Interference-Aware Energy Efficiency Maximization in 5G Ultra-Dense Networks

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***Abstract*— Ultra-dense networks can further improve the spectrum efficiency (SE) and the energy efficiency (EE). However, the interference avoidance and the green design are becoming more complex due to the intrinsic densification and scalability. It is known that the much denser small cells are deployed, the more cooperation opportunities exist among them. In this paper, we characterize the cooperative behaviors in the Nash bargaining cooperative game-theoretic framework, where we maximize the EE performance with a certain sacrifice of SE performance. We first analyze the relationship between the EE and the SE, based on which we formulate the Nash-product EE maximization problem. We achieve the closed-form sub-optimal SE equilibria to maximize the EE performance with and without the minimum SE constraints. We finally propose a CE2MG algorithm, and numerical results verify the improved EE and fairness of the presented CE2MG algorithm compared with the non-cooperative scheme.**

***Index Terms*— Cooperative game, energy efficiency (EE), ultra-dense networks, spectrum efficiency (SE).**

I. Introduction

THE fifth generation (5G) mobile communication systems are facing novel challenges due to promising mobile Internet and Internet of Things applications [1]. 5G should be with both spectrum efficiency (SE) and energy efficiency (EE) [2], [3], [8]–[12]. Increasing network densification is regarded as one of the powerful ways to jointly enhance them in a cost-effective manner [1], [4], [5]. However, ultra-dense deployment of small cells also introduces novel technical challenges, e.g., the interference. In order to avoid the interference and increase the SE, some useful observations of interference management were reported in [6], e.g., the more irregular and denser deployment of small cells, the higher gains in interference mitigation.

However, the interference and green design problems in the ultra-dense networks are becoming more complex due to the intrinsic densification and scalability. On one hand, ultra-dense small cells underlay the macrocell, which introduces complex interference. It is hard to analyze the interactive behaviors and strategic decision-making among different small cell eNBs (SeNBs). Meanwhile, the scalability is also challenging in specific hotspots. On the other hand, distributed resource management and interference control will be more effective in ultra-dense networks. However, the signaling overhead will always be high, which challenges and burdens the backhaul of the networks.

Different from the existing research on interference mitigation by improving the SE performance only, in this work, we study the EE maximization problem by exploring and exploiting various cooperative diversity gains. First, it is known that the denser the small cells are, the more cooperative diversity gains can be explored to mitigate interference, thus improving the SE and the EE performance [19]. Second, to explore the intrinsic characteristics of ultra-dense networks, game theory can well characterize the interactive behaviors and strategic decision-making among different small cell players. Meanwhile, game theory also facilitates the design of distributed resource management and interference control. To characterize the cooperative behaviors, we turn to the bargaining cooperative games. The main contributions are summarized as follows:

- We implement the mean field approach to characterize and mitigate the complex interference influence. This approach can be applied in the ultra-dense networks with easier analysis of the relationship between the EE and the SE.
- With the known interference scenario, we provide closed-form relationships between the SE and the EE in different cases. This clarifies the bargaining cooperative optimization problem formulated in this work.
- We propose a distributed cooperative interactive algorithm to approximate the optimal solutions, where both the efficiency and the fairness can be guaranteed.

The remainder of this article is structured as follows. In Section II, we illustrate the typical ultra-dense networks

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and clarify the technical challenges. In Section III, we analyze the optimal tradeoff between the SE and the EE, and discover how much revenues can be gained in the cooperative way. We formulate the cooperative EE maximization game (CE2MG) model in Section IV. The SE optimization to maximize the energy efficiency is analyzed without multiple constraints in Section V, and then with constraints in Section VI, respectively. Based on these analysis, we propose a distributed CE2MG algorithm. Finally, in Section VII we provide extensive numerical results to verify the improved EE performance and the fairness of the presented CE2MG algorithm compared with the non-CE2MG algorithm. We conclude this work in Section VIII.

II. Tradeoff, Densification, and Cooperative Bargaining Games

There are many seminal research attempting for the EE and the SE maximization, or tradeoff optimization, and we surveyed them in [28]. Meanwhile, various interference-aware management strategies have been designed for either SE or EE optimization [7]–[12]. However, it is not simple to simultaneously optimize both the EE and the SE [8], [12], [13]. Generally speaking, the optimal EE performance often leads to lower SE performance, and vice versa. Therefore, there exists an inherent tradeoff between the EE and the SE optimization. Existing studies on the tradeoff of the EE and the SE optimization can be divided into two categories: one is to characterize a closed-form relationship between the EE and the SE [15]; the other is to maximize the EE with the SE requirement or to maximize SE with an EE requirement. There have been several studies on the SE and the EE in ultra-dense networks, for instance, Yunas *et al.* [17] analyzed them with extreme densification levels of both indoor and outdoor scenarios. Zhang *et al.* [18] investigated the downlink performance of coordinated scheduling among different small cells. It was shown that given the number of antennas, the EE is a decreasing function with the increasing user density.

Most of surveyed work in this area concentrate on the tradeoff between the spectrum and the energy efficiency, and the resource sharing, power control, or scheduling, cell zooming, and other traffic-aware techniques are designed to achieve it. However, in our paper, we take a different perspective to maximize the EE performance by sacrificing some SE performance, which is motivated by the fact that sacrificing a certain spectrum efficiency can significantly improve the energy efficiency [29].

Moreover, we investigate the problem in ultra-dense networks [30], thus it is necessary to firstly know the desification concept. By now, there is not a standard concept of ultra-dense networks. Ultra-dense networks can be defined as those networks, where there are more cells than active users [31]. In other words, $\lambda_b \gg \lambda_u$, where λ_b is the density of access points, and λ_u is the density of users. Another definition of ultra-dense networks was solely given in terms of the cell density, irrespective of the users density. Lopez-Perez *et al.* [31] provided a quantitative measure of the density at which a network can be considered ultra-dense when the density of access points $\lambda_b \geq 10^3 \text{cells/km}^2$. In fact, the first definition converges to the second given that the active users density considered in dense urban scenarios is upper bounded by about 600 active users/km^2 [31].

In our previous work, we have studied the resource competition problem between primary and secondary users, where we formulated the interference-aware power control problem as a bargaining game in [20]. Meanwhile, joint channel and power allocation were investigated in [21], and [22] surveyed the problems which were modeled as a cooperative game in 5G heterogeneous cellular networks. More details of the advanced game models and their applications in wireless communications can be found in [26]. Previously, in [23], we presented an overview of the basics of the celebrated Nash bargaining solution and its extensions with geometric interpretations to help understand them and facilitate distributed algorithm design. Then, both symmetric and asymmetric cooperative game-theoretic frameworks were formulated with different tradeoffs incorporating an asymmetric unified -coefficient determined cooperative game model [20]. As a use case, a preference function was designed firstly incorporating both the SE and the EE.

As stated above, the SE and the EE are coupled with each other, and the complex interference influences should be well investigated to maximize the EE performance. To solve the EE maximization problem, we formulate a novel cooperative game-theoretic framework in this work. Different from the current investigations of tradeoff and optimization between the EE and the SE [24], [25], we answer two important questions:

- What are the characteristics of the optimal tradeoff between the EE and the SE with known interference situations;
- How to maximize the EE without sacrificing much of the SE, that is what the achievable SE performance is while maximizing the EE.

We formulate a novel cooperative energy efficient maximization game (CE2MG) model and propose a distributed CE2MG algorithm to achieve the optimal SE solution to maximize the EE performance for each small cell. We list the key notations and definitions in Table I.

III. Ultra-Dense Networks and Challenges

In this section, we illustrate ultra-dense networks, and discuss the characteristics of the SE and the EE, and clarify the interference and high energy consumption challenges affecting the SE and the EE performance.

A. Ultra-Dense Networks and Characteristics of SE and EE

We illustrate the ultra-dense heterogeneous small cell networks (ultra-dense networks) in Fig. 1. We assume that over the macrocell coverage, the small cells are densely deployed. SeNBs with the omni-directional antenna, coexist with the macrocell eNB (MeNB) via a shared spectrum access. We assume that L SeNBs share bandwidth with the MeNB, where severe inter-tier interference and intra-tier interference exist and thus impact the overall system performance. There are L cooperating SeNBs, and the concerned SeNBs are interchangeably called players in this paper.

TABLE I
SUMMARY OF THE KEY NOTATIONS AND DEFINITIONS

Notation	Meaning
$\mathcal{L}$, and ℓ	The SeNB set also called the player set $\mathcal{L} = \{1, ..., L\}$, and the index of each SeNB$_\ell$, $\ell \in \mathcal{L}$
m_ℓ, and p_ℓ^{est}	The number of SUEs, and the fixed energy consumption in each SeNB$_\ell$, $\ell \in \mathcal{L}$
c_ℓ^m, w_ℓ^m, and g_ℓ^m	The downlink capacity, undesirable power, and channel gain of SUE$_m$ in the SeNB$_\ell$, $\ell \in \mathcal{L}$
p_ℓ, ϖ_ℓ, and c_ℓ	The downlink transmission power, normalized undesirable power, and the system capacity of the SeNB$_\ell$, $\ell \in \mathcal{L}$
π_ℓ, and η_ℓ	The spectrum and energy efficiency of the SeNB$_\ell$, $\ell \in \mathcal{L}$

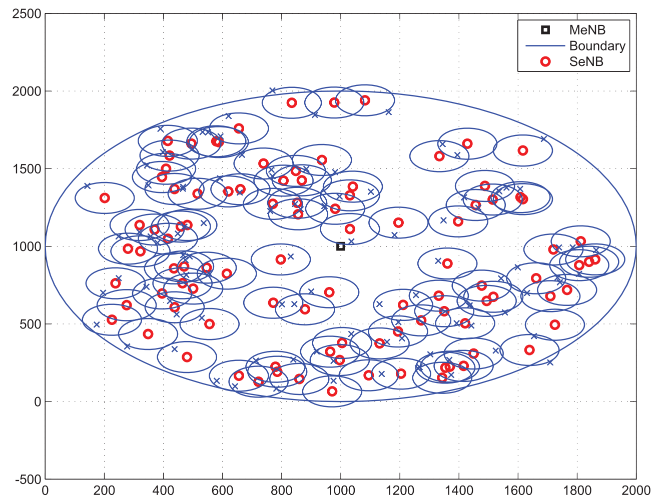

Fig. 1. A sketch of ultra-dense heterogeneous and small cell networks.

In the ultra-dense networks, we omit the effects and analysis of the MeNB on the dense SeNBs in Fig. 1. We concentrate on the investigation of the tradeoff between the EE and the SE, and the way to approach the EE maximization point without sacrificing much of SE of each SeNB$_\ell$, where $\ell \in \mathcal{L}$ and $\mathcal{L} = \{1, ..., L\}$. Meanwhile, here we assume that all L-SeNBs form a grand coalition, which is proved to be the most social optimal in the cooperative game theoretic discipline [26]. Then, we formulate the cooperative bargaining game theoretic framework of the EE maximization in ultra-dense networks.

As shown in Fig. 1, there exist several hotspots, which can represent a residential community or an office building. Each hotspot can be a grand coalition constituted by multiple SeNBs, where each SeNB is also called a player in a game. Each SeNB$_\ell$, $\ell \in \mathcal{L}$, will pursue the EE maximization with suitable SE settings. Without loss of generality, we use a specific SeNB as an example. For SeNB$_l, l \in L$ serving m_l small cell user equipments (SUEs), we assume that all the m_l SUEs can achieve a similar performance due to the limited coverage of the SeNBs. The downlink capacity of the mth SUE served by the SeNB$_l$ can be approximately represented by the Shannon capacity function as

$$c_\ell^m = W \ln\left(1 + \frac{p_\ell g_\ell^m}{\omega_\ell^m}\right), \tag{1}$$

where W, p_ℓ, g_ℓ^m are the spectrum bandwidth, the downlink transmission power, and the channel gain of the mth SUE served by the SeNB$_\ell$. ω_ℓ^m is the sum of inter- and intra-tier interference plus noise power, which is perceived by the mth SUE served by the SeNB$_\ell$. To simplify the following analysis, here we use the natural logarithm to compute the capacity, which results in the unit of Mnat/s/User. Furthermore, the capacity of each small cell can be expressed

$$\hat{c}_\ell^m = W \ln\left(1 + \frac{p_\ell}{\varpi_\ell}\right), \tag{2}$$

where ϖ_ℓ is termed as the average normalized undesirable power, which is given by

$$\varpi_\ell = \frac{1}{m_\ell} \sum_m \frac{\omega_\ell^m}{g_\ell^m}. \tag{3}$$

Here, ϖ_ℓ is the average normalized undesirable power with respect to the total number m_ℓ of associated SUEs with the SeNB$_\ell$.

With the above assumption of the similar capacity achieved by all associated SUEs in the small cell, it is known that if SeNB$_l$ serves m_l SUEs, then the total per-small cell capacity is

$$c_\ell = m_\ell \hat{c}_\ell^m. \tag{4}$$

In the ultra-dense small cell networks, as the number of small cells is very large, the distance between the small cell base station and the user is very small. Therefore, the channel condition and cell capacity for different small cells is very familiar with each other. Thus, we assume that there is not much difference for the capacity of different cells in the small cell network. In this paper, for simplicity of expression and highlighting the contribution of our work, we assume all users receive with similar capacity.

Certainly, this assumption also leads to other benefits. It can reduce the analysis, or in other words we can derive the closed-form solution to reflect the relationship between the EE and the SE. Based on which, the designed distributed algorithm can reduce the signaling overhead. Finally, this is in line with the cooperative game settings, where all the players rationally expect the utility maximization.

Most of the small cells are deployed by individual subscribers or the service providers, therefore each small cell owner cares more about the per-cell SE performance and the system EE performance. To investigate the relationship between them and to maximize the EE performance, we should first define the cell-specific spectrum and the energy efficiency as follows.

Definition 1: The SE π_ℓ of the SeNB$_\ell$ can be defined as

$$\pi_\ell = \frac{c_\ell}{W}, \tag{5}$$

where c_ℓ (Mnat/s) is the small cell capacity defined in (4), W (MHz) is the bandwidth, and the SE π_ℓ (nat/s/Hz).

The EE is another critical performance metric in dense networks due to rapidly rising costs and carbon footprint of energy consumption. Below we define the EE performance metric.

Definition 2: Without loss of generality, we define the EE η_ℓ of the SeNB$_\ell$ as

$$\eta_\ell = \frac{c_\ell}{\epsilon_\ell p_\ell + p_\ell^{\text{cst}}}, \tag{6}$$

where ϵ_ℓ represents the effectiveness of the power amplifier in the SeNB$_\ell$, where $0 \leq \epsilon_\ell \leq 1$. The term p_ℓ^{cst} is the constant circuit power consumption.

In the next section we clarify serious interference and high energy consumption challenges affecting the SE and EE performance metrics. Then, we identify the potential opportunities in ultra-dense networks through the metric analysis.

B. Summary of Research Challenges

We summarize the research challenges for the ultra-dense deployment of small cells as follows:

- Intra-tier interference among different small cells: For a full co-channel deployment, the fact of having multiple small cells close to each other increases interference, thereby degrading the achievable SE.
- High energy consumption: Increasing the number of small cells leads to an increase in the global energy consumption of the system. A high energy consumption is not desirable in the ultra-dense networks due to its impact on the environment and its high operating expenses.

Serious intra-tier interference and high energy consumption significantly affect the performance of both the SE and the EE. With interference-awareness of small cells, we characterize the relationship between the SE and the EE with interference and circuit energy consumption consideration for an individual small cell. Accordingly, we formulate an EE maximization problem via a cooperative game theoretic framework, which consists of the following contents in this work.

IV. Tradeoff Analysis and Opportunities of Cooperation Gains

In this section, we first derive the tradeoff relationship between the SE and the EE, which will help us explore opportunities of cooperation gains, and realize the EE-maximization problem.

A. Tradeoff Analysis Between SE and EE

We have the SE and the EE definitions in (5) and (6) respectively. It is useful to clarify the relationship between them, which can help us design an energy efficient ultra-dense network.

Theorem 1: The tradeoff relationship between spectrum and energy efficiency of each player ℓ, $\ell \in \mathcal{L}$ can be summarized as

$$\eta_\ell = \frac{\pi_\ell}{\epsilon_\ell \frac{\varpi_\ell}{W}\left(e^{\frac{\pi_\ell}{m_\ell}} - 1\right) + \frac{p_\ell^{\text{cst}}}{W}}. \tag{7}$$

Proof: Observed from (4) and (5), it is easy to conclude that

$$W\pi_\ell = m_\ell c_\ell^m, \tag{8}$$

with the assumptions of similar capacity achieved by all the SUEs associated to the SeNB$_\ell$. At this time, we compute the transmission power p_ℓ via (2) with $c_\ell^m = \frac{W\pi_\ell}{m_\ell}$ according to (8). At last, we achieve a closed-form transmission power p_ℓ as

$$p_\ell = \varpi_\ell \left(e^{\frac{\pi_\ell}{m_\ell}} - 1\right). \tag{9}$$

Furthermore, with (5) we know $c_\ell = \frac{\pi_\ell}{W}$. Substituting (9) into (6), and with necessary normalized transformation with respect to bandwidth W, we conclude this proof. ∎

Observing the closed-form of tradeoff relationship between the spectrum and the energy efficiencies in (7), we know that it is important and different from the literature, e.g., [34], because it characterizes the relationship between the EE and the SE. Meanwhile, here we can clearly see the effects of the number of small cells, the interference, and circuit power on the tradeoff. Moreover, we have the following conclusions.

- When $p_\ell^{\text{cst}} \neq 0$, if SE $\pi_\ell \to 0$, then the EE $\eta_\ell \to 0$; if SE $\pi_\ell \to \infty$, then $\eta_\ell \to 0$. Therefore, the EE η_ℓ is a first-increasing and then finally decreasing function of SE π_ℓ.
- When $p_\ell^{\text{cst}} = 0$, if SE $\pi_\ell \to 0$, then the EE $\eta_\ell = \frac{Wm_\ell}{\epsilon_\ell \varpi_\ell}$; if SE $\pi_\ell \to \infty$, then $\eta_\ell \to 0$.

Meanwhile, if we can always attain the optimal SE $\pi_\ell^\star$, $\ell \in \mathcal{L}$, then the transmission rate of the SeNB$_\ell$ is $c_\ell^\star = W\pi_\ell^\star$ can be determined with the above assumptions. Furthermore, the Theorem 1 leads to various cooperative gains to mitigate the interference, save the energy, and balance the traffic.

V. Cooperative Energy Efficiency Maximization Game (CE2MG) and Problem Formulation

With the above listed technical challenges, we have found that the identified cooperation benefits could deal with them. In this section, we explore such kinds of rich cooperation diversities to combat these challenges via a bargaining cooperative game theoretic framework. We formulate a cooperative energy efficient maximization game (CE2MG) model and propose a distributed CE2MG algorithm to achieve the optimal SE solution of each player maximizing system EE. Bargaining-based cooperative game can guarantee the performance of both the efficiency and the fairness among different SeNBs.

A. Cooperative Energy Efficiency Maximization Game (CE2MG)

It is known that the optimal Nash cooperative bargaining solution (NBS)-based control will achieve an optimal tradeoff between Nash fairness and Nash axiomatic efficiency under the framework of Nash axiomatic theory, which has been verified in our previous NBS-formulated work of [20]–[22]. In summary, the cooperative EE maximization game can be achieved by solving the Nash-product problem of $\{\pi_1^\star, \pi_2^\star\} = \max_{\eta_1 \geq \eta_1^{min}, \eta_2 \geq \eta_2^{min}} (\eta_1 - \eta_1^{min})(\eta_2 - \eta_2^{min})$, where η_1^{min} and η_2^{min}

are regarded as the disagreement points. Generally, η_1^{min} and η_2^{min} are set as the minimal energy efficiency requirements of the participating players.

With these basic statements of Nash bargaining game of a two-player case, we define the L-player cooperative EE maximization game as follows.

Definition 3: The L-player cooperative EE maximization game (CE2MG) is formulated as CE2MG $= \{\mathcal{L}, \mathcal{S}, \mathcal{U}\}$, with $\mathcal{L} = \{1, 2, ..., L\}$ as the player set, and L-SeNBs in ultra-dense networks are regarded as the players; $\mathcal{S}$ is the Cartesian product action space defined as $\mathcal{S} = \prod_{\ell=1}^{L} \mathcal{S}_\ell$, where $\mathcal{S}_\ell$ represents the available set of player ℓ, which will be further given with multiple constraints; $\mathcal{U} = \prod_{\ell=1}^{L} u_\ell$ is systematic utility function, which characterizes the player's EE preference regarding the tradeoff between EE and SE as described in (7).

The optimal total utility in the Nash-product form defined in the presented (CE2MG) framework guarantees both the efficiency and the fairness, which has been proved in [20]–[22]. In the following, we first define the cooperative bargaining solution of the CE2MG and investigate its properties. We use the straightforward achieved definition in game theory to describe the equilibrium behaviors.

Definition 4: An SE profile $\pi^\star = \left\{\pi_\ell^\star, \ell \in \mathcal{L}\right\}$ is a Nash cooperative bargaining solution if and only if no player can improve its utility by deviating unilaterally, i.e.,

$$u_\ell(\pi_\ell^\star, \pi_{-\ell}^\star) \geq u_\ell(\pi_\ell, \pi_{-\ell}^\star), \forall \ell \in \mathcal{L}, \forall \pi_\ell \in \mathcal{S}_\ell, \pi_\ell \neq \pi_\ell^\star.$$

Nash cooperative bargaining solutions for the CE2MG can be guaranteed by following the best response function.

Definition 5: We define the best response function (BRF) of player $\ell \in \mathcal{L}$ as the optimal SE profile that maximizes the EE utility when any SE profiles $\{\pi_{-\ell}, -\ell \in \mathcal{L}, -\ell \neq \ell\}$ of other players are given. Mathematically, it is represented as

$$\rho(\pi_{-\ell}) = \arg\max_{\pi_\ell \in S_\ell} u_\ell(\pi_\ell, \pi_{-\ell}).$$

Based on the definition of the BRF, it is known that the Nash cooperative bargaining solution of player $\ell \in \mathcal{L}$ is

$$\pi_\ell^\star = \rho(\pi_{-\ell}^\star).$$

At this time, the problem is transferred to derive the BRF to approach the cooperative bargaining equilibrium solution of player $\ell \in \mathcal{L}$. We will formulate the Nash-product EE maximization problem to achieve such a BRF.

B. Problem Formulation

In the general cooperative bargaining game-theoretic framework, the CE2MG is formulated as the following optimization problem

$$\mathbf{P1}: \quad \max \quad u = \prod_{\ell=1}^{L}\left(\eta_\ell - \eta_\ell^{\min}\right), \tag{10a}$$

$$\text{subject to } \pi \leq \sum_{\ell=1}^{L} \pi_\ell, \tag{10b}$$

$$\pi_\ell \leq \pi_\ell^{\max}, \ell = 1, ..., L. \tag{10c}$$

Objective (10a) is the Nash product function of the EE function defined in (7) and the minimum EE of the player ℓ, which is in line with the Nash bargaining game-theoretic framework. Constraint (10b) represents the summation of SE that is larger or equal to a minimum SE requirement π. π_ℓ and $\pi_\ell^{\max}$ are the SE and maximum SE of player ℓ, respectively. Each player is constrained by the individual rate limit in (10c).

This equivalent re-formulation is widely used to transfer the Nash product-based objective function into the utility-summation form using the logarithmic function [22]. It is known that this process does not change the convexity of the prime objective function in (10).

Moreover, and according to [22], the equivalent model of (10) is given by

$$\mathbf{P2}: \quad \max \quad \hat{u} = \sum_{\ell=1}^{L} \ln\left(\eta_\ell - \eta_\ell^{\min}\right), \tag{11a}$$

$$\text{subject to (10}b\text{) and (10}c\text{)}, \tag{11b}$$

where we compute $\hat{u} = \ln(u)$ as a natural logarithmic transformation that will not change the convexity of u. We will analyze the problem to achieve the specific BRF without and with the constraints in the following two sections.

VI. Optimal Spectral Efficiency Analysis Without Constraints

In this section, we first study the optimal SE to maximize the EE, where we omit the individual and system SE requirements. Then, some intrinsic characteristics between the EE and the SE are provided in this section.

We first characterize the optimal tradeoff between the EE and the SE aware of the interference situation. We will introduce the corresponding methods to estimate the interference power in the simulation section.

With (7) and to simplify the following expressions, we represent the EE as

$$\eta_\ell = \frac{\pi_\ell}{\varphi_\ell + \sigma_\ell}, \tag{12}$$

where $\varphi_\ell = \mu_\ell\left(e^{\frac{\pi_\ell}{m_\ell}} - 1\right)$ is a function of the SE π_ℓ, and here $\mu_\ell = \epsilon_\ell \frac{\varpi_\ell}{W}$ is the effective interference plus noise power spreading over all the bandwidth W, and $\sigma_\ell = \frac{p_\ell^{\text{cst}}}{W}$ is the normalized circuit power consumption. For each implementation of the EE maximization, we assume that both the normalized static power consumption σ_ℓ and the normalized interference power μ_ℓ are determined.

Finally, according to (11a), we have the following optimization problem without multiple constraints.

$$\mathbf{P3}: \max \quad \hat{u} = \sum_{\ell=1}^{L} \ln\left(\frac{\pi_\ell}{\varphi_\ell + \sigma_\ell} - \eta_\ell^{\min}\right). \tag{13}$$

Furthermore, we analyze the possible existence properties and the closed-form of SE to maximize the EE.

Theorem 2: There always exist optimal solutions of SE to maximize the EE with the known σ_ℓ, μ_ℓ, and fixed number of SUEs, m_ℓ, $\ell \in \mathcal{L}$ in **P3** defined in (13).

Proof: With the known σ_ℓ, μ_ℓ, and fixed number of SUEs, m_ℓ, $\ell \in \mathcal{L}$ in **P3** defined in (13), the individual EE-related utility function $u_\ell = \ln\left(\frac{\pi_\ell}{\varphi_\ell+\sigma_\ell} - \eta_\ell^{\min}\right)$ is a unimodal function with respect to π_ℓ. The detailed proof can be found in the Appendix A. ■

With the above conclusion, it is known that there always exist optimal solutions of the spectrum efficiency to maximize the EE. We further derive the optimal closed-form solutions.

Theorem 3: The optimal SE $\pi_\ell^\star$, $\ell \in \mathcal{L}$ to maximize the system EE can be mathematically achieved by solving the following equation:

$$\pi_\ell = m_\ell\left(1 - \frac{\mu_\ell - \sigma_\ell}{\mu_\ell} e^{-\frac{\pi_\ell}{m_\ell}}\right), \tag{14}$$

when $\pi_\ell \neq \eta_\ell^{\min}(\varphi_\ell + \sigma_\ell)$.

Proof: Here, we know that $\varphi_\ell = \mu_\ell\left(e^{\frac{\pi_\ell}{m_\ell}} - 1\right)$, and we can compute first-order partial differential function with respect to π_ℓ, yielding

$$\frac{\partial \varphi_\ell}{\partial \pi_\ell} = \frac{1}{m_\ell}\mu_\ell e^{\frac{\pi_\ell}{m_\ell}} = \frac{\varphi_\ell + \mu_\ell}{m_\ell}. \tag{15}$$

Further, we compute the partial differential function of the individual utility function u_ℓ with respect to SE π_ℓ, which is shown as

$$\begin{aligned}\frac{\partial u_\ell}{\partial \pi_\ell} &= \frac{(\varphi_\ell + \sigma_\ell) - \pi_\ell \frac{\partial \varphi_\ell}{\partial \pi_\ell}}{\left(\pi_\ell - \eta_\ell^{\min}(\varphi_\ell + \sigma_\ell)\right)(\varphi_\ell + \sigma_\ell)} \\ &= \frac{(\varphi_\ell + \sigma_\ell) - \frac{\pi_\ell}{m_\ell}(\varphi_\ell + \mu_\ell)}{\left(\pi_\ell - \eta_\ell^{\min}(\varphi_\ell + \sigma_\ell)\right)(\varphi_\ell + \sigma_\ell)}.\end{aligned} \tag{16}$$

In the above derivation process of (16), we use the result of (15). According to Theorem 2, we know that the optimal solutions always exist. Therefore, we conclude that if $\pi_\ell - \eta_\ell^{\min}(\varphi_\ell + \sigma_\ell) \neq 0$, which can be also represented as $\pi_\ell \neq \eta_\ell^{\min}(\varphi_\ell + \sigma_\ell)$, then we know that

$$(\varphi_\ell + \sigma_\ell) - \frac{\pi_\ell}{m_\ell}(\varphi_\ell + \mu_\ell) = 0, \tag{17}$$

will guarantee $\frac{\partial u_\ell}{\partial \pi_\ell} = 0$ in (16). At last, with (17) we know that

$$\pi_\ell = m_\ell \frac{\varphi_\ell + \sigma_\ell}{\varphi_\ell + \mu_\ell}. \tag{18}$$

We compute $\varphi_\ell + \mu_\ell = \mu_\ell e^{\frac{\pi_\ell}{m_\ell}}$ and $\varphi_\ell + \sigma_\ell = \mu_\ell e^{\frac{\pi_\ell}{m_\ell}} + \mu_\ell - \sigma_\ell$ according to $\varphi_\ell = \mu_\ell\left(e^{\frac{\pi_\ell}{m_\ell}} - 1\right)$, and substitute these two equations to (18), which concludes the proof. ■

Theorem 4: If the attained average SE $\frac{\pi_\ell}{m_\ell}$ of each SeNB$_\ell$, $\ell \in \mathcal{L}$ is large enough in the ultra-dense networks, then we conclude that the optimal SE $\pi_\ell^\star = m_\ell\sqrt{\frac{\sigma_\ell}{\mu_\ell}}$.

Proof: If the attained average SE $\frac{\pi_\ell}{m_\ell}$ of each SeNB$_\ell$, $\ell \in \mathcal{L}$ is large enough in the ultra-dense networks, then we know that

$$e^{-\frac{\pi_\ell}{m_\ell}} \approx 1 - \frac{\pi_\ell}{m_\ell} \tag{19}$$

with the first-order series approximation to $e^{-\frac{\pi_\ell}{m_\ell}}$. Substituting (19) into (14), we conclude the proof. ■

In summary, we conclude from Theorem 4 that more associated SUEs (m_ℓ), higher circuit power consumption (σ_ℓ), and less normalized undesirable power (μ_ℓ) mean the SeNB$_\ell$ should pursue more SE (π_ℓ) to maximize the EE.

VII. Analysis of Optimal Spectral Efficiency to Maximize Energy Efficiency

In this section, we solve the CE2MG problem with both the individual and the system SE constraints among different SeNB-players. We try to find the optimal SE of each player to maximize the system EE. We analyze the dual problem of predefined problem **P2** in (11), based on which we analyze the proposed CE2MG algorithm and the solution properties.

A. The Denoted Problem

In this section, we solve the problem **P2** in (11) by solving its Lagrangian relaxation problem, shown as

$$\mathbf{P4}: \max_{\lambda \geq 0, \kappa_\ell \geq 0, \ell \in \mathcal{L}} \xi, \tag{20a}$$

$$\text{subject to} \quad \pi_\ell \in \mathcal{S}_\ell, \ell \in \mathcal{L}, \tag{20b}$$

where

$$\mathcal{S}_\ell = \left\{\pi \leq \sum_{\ell=1}^{L} \pi_\ell, \pi_\ell \leq \pi_\ell^{\max}, \ell \in \mathcal{L}\right\}, \tag{21}$$

and ξ represents the Lagrangian function associated with **P2**. ξ is defined in (22), as shown at the bottom of this page.

B. Analysis of SE to Maximize EE

It is critical to derive the detailed best response function for the CE2MG model via solving (22).

Theorem 5: Again with all σ_ℓ, μ_ℓ, $\ell \in \mathcal{L}$ as constants, we conclude that

$$\pi_\ell = m_\ell \frac{(\varphi_\ell + \sigma_\ell)(\lambda - \kappa_\ell)\eta_\ell^{\min} + 1}{\frac{\varphi_\ell + \mu_\ell}{\varphi_\ell + \sigma_\ell} + m_\ell(\lambda - \kappa_\ell)}, \tag{23}$$

$$\begin{aligned}\xi &= \sum_{\ell=1}^{L} \ln\left(\frac{\pi_\ell}{\varphi_\ell + \sigma_\ell} - \eta_\ell^{\min}\right) - \lambda\left(\pi - \sum_{\ell=1}^{L}\pi_\ell\right) - \sum_{\ell=1}^{L}\kappa_\ell\left(\pi_\ell - \pi_\ell^{\max}\right) \\ &= \sum_{\ell=1}^{L}\left\{\ln\left(\frac{\pi_\ell}{\varphi_\ell + \sigma_\ell} - \eta_\ell^{\min}\right) + (\lambda - \kappa_\ell)\pi_\ell\right\} + \left(\sum_{\ell=1}^{L}\kappa_\ell \pi_\ell^{\max} - \lambda\pi\right)\end{aligned} \tag{22}$$

where we assume that the Lagrangian parameters λ, and κ_ℓ, $\ell \in \mathcal{L}$ are pre-determined according to KKT conditions of

$$\lambda\left(\pi - \sum_{\ell=1}^{L}\pi_\ell\right) = 0 \tag{24}$$
$$\sum_{\ell=1}^{L}\kappa_\ell\left(\pi_\ell - \pi_\ell^{\max}\right) = 0.$$

Proof: With respect to the SE π_ℓ, we derive the first-order derivation of the Lagrangian relaxed function in (22), which yields

$$\frac{\partial \xi}{\partial \pi_\ell} = \frac{(\varphi_\ell + \sigma_\ell) - \frac{\pi_\ell}{m_\ell}(\varphi_\ell + \mu_\ell)}{\left(\pi_\ell - \eta_\ell^{\min}(\varphi_\ell + \sigma_\ell)\right)(\varphi_\ell + \sigma_\ell)} - (\lambda - \kappa_\ell). \tag{25}$$

We solve the equation $\frac{\partial \xi}{\partial \pi_\ell} = 0$, and then with necessary calculation and we have

$$\begin{aligned}\frac{\pi_\ell}{m_\ell} &= \frac{(\varphi_\ell + \sigma_\ell) + (\lambda - \kappa_\ell)\,\eta_\ell^{\min}(\varphi_\ell + \sigma_\ell)^2}{\varphi_\ell + \mu_\ell + m_\ell(\lambda - \kappa_\ell)(\varphi_\ell + \sigma_\ell)} \\ &= \frac{(\varphi_\ell + \sigma_\ell)(\lambda - \kappa_\ell)\,\eta_\ell^{\min} + 1}{\frac{\varphi_\ell + \mu_\ell}{\varphi_\ell + \sigma_\ell} + m_\ell(\lambda - \kappa_\ell)}.\end{aligned} \tag{26}$$

With (26), therefore, we know (23) holds with KKT conditions of the Lagrangian parameters constrains of λ, and κ_ℓ, $\ell \in \mathcal{L}$ in (24). ■

Here, Theorem 5 is a general conclusion, with Theorem 3 as a special case of $\lambda = \kappa_\ell = 0$, for all $\ell \in \mathcal{L}$.

C. Distributed CE2MG Algorithm

With the above analysis, a distributed CE2MG algorithm is presented to achieve the optimal SE of each SeNB$_\ell$, $\ell \in \mathcal{L}$ to maximize the EE. The detailed implementation of the proposed distributed CE2MG algorithm is described as follows:

- **Initialization:** Initialize the related parameters including the total number (L) of SeNBs in the considered ultra-dense networks, the number of SUEs (m_ℓ) associated to each SeNB$_\ell$, the circuit power consumption p_ℓ^{cst}, the shared bandwidth W, the effectiveness of the power amplifier ϵ_ℓ, the minimum EE requirement $\eta_\ell^{\min}$, and the introduced Lagrangian factors of λ^0 , κ_ℓ^0 of the SeNB player $\ell \in \mathcal{L}$.
- **Observation:** To achieve the awareness of the interference environment, it is critical for each SeNB player $\ell \in \mathcal{L}$ to attain the undesirable power $\varpi_\ell = \frac{1}{m_\ell}\sum_m \frac{\omega_\ell^m}{g_\ell^m}$ in the form of (3) and $\sigma_\ell = \frac{p_\ell^{\mathrm{cst}}}{W}$. The critical estimation of ω_ℓ^m is given in Appendix B.
- **Decision:** Each SeNB player $\ell \in \mathcal{L}$ repeats the following steps until the convergence to achieve the optimal SE ($\pi_\ell^\star$), $\ell \in \mathcal{L}$:
  - First, estimate the initial SE via
  $$\pi_\ell^t = \ln\left(1 + \frac{p_\ell^t}{\varpi_\ell^t}\right).$$
  - Second, calculate the best response function
  $$\varphi_\ell^t = \mu_\ell^t\left(e^{\frac{\pi_\ell^t}{m_\ell}} - 1\right),$$
  where $\mu_\ell^t = \epsilon_\ell \frac{\varpi_\ell^t}{W}$.
  - Third, calculate the next available SE via
  $$\pi_\ell^{t+1} = m_\ell \frac{\left(\varphi_\ell^t + \sigma_\ell\right)\left(\lambda^t - \kappa_\ell^t\right)\eta_\ell^{\min} + 1}{\frac{\varphi_\ell^t + \mu_\ell}{\varphi_\ell^t + \sigma_\ell} + m_\ell\left(\lambda^t - \kappa_\ell^t\right)}$$
  - Finally, update the Lagrangian multipliers using
  $$\lambda^{t+1} = \lambda^t + \alpha\left(\pi - \sum_{\ell=1}^{L}\pi_\ell^t\right), \tag{27}$$
  and
  $$\kappa_\ell^{t+1} = \kappa_\ell^t + \beta\left(\pi_\ell^t - \pi_\ell^{\max}\right), \tag{28}$$
  where α and β are the adjustable step factors, which sensitively affect the convergence properties of the CE2MG algorithm.
- **Convergence:** Determine the termination condition using
$$\left\|\lambda^{t+1} - \lambda^t\right\| \le \varepsilon_1, \tag{29}$$
and
$$\left\|\kappa_\ell^{t+1} - \kappa_\ell^t\right\| \le \varepsilon_2, \tag{30}$$
where ε_1 and ε_2 are convergence precision settings.

For the proposed distributed CE2MG algorithm, we have the following remarks:

- First, the iteration equations of the Lagrangian multipliers (27) and (28) are achieved by the sub-gradient derivations.
- Second, with the convergence settings of (29) and (30) the proposed distributed CE2MG algorithm can converge after limited iterations, which is verified via numerical simulation.
- Third, in the observation step, each SeNB can achieve the real undesirable power in the form of (3) via the similar interference estimation method in [27], which is given in Appendix B.

VIII. Numerical Results

In this section, we first illustrate the simulation scenarios and the basic settings with multiple parameters. Then, we verify the convergence property, the effectiveness condition, and the improved performance of the proposed distributed CE2MG scheme. Finally, we evaluate the scheme with more settings, including the interference situation, different requirements, and extremely dense players.

A. Basic Simulation Settings

In this section, we present simulation results for an interference-limited downlink OFDMA ultra-dense network and observe the performance of the proposed algorithm in equilibrium. We set the system parameters as the bandwidth $W = 20$MHz, background noise power is $2e{-}13$ Watt with the noise power spectrum density as -174dBm/Hz. The average channel gain is $2.5e{-}10$, which is determined with the fading model of $148 + 40\log_{10}[d]$, where d is the maximum coverage radius of various SeNBs in the unit of km. The average number of SUE $m_\ell = 5$ associated to each SeNB$_\ell$, the effectiveness of the power amplifier $\epsilon_\ell = 0.8$, the minimum EE requirement

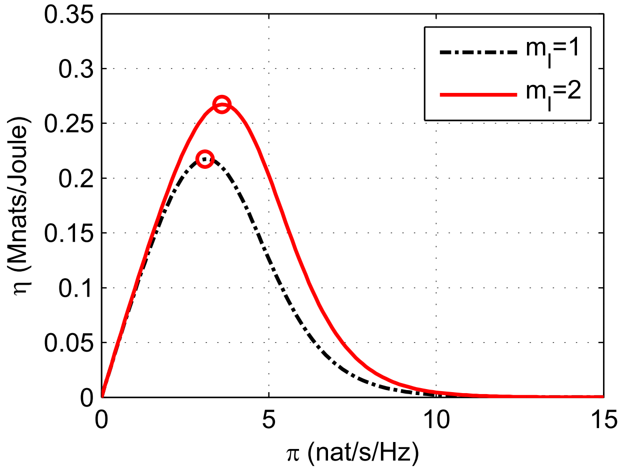

Fig. 2. Convergence verification and effectiveness.

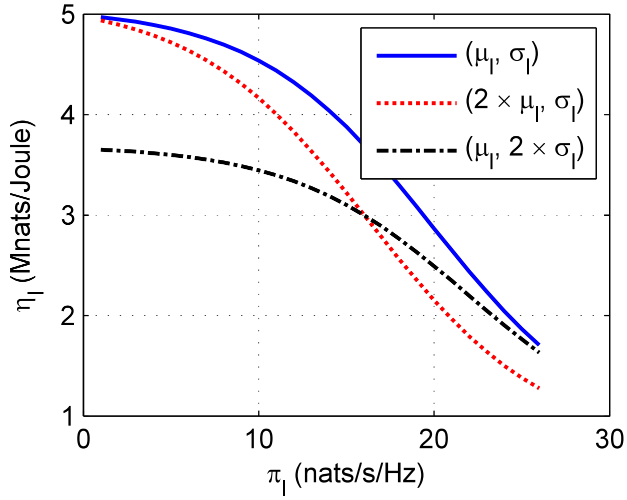

Fig. 3. Tradeoff relationships of the proposed CE2MG scheme with different assumptions.

$\eta_\ell^{\min} = 1$, and the introduced Lagrangian factors of $\lambda^0 = 1$, $\kappa_\ell^0 = 0.5$ of the SeNB player $\ell \in \mathcal{L}$. If it is not specified in the following subsection, then the numerical results are with these basic system settings. However, we will change some of them during the following simulations to reflect improved performance using the proposed algorithm.

B. Convergence Verification

First, we illustrate the iteration process of the proposed CE2MG scheme, where we use the settings of $m_\ell = 5$, $\eta_\ell^{min} = 1$ as the benchmark case. Meanwhile, we choose $m_\ell = 10, \eta_\ell^{min} = 1$, and $m_\ell = 5, \eta_\ell^{min} = 2$ as another two cases to reflect the effects of increasing number of SUEs and the increasing minimum EE requirement of the SeNB$_\ell$ on the final performance and the convergence rate. The EE performance of the proposed CE2MG scheme with the above different settings is described in Fig. 2 with respect to multiple iterations.

Observing Fig. 2, several conclusions can be made as follows:

- The proposed CE2MG scheme guarantees the convergence with different settings, different numbers of SUEs and various minimum energy efficiency requirements of the SeNB$_\ell$. We can see that the proposed CE2MG scheme will converge to a steady region of convergence around less than 1000 iterations.
- Increasing the number of SUEs in a specific SeNB$_\ell$ will help enhance the EE, which can be found from the performance curves of the settings of $m_\ell = 5, \eta_\ell^{min} = 1$ and $m_\ell = 10, \eta_\ell^{min} = 1$. Also, increasing more SUEs will make the convergence rate slow down since more implementations occur.
- An increasing minimum EE requirement of the SeNB$_\ell$ will help to improve the final performance, and there is little effect on the convergence rate, which can be found from the performance curves of the settings of $m_\ell = 5, \eta_\ell^{min} = 1$ and $m_\ell = 5, \eta_\ell^{min} = 2$.

Second, we investigate the tradeoff relationships of the proposed CE2MG scheme with different assumptions of fixed effective interference plus noise power μ_ℓ and the normalized static power consumption σ_ℓ, which is illustrated in Fig. 3.

Fig. 3 demonstrates three tradeoff relationships of the proposed CE2MG scheme with different assumptions of (μ_ℓ, σ_ℓ), $(2 \times \mu_\ell, \sigma_\ell)$, and $(\mu_\ell, 2 \times \sigma_\ell)$. We can make the following conclusions:

- Specific SeNB$_\ell$, e.g., $(2 \times \mu_\ell, \sigma_\ell)$, that bears more interference will achieve a low tradeoff between SE and EE, compared with the least value, e.g., (μ_ℓ, σ_ℓ). On the one hand, more interference means lower EE with the same SE, e.g., when specific SE $\pi_l = 10$, specific SeNB$_\ell$, e.g., (μ_ℓ, σ_ℓ) that bears less interference can achieve a 33% EE improvement, compared with the case of $(2 \times \mu_\ell, \sigma_\ell)$. On the other hand, more interference means lower SE with the same EE, e.g., when specific EE $\eta_l = 4$, specific SeNB$_\ell$, e.g., (μ_ℓ, σ_ℓ) that bears less interference can achieve a 40% SE improvement, compared with the case of $(2 \times \mu_\ell, \sigma_\ell)$.
- Specific SeNB$_\ell$, e.g., $(\mu_\ell, 2 \times \sigma_\ell)$ that consumes more power will achieve a low tradeoff between SE and energy efficiency, compared with the least value, e.g., (μ_ℓ, σ_ℓ). On the one hand, more power consumption means lower EE with the same SE, e.g., when specific SE $\pi_l = 10$, specific SeNB$_\ell$, e.g., (μ_ℓ, σ_ℓ) that is with less power consumption can achieve a 100% EE improvement, compared with the case of $(\mu_\ell, 2 \times \sigma_\ell)$. On the other hand, more power consumption means lower SE with the same EE, e.g., when specific EE $\eta_l = 3$, specific SeNB$_\ell$, e.g., (μ_ℓ, σ_ℓ) that consumes less power can achieve a 25% spectrum efficiency improvement, compared with the case of $(\mu_\ell, 2 \times \sigma_\ell)$.

C. Performance Evaluation With Dense Deployments of Small Cells

In this subsection, we first characterize the effects of densification (e.g., the increasing number L of small cells) on the system EE η and the average EE η_ℓ of specific SeNB$_\ell$, $\ell \in \mathcal{L}$. At last, we investigate the fairness among different SeNBs,

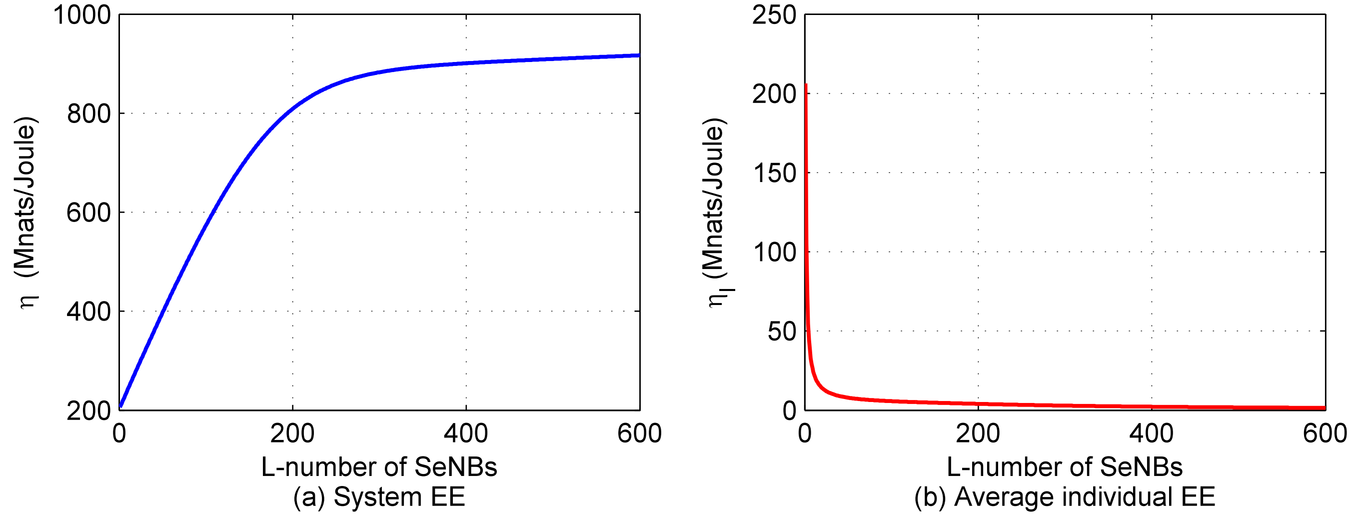

Fig. 4. Effects of densification on the system EE (a) and the average EE (b).

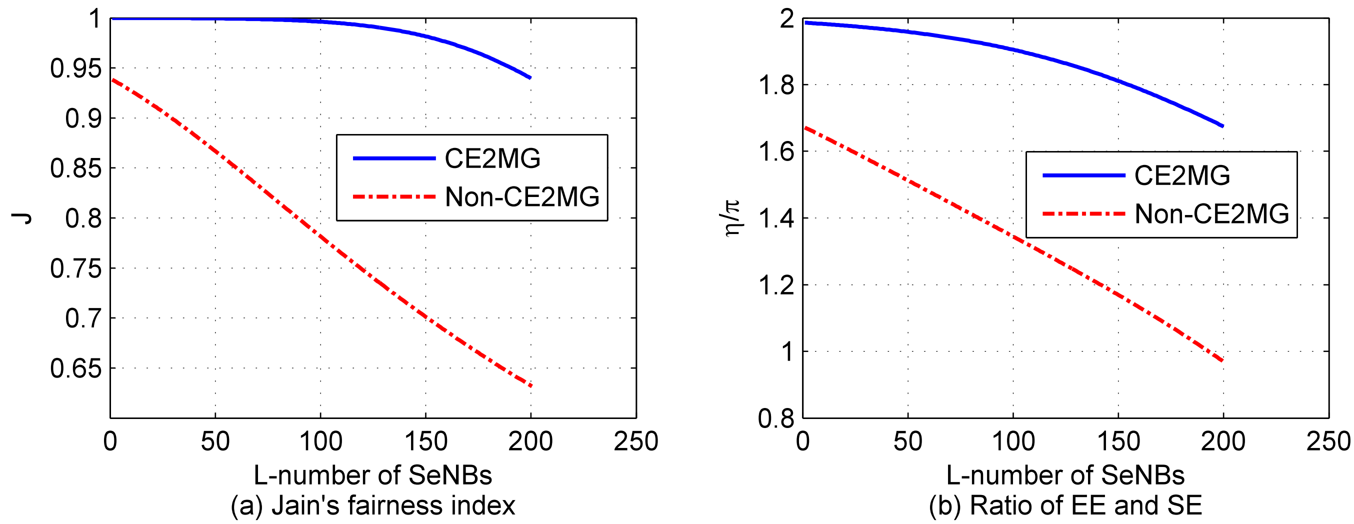

Fig. 5. Performance of the Jain's fairness index (a) and the EE vs. SE tradeoffs (b).

and the optimal tradeoff of our proposed CE2MG scheme compared with the selfish non-CE2MG scheme.

First, we observe the effects of densification (e.g., the increasing number L of small cells) on system EE η and average energy efficiency η_ℓ of specific SeNB$_\ell$, $\ell \in \mathcal{L}$. Without loss of generality, we illustrate both the system EE and the average EE performance with respect to the increasing number L of small cells via Fig. 4.

We conclude that the system EE η keeps increasing with the increasing number L of small cells, which can be seen in Fig. 4; however, the rate and the speed are not the same. At first, a dramatic increment of the system EE η until the number (L) of small cells is about 200 and after that, there is little increment in the SE. On the other hand, we depict the curve of the individual average energy efficiency (η_ℓ) with respect to the increasing number (L) of small cells. We can see that η_ℓ keeps decreasing and then reaches a flat performance, which can be seen in Fig. 4.

Second, we investigate the fairness among different SeNBs, and the optimal tradeoff of our proposed CE2MG scheme compared with the selfish Non-CE2MG scheme. Here, the non-CE2MG scheme can be regarded as the conventional non-cooperative method. Here, we assume that the SeNBs always selfishly expect more spectrum efficiency have the same strategy as the non-cooperative players in the CE2MG model. In addition, the corresponding scheme is termed as Non-CE2MG scheme. When analyzing the formulated CE2MG, fairness is an important issue in the multiple players case. Here, in this paper, fairness means that L SeNB players will achieve a fair improved performance with respect to either energy or spectrum efficiency. Here, we use Jain's fairness index as the criterion of fairness. This is mathematically computed as

$$J = \frac{\left(\sum_{\ell=1}^{L} \eta_\ell\right)^2}{L \sum_{\ell=1}^{L} \eta_\ell^2},$$

where we use the achieved EE η as the measurement metric. Here, we compute the fairness index of J from $L = 1$, and then we compute it again with one SeNB participating into the CE2MG gaming process until the total number of SeNBs is 200. The fairness index of the proposed CE2MG scheme and the benchmark Non-CE2MG are illustrated in Fig. 5. Here, we conclude that both the presented CE2MG scheme and the Non-CE2MG scheme will achieve decreasing fairness due to the increasing number of small cells. Meanwhile, the presented CE2MG scheme always outperforms the Non-CE2MG scheme. For instance, the presented CE2MG scheme will achieve a 30% fairness improvement compared with the Non-CE2MG scheme when the number of small cells is 200. Furthermore, we can see that the presented CE2MG scheme will maintain this advantage with a larger number of small cells.

At last, we show the tradeoffs in the measurement of $\frac{\eta}{\mu}$ in Fig. 5. Also, here the presented CE2MG scheme significantly outperforms the Non-CE2MG scheme. For instance, the presented CE2MG scheme will achieve at least 30% tradeoff improvement compared with the Non-CE2MG scheme when the number of small cells is 150. Also, we can see that the presented CE2MG scheme will maintain this advantage with a larger number of small cells. Overall, with an increasing number of players, both the fairness and the tradeoff decrease; however, the presented CE2MG scheme significantly outperforms the Non-CE2MG scheme due to the exploring of the cooperation gains.

IX. Conclusion

Ultra-dense networks enhance the system capacity via exploring both spatial and frequency diversities. The interference and green design problems were more complex due to the intrinsic densification and scalability, and at the same time it requires distributed control with reduced signaling overhead. Different from the popular research on interference mitigation on improving the SE performance, in this work, we studied the energy efficiency maximization problem by exploring and exploiting various cooperative diversity gains. In this work, we formulated a cooperative energy efficiency maximization game (CE2MG) model and proposed a distributed CE2MG algorithm to achieve the optimal SE solution of each small cells to maximize the system energy efficiency. We characterized the optimal tradeoff relationship between energy efficiency and spectrum efficiency with the emphasis of circuit power consumption of small cell and the known interference situations. Based on the derived tradeoff function, we proposed an energy efficiency utility function to maximize energy efficiency without sacrificing more spectrum efficiency. Numerical results verified the convergence properties, the tradeoff relationship, the improved system energy efficiency performance and the fairness of the presented CE2MG algorithm compared with non-CE2MG solution.

Appendix A
Unimodal Function Proof of the Formulated Utility Function

We can prove the designed utility function is a unimodal function, which guarantees the existence of the maximum point. For the designed utility function $u_\ell = \ln(\frac{\pi_\ell}{\phi_\ell+\sigma_\ell} - \eta_\ell^{\min})$, we know the spectrum efficiency $\pi_\ell = \frac{c_\ell}{W}$, and $\pi_\ell \geq 0$; at the same time, $\phi_\ell = \mu_\ell(e^{\frac{\pi_\ell}{m_\ell}} - 1) \geq 0$, and $\mu_\ell = \frac{\varepsilon_\ell \varpi_\ell}{W} \geq 0$, $\sigma_\ell = \frac{p_\ell^{cst}}{W} \geq 0$, and $m_\ell \geq 0$. In the designed utility function $u_\ell = \ln(\frac{\pi_\ell}{\phi_\ell+\sigma_\ell} - \eta_\ell^{\min})$, we know $\frac{\pi_\ell}{\phi_\ell+\sigma_\ell} \overset{\phi_\ell+\sigma_\ell>0}{>} \eta_\ell^{\min} \Rightarrow \pi_\ell > (\phi_\ell+\sigma_\ell)\eta_\ell^{\min}$, which can help first determine the value range of $\pi_\ell = \frac{c_\ell}{W}$. We define the function $F(\pi_\ell) = \pi_\ell - (\mu_\ell(e^{\frac{\pi_\ell}{m_\ell}} - 1) + \sigma_\ell)\eta_\ell^{\min}$, and then we have $\frac{\partial F(\pi_\ell)}{\partial \pi_\ell} = 1 - \frac{\mu_\ell \eta_\ell^{\min}}{m_\ell} e^{\frac{\pi_\ell}{m_\ell}}$ with $\frac{\pi_\ell}{m_\ell} \geq 0$. There exist two cases:

- **Case 1:** when $\frac{\mu_\ell \eta_\ell^{\min}}{m_\ell} \geq 1$, we know $\frac{\partial F(\pi_\ell)}{\partial \pi_\ell} < 0$ always holds. It is known that $\frac{\pi_\ell}{\phi_\ell+\sigma_\ell} - \eta_\ell^{\min}|_{\pi_\ell=0} \leq 0$ when $\pi_\ell = 0$, which tells that there does not exist $\pi_\ell > 0$ to achieve the $u_\ell > 0$.
- **Case 2:** when $\frac{\mu_\ell \eta_\ell^{\min}}{m_\ell} < 1$, we have $\frac{\partial F(\pi_\ell)}{\partial \pi_\ell} = 1 - \frac{\mu_\ell \eta_\ell^{\min}}{m_\ell} e^{\frac{\pi_\ell}{m_\ell}} = 0$. We obtain $\pi_\ell = m_\ell \ln(\frac{m_\ell}{\mu_\ell \eta_\ell^{\min}})$, and we can see that $\frac{\partial F(\pi_\ell)}{\partial \pi_\ell}$ will decrease with π_ℓ. Meanwhile, we know $\frac{\partial F(\pi_\ell)}{\partial \pi_\ell}|_{\pi_\ell=0} > 0$, which leads to $F(\pi_\ell)$ is a unimodal function.

In this case 2, with $\pi_l = m_l \ln(\frac{m_l}{\mu_l \eta_l^{\min}})$, we have $F(\pi_l)|_{\pi_l = m_l \ln(\frac{m_l}{\mu_l \eta_l^{\min}})} = m_l \ln(\frac{m_l}{\mu_l \eta_l^{\min}}) - m_l + \mu_l \eta_l^{\min} - \sigma_l \eta_l^{\min}$. This concludes that when $F(\pi_l)|_{\pi_l = m_l \ln(\frac{m_l}{\mu_l \eta_l^{\min}})} \geq 0$, $F(\pi_l)$ will exist with the potential π_l.

As follows, we further prove the designed utility function $u_l = \ln(\frac{\pi_l}{\phi_l+\sigma_l} - \eta_l^{\min})$ is a unimodal function. We assume that $G(\pi_l) - \eta_l^{\min} > 0$, where $G(\pi_l) = \frac{\pi_l}{\phi_l+\sigma_l}$, thus leading to $u_l = \ln(G(\pi_l) - \eta_l^{\min})$. We have $\frac{\partial u_l}{\partial \pi_l} = \frac{1}{G(\pi_l)-\eta_l^{\min}} \frac{\partial G(\pi_l)}{\partial \pi_l}$. Here, we know $G(\pi_l) - \eta_l^{\min} > 0$; therefore, if $\frac{\partial u_l}{\partial \pi_l} > 0$, then $\frac{\partial G(\pi_l)}{\partial \pi_l} > 0$. Furthermore, $\frac{\partial G(\pi_l)}{\partial \pi_l} = \frac{\phi_l+\sigma_l-(\partial \phi_l/\partial \pi_l)\pi_l}{(\phi_l+\sigma_l)^2} = \frac{K(\pi_l)}{(\phi_l+\sigma_l)^2}$.

By now the problem is to prove whether the numerator is larger than 0. We further have $K(\pi_l) = \mu_l(1 - \frac{\pi_l}{m_l})e^{\frac{\pi_l}{m_l}} - \mu_l + \sigma_l$, and we know $\frac{\partial K(\pi_l)}{\partial \pi_l} = [\frac{1}{m_l}e^{\frac{\pi_l}{m_l}} - (\frac{1}{m_l}e^{\frac{\pi_l}{m_l}} + \frac{\pi_l}{m_l^2}e^{\frac{\pi_l}{m_l}})]\mu_l = -\frac{\pi_l \mu_l}{m_l^2}e^{\frac{\pi_l}{m_l}} < 0$. It is easy to conclude that $K(\pi_l)$ is a monotonic function. Meanwhile, with the definition of $K(\pi_l)$, we know when $\pi_l = 0$, $K(\pi_l) = \sigma_l > 0$; when $\pi_l \to -\infty$, $K(\pi_l) \to -\infty$. We can conclude that $G(\pi_l) = \frac{\pi_l}{\phi_l+\sigma_l}$ a unimodal function.

In summary, we know the designed utility function $u_l = \ln(\frac{\pi_l}{\phi_l+\sigma_l} - \eta_l^{\min})$ is a unimodal function.

Appendix B
Estimation of Undesirable Interference Power

Here, we estimate the undesirable interference power value of ω_ℓ^m in a mean field approach [27]. We have

$$\omega_\ell^m = \sum_{\ell'=1, \ell' \neq \ell}^{L} p'_\ell g_{\ell',\ell}^m = (L-1)\,\hat{p}'_\ell \hat{g}_{\ell',\ell}^m,$$

where p'_ℓ, $g_{\ell',\ell}^m$ represent the downlink power of other SeNBs $\ell' \in \mathcal{L}, \ell' \neq \ell$ except the SeNB$_\ell$, and the interference gain from the SeNB$'_\ell$ to the SeNB$_\ell$. In addition, $\hat{p}'_\ell$ is the known test transmit power, and $\hat{g}^m_{\ell',\ell}$ defines the mean interference channel gain, which can be estimated by the following idea.

If we test using $\hat{p}'_\ell$, then the received power at SeNB$_\ell$ is

$$p'^R_\ell = \hat{p}'_\ell g^m_{\ell',\ell} + \omega^m_\ell,$$

where $g^m_{\ell',\ell}$ is the known effective channel gain, and $\hat{p}'_\ell g^m_{\ell',\ell}$ is the effective received power, and ω^m_ℓ is the received interference power from all the other SeNBs $\ell' \in \mathcal{L}, \ell' \neq \ell$.

With the above two equations we can derive the only unknown variable $\hat{g}^m_{\ell',\ell}$ as

$$p'^R_\ell = \hat{p}'_\ell g^m_{\ell',\ell} + (L-1)\,\hat{p}'_\ell \hat{g}^m_{\ell',\ell},$$

and we have

$$\hat{g}^m_{\ell',\ell} = \frac{p'^R_\ell - \hat{p}'_\ell g^m_{\ell',\ell}}{(L-1)\,\hat{p}'_\ell}.$$

That is, each SeNB can achieve the undesirable power in the form of (3) via the interference estimation method in [27]. This method can approximate the real interference with the increasing number of SeNBs.

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