pure-strategy NE. Furthermore, according to the relationship between the potential function and the network-centric optimization objective, Theorem 1 is proved. ■

Theorem 2: For any network topology, the aggregate interference level of all the players at any NE point is bounded by $U(a_{\mathrm{NE}}) \geq -(\sum_{n \in \mathcal{N}} \sum_{j \in \mathcal{J}_n} K_n K_j / M)$.

Proof: The proof follows the methodology in [21]. For any pure-strategy NE $a_{\mathrm{NE}} = (a_1^*, \ldots, a_N^*)$, the following inequality holds for each player n $\forall n \in \mathcal{N}$:

$$u_n\left(a_n^*, a_{\mathcal{J}_n}^*\right) \geq u_n\left(\bar{a}_n, a_{\mathcal{J}_n}^*\right) \; \forall \bar{a}_n \in \mathcal{A}_n, \bar{a}_n \neq a_n^* \tag{20}$$

which is obtained according to the definition given in (6). Based on (20), it follows that:

$$C_M^{K_n} \times u_n\left(a_n^*, a_{\mathcal{J}_n}^*\right) \geq \sum_{\bar{a}_n \in \mathcal{A}_n} u_n\left(\bar{a}_n, a_{\mathcal{J}_n}^*\right) \tag{21}$$

where $C_M^{K_n}$ is the number of K_n-combinations of the channel set $\mathcal{A}_n$ (note that $|\mathcal{A}_n| = M$). It is seen that $\sum_{\bar{a}_n \in \mathcal{A}_n} u_n(\bar{a}_n, a_{\mathcal{J}_n}^*)$ represents the aggregate experienced interference level of player n as if it would access all possible channel profiles simultaneously while the neighboring users still only transmit on their chosen channels. As a result, it can be calculated as follows:

$$\sum_{\bar{a}_n \in \mathcal{A}_n} u_n\left(\bar{a}_n, a_{\mathcal{J}_n}^*\right) = -C_{M-1}^{K_n - 1} \sum_{j \in \mathcal{J}_n} K_j \tag{22}$$

where $|\mathcal{J}_n|$ is the number of neighboring users of user n. Thus, (21) can be rewritten as

$$u_n\left(a_n^*, a_{\mathcal{J}_n}^*\right) \geq -\frac{C_{M-1}^{K_n-1}}{C_M^{K_n}} \sum_{j \in \mathcal{J}_n} K_j = -\frac{1}{M} \sum_{j \in \mathcal{J}_n} K_n K_j. \tag{23}$$

Finally, it follows that

$$U(a_{\mathrm{NE}}) = \sum_{n \in \mathcal{N}} u_n\left(a_n^*, a_{\mathcal{J}_n}^*\right) \geq -\frac{\sum\limits_{n \in \mathcal{N}} \sum\limits_{j \in \mathcal{J}_n} K_n K_j}{M} \tag{24}$$

which proves Theorem 2. ■

Theorem 2 characterizes the achievable interference bound of the formulated spectrum access game. Some further discussions are given in the following.

- If all the players choose only one channel for transmission, i.e., $K_n = 1$ $\forall n \in \mathcal{N}$, we have $U(a_{\mathrm{NE}}) \geq -(\sum_{n \in \mathcal{N}} |\mathcal{J}_n| / M)$, which corresponds to the result for single channel opportunistic spectrum access obtained in [21].
- When the number of available channels increases, the bounded aggregate interference level decreases. In particular, if the number of channels becomes sufficiently large, i.e., $M \to \infty$, we have $U(a_{\mathrm{NE}}) \to 0$. In this case, the spectrum resources are abundant, and mutual interference among the players are completely eliminated. Moreover, when the network becomes sparse, i.e., decreasing $|\mathcal{J}_n|$, the bounded aggregate interference level also decreases.

B. Autonomous Best Response Learning

As the distributed spectrum access problem now formulated as an exact potential game, the BR algorithm [23] can be applied to achieve NE of the game. However, there is a limitation of standard BR algorithm: Only one player is randomly selected to update its action in each iteration. Thus, the convergence speed is very slow as the network becomes dense. To overcome this problem, we propose an autonomous BR learning algorithm, which converges to the NE rapidly.

The key idea is that multiple users are autonomously selected to update their selections simultaneously. Specifically, due to the local interference among the users, multiple nonneighboring can use the BR rule to update their channel selections [20], [22]. To achieve this, we assume that there is a common control channel (CCC) available and a 802.11 DCF-like contention mechanism can be applied at each CR user. Specifically, each SAP has three states, i.e., *free*, *active*, and *inactive*, as shown in Fig. 2, and only the active users have the opportunities to update their channel selections. A brief description of state transition is as follows[2]:

- A free SAP generates a backoff timer according to uniform distribution on an interval, e.g., $[0, \tau_{\max}]$ for some fixed parameter $\tau_{\max}$. If the backoff timer expires, it becomes active. Then, it broadcasts an updating request message (URM).

[2]Note that some similar but preliminary coordination mechanisms have been proposed in [20] and [22]. In this paper, an improved mechanism is proposed.

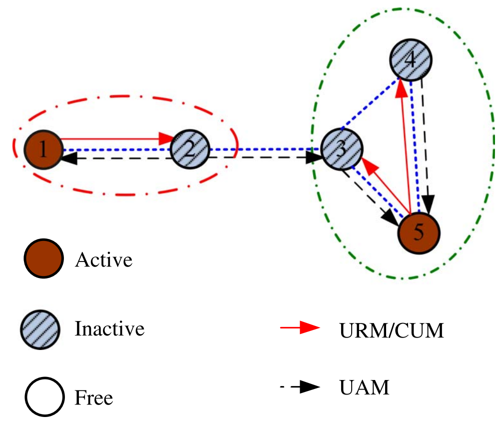

Fig. 2. Illustrative diagram of the proposed autonomous BR learning algorithm. Using the autonomous contention mechanism, users 1 and 5 can update channel selections simultaneously.

- If a free SAP hears a URM before its backoff timer expires, it freezes the timer immediately, enters into the inactive state, and responds with an updating announce message (UAM).
- If a free SAP hears a UAM before its backoff timer expires, it also freezes the timer immediately, enters into inactive state, and keep silent until the next period.
- When an active SAP receives a UAM from its neighbors, it updates their channel selection using the BR rule. After the updating, it broadcasts a channel updating message (CUM) to announce its new channel selection, and they become free again.
- On hearing a CUM message, the inactive users turn to be free again.

Based on the given state transition, the proposed autonomous learning algorithm is described in Algorithm 1.

Algorithm 1 *Autonomous best response algorithm*

1) Initialization: All the SAPs exchange information (the channel selection) with its neighbors.

2) All SAPs repeatedly perform the following procedure:

Based on the information of its neighbors, each SAP n calculates the best action selection n as follows:

$$a_n^{(b)}(i-1) = \arg \max_{a_n \in A_n} u_n \left(a_n, a_{\mathcal{J}_n}(i-1)\right) \tag{25}$$

where $a_{\mathcal{J}_n}(i-1)$ is the action profiles of its neighboring SAPs in the $(i-1)$th iteration. That is, SAP n finds the action $a_n^{(b)}(i-1)$ that maximizes its utility function given the action profiles of the neighboring SAPs.

If $a_n^{(b)}(i-1)$ is better than the current selection $a_n(i-1)$, SAP n contends for an updating opportunity using the autonomous coordination mechanism. If the contention is successful, it updates its channel selection as $a_k(i) = a_k^{(b)}(i-1)$; otherwise, it keeps the current action unchanged.

Theorem 3: The proposed autonomous BR learning algorithm converges to a pure-strategy NE point of the formulated spectrum access game $\mathcal{F}$ in finite steps. Therefore, the aggregate interference level in the small-cell networks is globally or locally minimized.

Proof: It is seen that each updating user always makes its utility function increasing. As the updating users are nonneighboring, the potential function, as specified by (8), is also increasing. Since the potential function is upper bounded (the maximum value is zero), the algorithm will finally converge to a global or local maximum point of the potential function in finite steps. Thus, Theorem 3 is proved. ■

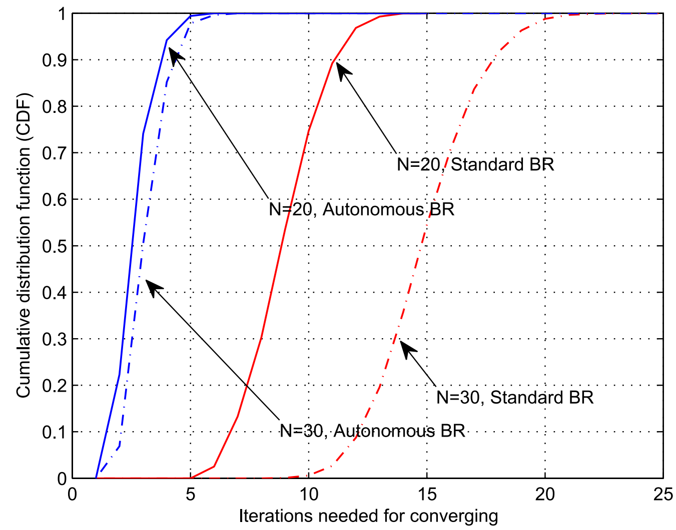

Fig. 3. Convergence speed comparison between the standard BR and the autonomous BR. The number of channels is $M = 5$.

Surely, we can achieve the global optimal solutions as the potential function coincides with the objective function of the centralized problem $P1$, using the spatial adaptive play [20] or B-logit learning [21]. However, the convergence speed of the optional algorithms is slow. Therefore, to make it more practically, it should balance the tradeoff between convergence speed and performance, which is the motivation of the proposed autonomous learning algorithm.

Remark: Some discussions on the practical implementations of the autonomous BR learning algorithm are listed in the following. First, note that it is assumed that the users are truthful in exchanging information and are obedient in executing the contention mechanism and the BR update rule. In essence, the presented game-theoretic solution follows the so-called "engineering agenda" of game theory, i.e., using games as a tool for distributed control [24]. Second, only at the initialization phase, each SAP needs to know the current channel selection profiles of neighboring SAPs. In practice, information exchange among neighboring SAPs can be achieved via the backhaul network or the X2 interference. Third, as the algorithm begins to iterate, the users broadcast their new selections in the CUM message, which means that information exchange is no longer intentionally needed.

IV. Simulation Results and Discussion

We consider a small-cell network deployed in a square region. When there are 20 small cells, the square region is 200 m × 200 m. When the number of small cells increases, the square region increases proportionally to keep the same density. The coverage distance of each small cell is 20 m, and the interference distance is 60 m. For presentation, the load of each cell is randomly chosen from a load set $L = \{1, 2, 3\}$.

To begin with, we compare the convergence speed of the autonomous BR and the standard BR. In the standard BR, only one active user is scheduled to update its action in each iteration, which can be achieved by token or a gateway. There are five channels available in the network, and the comparison results of the cumulative distribution function of the iterations needed for converging are shown

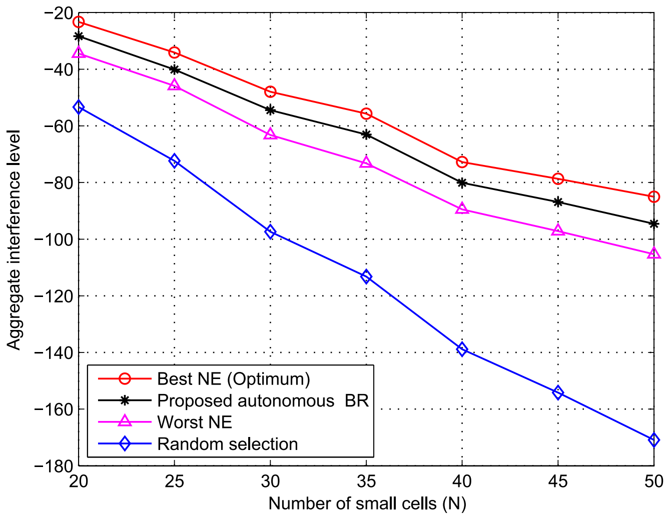

Fig. 4. Aggregate interference level when varying the number of small cells. The number of channels is $M = 5$.

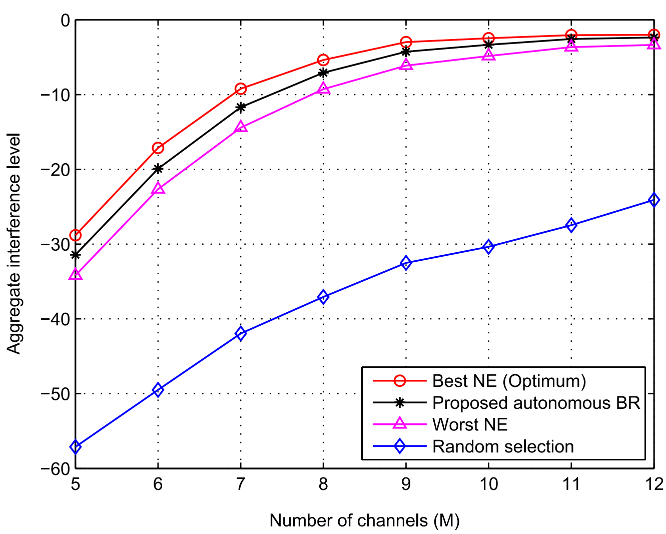

Fig. 5. Aggregate interference level when varying the number of channels. The number of small cells is $N = 20$.

in Fig. 3. The results are obtained by simulating five different network topologies and 1000 independent trials for each network topology. It is noted from the figure that, for the same size network, e.g., $N = 20$ or $N = 30$, the iterations needed for converging of the autonomous BR learning algorithm are significantly decreased. Furthermore, when the network scales up from $N = 20$ to $N = 30$, the convergence speed of the autonomous BR is slightly decreased, whereas that of the standard BR is largely decreased. The reason is that multiple nonneighboring users can update simultaneously. Thus, the proposed autonomous BR algorithm is particularly suitable for large-scale networks.

Second, the aggregate interference level when varying the number of small cells is shown in Fig. 4. The best and worst NE are obtained in a quasi-centralized manner. Specifically, assume there is an omnipotent genie, which knows the cell loads and the interference relation between the SAPs. We run the standard BR learning algorithm 1000 times and then choose the best (worst) result, respectively. According to Theorem 1, the best NE also serves as global minimum for the formulated dynamic spectrum access game. It is noted from the figure that as the network scale increases, the aggregate level increases, as can be expected. More importantly, it is noted that the performance of the proposed autonomous BR algorithm is close to the optimum solution. Moreover, the game-based solution significantly outperforms the random selection strategy. In addition, the aggregate interference level when varying the number of channels is shown in Fig. 5. It is noted that as the number of channels increases, the interference level decreases as can be expected. In particular, as the number of channels is large, e.g., $M > 9$, the interference level becomes moderate. Moreover, the performance of the autonomous BR algorithm is close to the optimum.

To summarize, the simulation results show that the proposed game-theoretic converges rapidly, and its performance is close to the optimum solution. More importantly, it is scalable when increasing the number of small cells, which means that it is suitable in large-scale networks.

V. Conclusion

In this paper, we have investigated the problem of load-aware spectrum access for small-cell networks, using a graphical game approach. Compared with existing work, we took the features of different cell loads and local interference relationship into account. It is proved that the formulated spectrum access game is an exact potential game with the aggregate interference level as the potential function, and NE of the game corresponds to the global or local optima of the original problem. Moreover, a lower bound of the aggregate interference level was rigorously derived. Then, we proposed an autonomous BR learning algorithm to converge toward the NE of the game. It is shown that the proposed learning algorithm converges rapidly, and its performance is close to the optimum solution.

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Achievable Rate of Rician Large-Scale MIMO Channels With Transceiver Hardware Impairments

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***Abstract*—Transceiver hardware impairments (e.g., phase noise, in-phase/quadrature-phase imbalance, amplifier nonlinearities, and quantization errors) have obvious degradation effects on the performance of wireless communications. While prior works have improved our knowledge of the influence of hardware impairments of single-user multiple-input multiple-output (MIMO) systems over Rayleigh fading channels, an analysis encompassing the Rician fading channel is not yet available. In this paper, we pursue a detailed analysis of regular and large-scale (LS) MIMO systems over Rician fading channels by deriving new closed-form expressions for the achievable rate to provide several important insights for practical system design. More specifically, for regular MIMO systems with hardware impairments, there is always a finite achievable rate ceiling, which is irrespective of the transmit power and fading conditions. For LS-MIMO systems, it is interesting to find that the achievable rate loss depends on the Rician K-factor, which reveals that the favorable propagation in LS-MIMO systems can remove the influence of hardware impairments. However, we show that the nonideal LS-MIMO system can still achieve high spectral efficiency due to its huge degrees of freedom.**

***Index Terms*—Achievable rate, hardware impairments, large-scale (LS) multiple-input multiple-output (MIMO), Rician fading channels.**

I. Introduction

By employing multiple antennas at the transceiver, wireless systems can significantly increase spectral efficiency and transmission reliability. The capacity of single-user multiple-input multiple-output (MIMO) systems has been well investigated in the literature [1], [2]. However, most prior works assume that ideal hardware is available at both the transmitter and the receiver, which is unrealistic in practice, whereas the performance of practical MIMO systems is usually affected by transceiver hardware impairments, such as phase noise, in-phase/quadrature-phase imbalance, amplifier nonlinearities, and

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