# Distributed TOA-Based Positioning in Wireless Sensor Networks: A Potential Game Approach

Mingxing Ke, Yuhua Xu, *Member, IEEE*, Alagan Anpalagan, *Senior Member, IEEE*, Dianxiong Liu, and Yuli Zhang

***Abstract*— In this letter, we solve the problem of positioning based on the time-of-arrival technique in wireless sensor networks. Different from the traditional estimation methods, the position of a target node is considered as a strategy of anchor nodes, and the cognition information is introduced to enrich positional information. After proposing the concept of consensus of cognition and consensus of measurements, we re-investigate the positioning problem from a game-theoretic perspective. It is proved that the proposed game is an exact potential game, which exhibits attractive properties about the Nash equilibrium (NE). Then, the best response algorithm is used to achieve the NE point. The simulation results indicate that the performance of positioning with a suitable tradeoff parameter is better than the performance of recursive least-squares algorithm in terms of positional accuracy and convergence rate.**

***Index Terms*— Distributed positioning algorithms, TOA, potential game.**

## I. Introduction

POSITIONING is a key aspect for many applications in wireless sensor networks (WSNs). Recently, centralized and distributed algorithms are two main methods to estimate the positions of target nodes. In centralized algorithms, the central node is burdened with a large number of calculations, and anchor nodes' traffic represents a system bottleneck [1]. On the other hand, in distributed algorithms, each anchor node can exchange information with neighboring anchors and determine the target node position locally. Consequently, distributed algorithms are more preferable, because they require less computations and avoid communication bottlenecks.

However, even though the distributed algorithms have been investigated extensively, there are still some unresolved problems. Recently, some works [2] and [3], have provided an overview of various positioning techniques in WSNs. Namely, the least squares (LS), which represents a simple method, is widely used for positioning, but its accuracy is poor in NOLS scenarios [4]. In order to improve the accuracy, the particle filter (PF) was used in [5], but it required a multiple calculations and was not suitable in normal WSN applications. Moreover, [6] presented a recursive least squares (RLS) approach, but the convergence rate was affected by the initial position of the target node. Therefore, the above algorithms did not solve the mentioned problems effectively.

Since the distributed nature coincides nicely with the framework of non-cooperative games [7], the game approach has been used in the positioning [8]. While compared with the traditional methods in [4]–[8] which just depend on measurements to obtain the position of target nodes, we introduce the cognition information of anchor nodes to enrich the positional information in our work. After proposing the concept of consensus of cognition and measurements, we design an effective utility function and prove that it transforms the positioning problem to the exact potential game. Note that, not only the positional information but also the scenarios and points of view to construct the utility functions in our work are different with the exist work [8]. Due to particularly good characteristics of exact potential game, the Nash equilibrium (NE) point can be considered as an optimal or sub-optimal solution of the problem. In order to achieve such solution, the best response (BR) learning algorithm is used. The simulation results show that the proposed algorithm can reach the NE in a finite number of steps and, when a suitable trade-off parameter is used, the error and convergence time of positioning are less than those in RLS algorithm.

## II. System Model for TOA-Based Positioning

The WSN structure consists of $N$ anchor nodes with known positions and a target node with unknown positions (see Fig. 1). Here, we consider two-dimensional coordinates: $\mathbf{p}_i = [p_i^x, p_i^y], i = 1, 2, \ldots, N$ and $\mathbf{a} = [x, y]$, which represent two kinds nodes coordinates. Further, we assume that anchor and target nodes are static during positioning and consider a fully connect network.

The TOA positioning techniques based on ultra wide-band (UWB) [9] technology can potentially achieve very high indoor position accuracy; hence, TOA-based positioning is adopted in this work. A widely used statistical model for TOA characterization is given by:

$$\hat{d}_i = d_i + n_i + b_i = ct_i, \tag{1}$$

where $\hat{d}_i$ and $d_i$ are the estimated distance and real distance between the $i^{th}$ anchor node and the target node, respectively. $t_i$ is the TOA of the signal. $c$ is the speed of light. $n_i \sim N(0, \sigma_i^2)$ is the additive white Gaussian noise (AWGN) with variance $\sigma_i^2$. $b_i$ is the positive distance bias introduced by the blockage of direct path, and it is defined as

$$b_i = \begin{cases} 0, & \text{if LOS} \\ \Psi_i, & \text{if NLOS.} \end{cases} \tag{2}$$

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M. Ke, Y. Xu, D. Liu, and Y. Zhang are with the College of Communications Engineering, PLA Army Engineering University, Nanjing 210007, China, and also with the Science and Technology on Communication Networks Laboratory, Shijiazhuang 050002, China (e-mail: lgdxkemingxing@163.com; yuhuaenator@gmail.com; dianxiongliu@163.com; yulipkueecs08@126.com).

A. Anpalagan is with the Department of Electrical and Computer Engineering, Ryerson University, Toronto, ON M5B 2K3, Canada (e-mail: alagan@ee.ryerson.ca).

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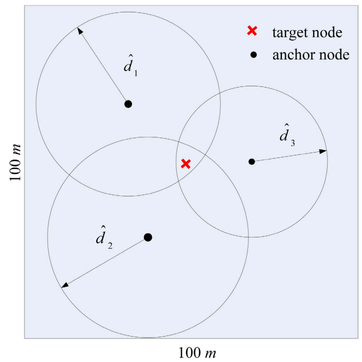


Fig. 1. The WSN nodes.

In the literature, the bias term $\Psi_i$ in NLOS environments is modeled in different ways, i.e. as Gaussian distributed, exponentially distributed, or using an empirical model based on measurements [1]. In our work for simplicity, we consider NLOS environment as non-negative Gaussian distributed. In addition, all noise terms are zero-mean and independent random Gaussian variables. The noise measurement between the target node and the $i^{th}$ anchor node is defined as:

$$\hat{d}_i = (x - p_i^x)^2 + (y - p_i^y)^2, \quad i = 1, \ldots, M. \tag{3}$$

The TOA-based positioning can be recognized as the estimation of target node positions based on noisy and biased distance measurements and anchor node positions. We re-investigate the above from a game-theoretic perspective.

## III. Game Model for Positioning

The formulated positioning game is formally defined by $\Gamma = (\mathcal{N}, A_i, u_i)$, where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the player set, which is the set of anchor nodes, $A_i$ is the strategy space set, $u_i$ is the utility function of $i$. Denote $a_i \in A_i$ as a strategy of player $i$ for target node; moreover, denote $a_{-i}$ as a strategy profile of all players except player $i$. Before we define the utility function, we have to introduce two basic characteristics for adopting a strategy, namely the consensus of cognition and the consensus of measurements.

- *The consensus of cognition*: the decisions on target node positions by different anchor nodes should be as similar as possible. In other word, the action $a_i$ of player $i$ (anchor node) should be consistent with all other players for the target node. The consensus of cognition is defined by:

$$U_i^c(a_i, a_{-i}) = (N-1)^{-1} \sum_{m=1, m \neq i}^{N} \|a_i - a_m\|^2, \tag{4}$$

  where $\| \bullet \|$ is the Euclidean norm, and when it is multiplied by $(N-1)^{-1}$, it denotes the average cognition of anchor nodes. This characteristic reflects that when anchor nodes have cognition, they can give judgements of the targets' positions as one part of the position information then cooperate to acquire the positions of the target node. It also contains the interaction information between anchor nodes, i.e., $a_m, m \neq i$.
- *The consensus of physical measurements*: the distance between nodes is proportional to the signal travel time. If the results of TOA-based measurement is larger, the distance between nodes is also larger. The consensus of measurements is given by:

$$U_i^p(a_i, a_{-i}) = (\|a_i - p_i\| - ct_i)^2, \tag{5}$$

  where $p_i$ is the position of anchor node $i$. This representation is consistent with the physical nature of signal transmission. This characteristic contains the measurement information of the single anchor node $i$, i.e., $t_i$.

According to the above mentioned characteristics, the distributed anchor nodes can sense and iteratively compute the coordinates of the target node.

In our work, we define the utility function of $i$ as follows:

$$\begin{aligned} u_i&(a_i, a_{-i}) \\ &= (\zeta - 1)U_i^c(a_i, a_{-i}) - \zeta U_i^p(a_i, a_{-i}) \\ &= \frac{\zeta - 1}{(N-1)} \sum_{m=1, m \neq i}^{N} \|a_i - a_m\|^2 - \zeta(\|a_i - p_i\| - ct_i)^2, \end{aligned} \tag{6}$$

where $\zeta$ determines a trade-off between two basic characteristics. Note that those two basic characteristics interrelate with each other and to ensure the complete positional information, the $\zeta$ is set as $0 < \zeta < 1$. In this game, each anchor node maximizes its utility function. Thus, for anchor node $i$ the optimal objective can be expressed as:

$$\Gamma : \max_{a_i \in A_i} u_i(a_i, a_{-i}), \quad \forall i \in \mathcal{N}. \tag{7}$$

In addition, the network optimal objective for target node is defined by:

$$\begin{aligned} &\max U(a_1, a_2, \ldots, a_N) \\ &= \max \sum_{i=1}^{N} \left\{ \frac{\zeta - 1}{N-1} \times \sum_{m=1, m \neq i}^{N} \|a_i - a_m\|^2 - \zeta(\|a_i - p_i\| - ct_i)^2 \right\}. \end{aligned} \tag{8}$$

*Definition 1 (Nash Equilibrium [10]):* A positioning profile $a^* = (a_1^*, \ldots, a_N^*)$ is a pure strategy Nash equilibrium (NE) if and only if no player can improve its utility by unilateral deviation, i.e., if the following condition is satisfied:

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*), \quad \forall i \in \mathcal{N}, \quad \forall a_i \in A_i, \quad a_i \neq a_i^*. \tag{9}$$

Namely, $\Gamma$ is a continuous game, if the strategy set $A_i$ is a continuous interval of $R$ and if each utility function $u_i(a_i, a_{-i})$ is continuous and differentiable.

*Definition 2 (Exact Potential Game [10]):* The game $\Gamma$ is an exact potential game if and only if a potential function $\Phi(a_i, a_{-i})$, $\forall i \in \mathcal{N}$ satisfies the following condition:

$$\begin{aligned} \Phi(a_i^*, a_{-i}) - \Phi(a_i, a_{-i}) &= u_i(a_i^*, a_{-i}) - u_i(a_i, a_{-i}), \\ &\forall a_i, \quad a_i^* \in A_i, \quad \forall a_{-i} \in A_{-i}. \end{aligned} \tag{10}$$

Furthermore, the positioning game is characterized by the following Lemma.

*Lemma 1: For a given positioning model, $\Gamma$ is an exact potential game that has at least one pure strategy NE; moreover, all pure strategy NE points of $\Gamma$ maximize the network utility $U$ globally or locally.*

**Algorithm 1** BR-Based Distributed Positioning Algorithm

**Step 1)** Initially, set $n = 1$ and let each anchor node choose a random position within the communication scope as the first positioning strategy for the target node; set $\tau = 5$ to ensure convergent.

**Step 2)** All anchor nodes exchange information with their neighbors.

**Step 3)** For $i = 1 : N$ All other anchor nodes repeat their strategies, i.e., $a_{-i}(n+1) = a_{-i}(n)$. Meanwhile, by obtaining the information from the neighbors, anchor node $i$ calculates the maximum point of utility function as next strategy $a_i(n+1)$ which satisfies:

$$a_i(n+1) = \arg\max_{a_i \in A_i} u_i(a_i, a_{-i}|n). \quad (16)$$

In (6), it can be seen that the $u_i$ is a convex function of $a_i$. Then the first and second order partial derivatives of $x_i$ and $y_i$ can be obtained and we combine them to solve the solution which is expressed as (17), as shown at the bottom of previous page. In the right of (17), they are both positive or negative and where

$$P_x = \frac{(1-\zeta)}{N-1} \sum_{m=1, m\neq i}^{N} x_m(n) + \zeta p_i^x, \quad (18)$$

$$P_y = \frac{(1-\zeta)}{N-1} \sum_{m=1, m\neq i}^{N} y_m(n) + \zeta p_i^y. \quad (19)$$

end

**Step 4)** If $\|a_i(n+1) - a_i(n)\| < \varepsilon$ and in the next $\tau$ iterations, $a_i$ also satisfies this inequality, stop; else go to Step 2.

*Proof:* we construct a potential function as follows:

$$\Phi(a_1, a_2, \ldots, a_N) = \sum_{i=1}^{N} \Big\{ (\zeta - 1)(N-1)^{-1} \times \sum_{m=1, m\neq i}^{N} \|a_i - a_m\|^2 - \zeta(\|a_i - p_i\| - ct_i)^2 \Big\}, \quad (11)$$

where $\Phi(a_1, a_2, \ldots, a_N)$ is the network utility given by (8). When one player $k$ changes its strategy from $a_k$ to $a_k^*$, the change of its individual utility function caused by the unilateral change is given by (12), as shown at the bottom of this page.

The change of the potential function caused by the unilateral change of player $k$ is given by (13), as shown at the bottom of this page, where $a_i^* = a_i \ (i \neq k)$.

For the player $i \ (i \neq k)$, its action does not change with the unilateral change of $k$. Hence, we obtain (14), as shown at the bottom of this page.

Then, from (12) to (14), we have the following equation:

$$\Phi(a_k^*, a_{-k}) - \Phi(a_k, a_{-k}) = u_k(a_k^*, a_{-k}) - u_k(a_k, a_{-k}). \quad (15)$$

Consequently, the change of individual utility function caused by unilateral deviation of an arbitrary player is the same as the change of the potential function. Significantly, when $\Delta a_k \to 0 (\Delta a_k = \|a_k^* - a_k\|)$, it still holds (15). According to the definition 2, the game $\Gamma$ is an exact potential game and $\Phi$ can serve as the potential function. The exact potential game exhibits the following important properties [10]: (i) any exact potential game has at least one pure strategy NE; (ii) any global or local maxima of the potential function constitutes a pure strategy NE.

Therefore, the global maximum of the potential function or the network utility is a pure positioning strategy NE point of the proposed game. Moreover, the potential game might have a local maxima that corresponds to other sub-optimal NE points. In conclusion, Lemma 1 is proved. ■

In potential games, there are several decision rules for finding a pure NE, such as spatial adaptive play (SAP) [11], fictitious play (FP) [12] and best response (BR) [13], etc. In our work, we adopt the BR learning algorithm to achieve the convergent solution. First, we extend the pure strategy to a mixed strategy form where $a_i(n)$ for player $i$ at iteration $n$ is denoted. The basic BR concept is that only one player is selected to update its selection according to the best strategy while all other players repeat their selections. However, in BR of positioning game, the anchor nodes are selected in a round-robin manner to update their strategies by re-choosing of the maximum points of utility function as the next iteration strategy. This process is repeated until any stop criterion is met (see Algorithm 1). In step 4, $\varepsilon$ is the threshold used to judge on the algorithm convergence.

*Lemma 2: For the positioning game, the BR learning algorithm can guarantee the end solution closer to an actual NE in a finite number of step.*

*Proof:* please refer to [10, Sec. 2.2] for detailed derivation.

Therefore, the result of BR will converge to the optimal or sub-optimal solution.

*Remark:* Here in our work, the exploration focus is on the feasibility and preliminary results of the positioning and our further research will focus on the design optimization.

$$u_k(a_k^*, a_{-k}) - u_k(a_k, a_{-k}) = \frac{\zeta-1}{N-1} \sum_{\substack{m=1 \\ m\neq i}}^{N} \|a_k^* - a_m\|^2 - \zeta(\|a_k^* - p_k\| - ct_k)^2 - \Big[\frac{\zeta-1}{N-1} \sum_{\substack{m=1 \\ m\neq i}}^{N} \|a_k - a_m\|^2 - \zeta(\|a_k - p_k\| - ct_k)^2\Big], \quad (12)$$

$$\Phi(a_k^*, a_{-k}) - \Phi(a_k, a_{-k}) = \sum_{i=1}^{N} \Big[\frac{\zeta-1}{N-1} \sum_{m=1, m\neq i}^{N} \|a_i^* - a_m\|^2 - \zeta(\|a_i^* - p_i\| - ct_i)^2\Big] - \sum_{i=1}^{N} \Big[\frac{\zeta-1}{N-1} \sum_{m=1, m\neq i}^{N} \|a_i - a_m\|^2 - \zeta(\|a_i - p_i\| - ct_i)^2\Big], \quad (13)$$

$$\sum_{\substack{i=1 \\ i\neq k}}^{N} \Big[\frac{\zeta-1}{N-1} \sum_{m=1, m\neq i}^{N} \|a_i^* - a_m\|^2 - \zeta(\|a_i^* - p_i\| - ct_i)^2\Big] - \sum_{\substack{i=1 \\ i\neq k}}^{N} \Big[\frac{\zeta-1}{N-1} \sum_{m=1, m\neq i}^{N} \|a_i - a_m\|^2 - \zeta(\|a_i - p_i\| - ct_i)^2\Big] = 0, \quad (14)$$

$$a_i(n+1) = [x_i(n+1), y_i(n+1)] = \Big\{ P_x \pm \frac{\zeta ct_i(P_x - p_i^x)}{\sqrt{(P_x - p_i^x)^2 + (P_y - p_i^y)^2}}, P_y \pm \frac{\zeta ct_i(P_y - p_i^y)}{\sqrt{(P_x - p_i^x)^2 + (P_y - p_i^y)^2}} \Big\} \quad (17)$$

TABLE I

DIFFERENT RMSE IN THE $20^{th}$ ITERATION

| $\zeta$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | $RLS$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $RMSE(m)$ | 8.733 | 5.963 | 3.992 | 2.748 | 1.894 | 1.795 | 1.483 | 1.378 | 1.435 | 1.736 |

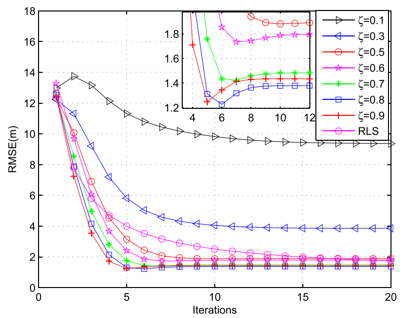


Fig. 2. The RMSEs of BR for different $\zeta$ values and RLS [6].

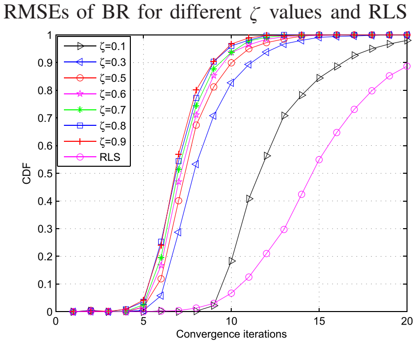


Fig. 3. The CDF of convergence time.

## IV. SIMULATION RESULTS

The proposed algorithm is validated using a WSN illustrated in Fig. 1. The WSN consists of $N = 5$ anchor nodes and one target node, randomly distributed within 100m × 100m region. At each iteration, all anchor nodes update their strategies sequentially and the result is the mean value of the decision values of the related anchor nodes. We consider that the communication and ranging mode are IEEE 802.15.4a compliant. The target position is estimated using anchor nodes and according to the TOA-based protocol explained in [14]. The variance of range measurements is $\sigma_i^2 = 1$, and the non-line-of-sight is $b_i = |n'_i|$ where $n'_i \sim N(0, 2)$. The threshold is set as $\varepsilon = 0.01$.

The results in Fig. 2 represent the averaged values of 1000 independent Monte Carlo simulations for each $\zeta$. We also analyzed the estimation results in terms of RLS algorithm which was used in [6]. The root mean square error (RMSE) values in the $20^{th}$ iteration are presented in Table I. The minimal RMSE for $\zeta = 0.8$ after 10 iterations is 1.378m. Here, we consider the same value of $\zeta$ for all nodes, but it can be adjusted for different nodes depending on their cognition and measurements. As it can be seen in Table I, when $\zeta$ is equal to 0.7, 0.8 or 0.9, the RMSEs of best response are smaller than those of RLS in 20 iterations. We also obtained the positioning performances in different NLOS models, such as the exponentially distributed, the uniform-gaussian mix distributed, etc., and they showed the similar results.

The cumulative distribution functions (CDF) of convergence time for different $\zeta$ is shown in Fig. 3, wherein it can be seen that for higher $\zeta$ the convergence is faster. For instance, given 80% confidence, the convergence occurs at about $8^{th}$ iteration for $\zeta = 0.9$, whereas it is about $14^{th}$ iteration for $\zeta = 0.1$ and at $18^{th}$ iteration in RLS. Therefore, $\zeta$ represents a trade-off between convergence time and positional accuracy. The optimality of $\zeta$ in different situations and the power consumption of different algorithms will be next key issues in further research.

## V. CONCLUSION

In this letter, we re-investigated the distributed positioning problem as a game using the concept of consensus of cognition and consensus of measurements, and the game was proved to be an exact potential game. In addition, it was shown that in the positioning game, a NE point of the position can be achieved using the information exchange among anchor nodes by best response algorithm. The simulation results have indicated that the performance of positioning with a suitable parameter is better than the performance of RLS algorithm.

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