Multi-Hop Cooperative Transmission Using Fountain Codes over Rayleigh Fading Channels

Tran Trung Duy, Alagan Anpalagan, and Hyung-Yun Kong

Abstract: **In this paper, we study multi-hop cooperative transmission protocols using fountain codes. The proposed protocols can reduce the end-to-end delay and number of stages compared to those in conventional multi-hop transmission. Various Monte-Carlo simulations are presented to evaluate and compare performance of the protocols over Rayeigh fading channels.**

Index Terms: **Cooperative communication, fountain codes, mutli-hop networks, Rayleigh fading channel.**

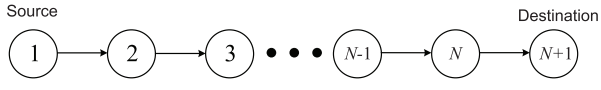

Fig. 1. The N-hop route from the source to the destination.

I. INTRODUCTION

Rateless codes or fountain codes [1] have drawn much attention due to their simple implementation and adaptation to channel conditions. At a transmitter using fountain codes, a limitless stream of encoded packets can be generated and transmitted to receivers. At the intended receivers, the original data can be recovered if the received information exceeds the amount of information of the original data [2]. Therefore, fountain codes avoid the need for feedback channel and do not require channel state information (CSI) at the receivers.

To exploit the broadcast nature of the wireless channel, cooperative communication can be efficiently used [3]–[7]. So far, most of works related to cooperative communication have been developed on single-hop networks. However, multi-hop relaying in which a source communicates with a destination via a number of relays has become a promising technique for application in the current and future wireless systems. Recently, multi-hop cooperative transmission protocols have been studied in several literatures [8]–[10] in the view points of implementation issues and performance evaluations. In such protocols, relays along the primary route can employ cooperative diversity transmission to improve end-to-end performance. However, these protocols increase the complexity of decoding operations at relays and are not able to reduce the end-to-end delay, compared to those in conventional multi-hop transmission. Recently, some works related to cooperative communication employing fountain codes have been reported [11], [12]. These proposed models have been shown to provide higher capacity as well as reduced transmission time, compared to those in conventional cooperative communication.

In this paper, we propose cooperative multi-hop transmission protocols using fountain codes to reduce end-to-end transmission time. First, we assume that there is a route between a source and a destination, which is established by the network layer. The source and relays along this route employ cooperative diversity to transmit the encoded data generated by fountain codes right to destination. The operations of the proposed protocols are realized in stages. At each stage, the relay which first recovers the original data replaces the previous source to forward the encoded data to the destination. On the other hand, if the first successfully decoding node is the destination, the data transmission ends at that stage. The performance of these protocols is evaluated via Monte-Carlo simulations and analyses. Results show that the proposed protocols reduce end-to-end transmission time as well as the number of stages, compared to conventional multi-hop transmission (CMT).

The rest of the paper is organized as follows. The system model and the proposed schemes are described in Section II. In Section III, the performances of the protocols are analyzed. The simulation results are presented in Section IV. Finally, the paper is concluded in Section V.

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II. SYSTEM MODEL

In Fig. 1, an N-hop route which is established by the network layer consists of nodes $1, 2, \cdots, N, N+1$, where node 1 is the source, node $N+1$ is the destination, and the remaining nodes are relays. We denote ψ as the amount of information of the original data. Similar to [2] and [7], it is assumed that a receiver can successfully decode the original data, if the received information is larger or equal to ψ.

A. Conventional Multi-Hop Transmission Protocol Using Fountain Codes (CMT)

In this protocol, the original data is relayed hop-by-hop from the source to the destination. In particular, at the jth hop, $1 \leq j \leq N$, node j uses fountain codes to encode the original data and transmit the encoded data to node $j+1$. As soon as node $j+1$ sufficiently receives amount of information ψ for reliably decoding the original data, it sends an acknowledgement message (ACK) to node j to inform the successful reception and this node ceases the transmission of the encoded data. Next, node $j+1$ decodes the original data from the received encoded data. It then re-encodes the original data using fountain codes and transmits the encoded data to node $j+2$. This procedure continues until the original data is successfully received by the destination.

B. The First Multi-Hop Cooperative Transmission Protocol Using Fountain Codes (MCT-1)

The data transmission process in the multi-hop cooperative transmission (MCT)-1 protocol can be realized as follows:

- Step 1: In the first stage, the source encodes the original data using fountain codes and transmits the encoded data to relays and the destination. If the destination is the first node sufficiently receiving amount of information ψ, the operation goes to step 3. In the case that the first node is a relay, e.g., i_1, $2 \leq i_1 < N+1$, the operation realizes to step 2.
- Step 2: Relay i_1 informs the source and the other relays by generating the ACK message. As soon as the source receives this message, it ceases the transmission of the encoded data. Next, relay i_1 decodes the original data and re-encodes it using fountain codes. It replaces the source to transmit the encoded data at the stage 2. It is noted that in this stage, only the destination and relays between relay i_1 and the destination are allowed to accumulate the encoded information in order to recover the original data. This means that relays $2, 3, \cdots, i_1 - 1$ will release the received information and no longer participate in decoding. Similar to stage 1, if the destination is the first node which can recover the original data, the operation goes to step 3. Otherwise, the operation repeats step 2 and in this case, a relay, e.g., i_2, $i_1 < i_2 < N+1$, which firstly decodes the original data, replaces relay i_1 to transmit the encoded data to the destination and relays between it and the destination at next stage.
- Step 3: The destination sends the ACK message to the source and all the relays to inform the status. In this case, the data transmission ends and the relays release the received data from their buffers.

C. The Second Multi-Hop Cooperative Transmission Protocol Using Fountain Codes (MCT-2)

Similar to the MCT-1 protocol, the operation of the MCT-2 protocol is also realized by three steps as follows:

- Step 1: This step is same with that of the MCT-1 protocol.
- Step 2: Relay i_1 informs the source and the other relays by generating the ACK message. After receiving this message, the source ceases the transmission of the encoded data and the other relays release the received encoded data from their buffers. In this protocol, only the destination is allowed to restore the received encoded information at the previous stages for recovering the original data while the relays must release them at the end of each stage. Also, by using fountain codes to encode the original data, relay i_1 replaces the source to transmit the encoded information which can be received and accumulated by the destination and relays between it and the destination. Then, step 3 will be realized if the destination is the first node sufficiently receiving amount of information ψ. Otherwise, the operation repeats step 2 with a new source, e.g., i_2, $i_1 < i_2 < N+1$.
- Step 3: This step is same with that of the MCT-1 protocol.

As compared with the MCT-1 protocol, the MCT-2 protocol reduces the number of relays that participate in the data transmission by only allowing the destination to restore the encoded information received at all stages. Hence, this protocol also reduces the number of stages, the number of re-encoding and decoding operations at the relays and the amount of encoded data stored in their buffers.

D. The Third Multi-Hop Cooperative Transmission Protocol Using Fountain Codes (MCT-3)

Similarly, the data transmission process of the MCT-3 protocol is described as follows:

- Step 1: This step is same with that of the MCT-1 protocol.
- Step 2: Relay i_1 informs the source and the other relays by generating the ACK message. After receiving this message, the source ceases the transmission of the encoded data and all the nodes including the destination and the other relays release the received encoded data from their buffers. At the next stage, relay i_1 becomes a new source and uses fountain codes to transmit the original to the destination and relays between it and the destination. Similar to the MCT-1 and MCT-2 protocols, if the destination is the first node sufficiently receiving amount of information ψ, the operation goes to step 3. Otherwise, step 2 is repeated by a new source, e.g., i_2, $i_1 < i_2 < N+1$.
- Step 3: This step is same with that of the MCT-1 protocol.

Because it is required that all the nodes have to release the received encoded data after each stage, the amount of data stored in buffers of the destination and relays can be significantly reduced. In addition, when reducing the stored data, the system requires less storage hardware and reduces backup and recovery times. Hence, as compared with the MCT-1 and MCT-2 protocols, the implementation of this protocol is easier but the end-to-end transmission time is longer.

The implementation of the proposed protocols is more complicate than that of the CMT protocol because it is assumed that the ACK message can be received at far distances. However, if we use powerful coding scheme for this message, the source and the relays can overhear it at a pretty far distance. In addition, if the number of hops between the source and the destination is large, we can divide the primary route into smaller multi-hop ones and then apply the proposed protocols on them.

III. PERFORMANCE ANALYSIS

A. Rayleigh Fading Channels

It is assumed that all nodes are equipped with a single antenna and operate in half-duplex mode. We also assume that no perfect CSI is known by the transmitters or receivers. For the data transmission between a transmitter t, $t \in \{1, 2, \cdots, N\}$, and a receiver r, $r \in \{2, 3, \cdots, N+1\}$, the data received at node r due to the transmission of node t is given by

$$y_{r,t} = \sqrt{P} h_{r,t} x + \eta_r \tag{1}$$

where P is the transmitted power of node t, x is the transmitted data, η_r is an additive white Gaussian noise (AWGN) with zero mean and variance N_0, and $h_{r,t}$ is the channel between node t and r and is modeled as flat Rayleigh fading.

The instantaneous received signal-to-noise ratio (SNR) at node r is expressed by

$$\gamma_{r,t} = \frac{P|h_{r,t}|^2}{N_0} = \overline{\gamma}|h_{r,t}|^2 \quad (2)$$

where $\overline{\gamma} = P/N_0$ is the average transmit SNR.

To take path loss into account, we model the variance of channel coefficient $h_{r,t}$ as a function of the distance between two nodes [6] as

$$\sigma^2_{r,t} = {d_{r,t}}^{-\beta} \quad (3)$$

where β is the path loss exponent that varies from 2 to 6, and $d_{r,t}$ is the distance between node t and node r.

Because $h_{r,t}$ has a Rayleigh fading distribution, $|h_{r,t}|^2$ has an exponential distribution. From (2) and (3), it is obvious that $\gamma_{r,t}$ is also an exponential random variable with the parameter $\lambda_{r,t}$ which is given by

$$\lambda_{r,t} = \frac{{d_{r,t}}^{\beta}}{\overline{\gamma}}. \quad (4)$$

Next, we will analyze the performances of the above-mentioned protocols in terms of end-to-end delay and average number of stages. For the proposed protocols, we assume that the data transmission process is realized in K stages $(1 \leq K \leq N)$ and the relays chosen at each stage are $i_1, i_2, \cdots, i_{K-1}$ with $2 \leq i_1 < i_2 < \cdots < i_{K-1} \leq N$.

B. End-to-End Delay

B.1 The CMT Protocol

In the CMT protocol, the number of stages is equal to the number of hops N. Consider the jth hop $(1 \leq j \leq N)$ between node j and node $j+1$; similar to [11, (4)], the required time for node $j+1$ to sufficiently receive amount of information ψ is calculated by

$$t_j^{\mathrm{CMT}} = \frac{\psi}{\log_2(1+\gamma_{j,j+1})}. \quad (5)$$

Therefore, the total transmission time from the source to the destination is formulated by

$$T^{\mathrm{CMT}} = \sum_{j=1}^{N} t_j^{\mathrm{CMT}} = \sum_{j=1}^{N} \frac{\psi}{\log_2(1+\gamma_{j,j+1})}. \quad (6)$$

From (6), the average end-to-end delay in CMT protocol is determined by

$$E\left\{T^{\mathrm{CMT}}\right\} = \sum_{j=1}^{N} E\left\{t_j^{\mathrm{CMT}}\right\} = \sum_{j=1}^{N} E\left\{\frac{\psi}{\log_2(1+\gamma_{j,j+1})}\right\}. \quad (7)$$

B.2 The MCT-1 Protocol

Because node i_1 is the first node to receive sufficient ψ information from the source, the channel between the source and node i_1 is best and hence $\gamma_{1,i_1} = \max_{j=2,3,\cdots,N+1}(\gamma_{1,j})$. Similar to (5), the time required for the first stage is

$$t_1^{\mathrm{MCT-1}} = \frac{\psi}{\log_2(1+\gamma_{1,i_1})}. \quad (8)$$

It is also noted from (8) that the time required for the first stage in all the proposed protocols are the same.

For the jth stage, $1 < j \leq K$, the delay time is calculated by

$$t_j^{\mathrm{MCT-1}} = \frac{\psi - \sum\limits_{m=0}^{j-2} t_m^{\mathrm{MCT-1}} \log_2\left(1+\gamma_{i_m,i_j}\right)}{\log_2\left(1+\gamma_{i_{j-1},i_j}\right)} \quad (9)$$

where $i_0 = 1$, $i_K = N+1$, $t_m^{\mathrm{MCT-1}}\,(1 \leq m \leq j-1)$ is transmission time of stage m, and $\psi - \sum_{m=0}^{j-2} t_m^{\mathrm{MCT-1}} \cdot\log_2\left(1+\gamma_{i_m,i_j}\right)$ presents the remaining information which needs to be collected by node i_j. Therefore, the total transmission time in the MCT-1 protocol is added as follows:

$$\begin{aligned} T^{\mathrm{MCT-1}} &= \sum_{j=1}^{K} t_j^{\mathrm{MCT-1}} \\ &= \frac{\psi}{\log_2(1+\gamma_{1,i_1})} \\ &+ \sum_{j=2}^{N} \frac{\psi - \sum\limits_{m=0}^{j-2} t_m^{\mathrm{MCT-1}} \log_2\left(1+\gamma_{i_m,i_j}\right)}{\log_2\left(1+\gamma_{i_{j-1},i_j}\right)}. \end{aligned} \quad (10)$$

B.3 The MCT-2 Protocol

For the jth $(1 \leq j \leq K)$ stage, the required time is calculated as

$$t_j^{\mathrm{MCT-2}} = \begin{cases} \dfrac{psi}{\log_2(1+\gamma_{1,i_1})}, & \text{if } j=1 \\ \dfrac{\psi}{\log_2\left(1+\gamma_{i_{j-1},i_j}\right)}, & \text{if } 1<j<K \\ \dfrac{\psi - \sum\limits_{m=0}^{K-2} t_m^{\mathrm{MCT-2}} \log_2(1+\gamma_{i_m,N+1})}{\log_2\left(1+\gamma_{i_{K-1},N+1}\right)}, & \text{if } j=K \end{cases} \quad (11)$$

where $t_m^{\mathrm{MCT-2}}$ is the transmission time of stage m. Hence, the total end-to-end delay in the MCT-2 protocol is given as

$$\begin{aligned} T^{\mathrm{MCT-2}} &= \sum_{j=1}^{N} \tau_j^{\mathrm{MCT-2}} \\ &= \frac{\psi}{\log_2(1+\gamma_{1,i_1})} + \sum_{j=2}^{K-1} \frac{\psi}{\log_2\left(1+\gamma_{i_{j-1},i_j}\right)} \\ &+ \frac{\psi - \sum\limits_{m=0}^{K-2} t_m^{\mathrm{MCT-2}} \log_2(1+\gamma_{i_m,N+1})}{\log_2(1+\gamma_{i_K,N+1})}. \end{aligned} \quad (12)$$

B.4 The MCT-3 Protocol

For the MCT-3 protocol, the total end-to-end transmission time is given as

$$T^{\mathrm{MCT-3}} = \sum_{j=1}^{K} t_j^{\mathrm{MCT-3}} = \sum_{j=1}^{K} \frac{\psi}{\log_2\left(1+\gamma_{i_{j-1},i_j}\right)} \tag{13}$$

where $t_j^{\mathrm{MCT-3}}$ is the time transmission at stage j and is calculated as $t_j^{\mathrm{MCT-3}} = \psi/\log_2\left(1+\gamma_{i_{j-1},i_j}\right)$.

Lemma 1: The end-to-end delay of the MCT-3 protocol is always less than or equal to that of the CMT protocol: $T^{\mathrm{MCT-3}} \leq T^{\mathrm{CMT}}$.

Proof: It is easy to see that $t_j^{\mathrm{MCT-3}} \leq t_j^{\mathrm{CMT}} (1 \leq j \leq K)$, hence we have

$$\begin{aligned} T^{\mathrm{MCT-3}} &= \sum_{j=1}^{K} t_j^{\mathrm{MCT-3}} \\ &\leq \sum_{j=1}^{K} t_j^{\mathrm{CMT}} \leq \sum_{j=1}^{T} t_j^{\mathrm{CMT}} = T^{\mathrm{CMT}}. \end{aligned} \tag{14}$$

Therefore, (14) proves Lemma 1. □

On the other hand, due to the operation process of the MCT-1, MCT-2, and MCT-3 protocols, we have the following inequality: $T^{\mathrm{MCT-1}} \leq T^{\mathrm{MCT-2}} \leq T^{\mathrm{MCT-3}}$.

C. Average Number of Stages

Lemma 2: The average number of stages in the MCT-3 protocol can be calculated as $L = A/B$, where A and B are respectively given by

$$\begin{aligned} A = &\Pr\left[\gamma_{1,N+1} = \max_{j=2,3,\cdots,N}(\gamma_{1,j})\right] \\ &+ \sum_{K=1}^{N-1} \sum_{\substack{i_1,i_2,\cdots,i_K=1 \\ i_1<i_2<\cdots<i_K}}^{N} \\ &\left\{(K+1)\prod_{m=1}^{K}\Pr\left[\gamma_{i_{m-1},i_m} = \max_{k=i_{m-1}+1,\cdots,N+1}(\gamma_{i_m,i_k})\right]\right\} \end{aligned} \tag{15}$$

$$\begin{aligned} B = &\Pr\left[\gamma_{1,N+1} = \max_{j=2,3,\cdots,N}(\gamma_{1,j})\right] \\ &+ \sum_{K=1}^{N-1} \sum_{\substack{i_1,i_2,\cdots,i_K=1 \\ i_1<i_2<\ldots<i_K}}^{N} \\ &\left\{\prod_{m=1}^{K}\Pr\left[\gamma_{i_{m-1},i_m} = \max_{k=i_{m-1}+1,\cdots,N+1}(\gamma_{i_m,i_k})\right]\right\} \end{aligned} \tag{16}$$

where $\Pr\left[\gamma_{1,N+1} = \max_{j=2,3,\cdots,N}(\gamma_{1,j})\right]$ and $\Pr\left[\gamma_{i_m,i_{m+1}} = \max_{k=i_m+1,\cdots,N+1}(\gamma_{i_m,i_k})\right]$ are calculated similarly to (19) in Appendix.

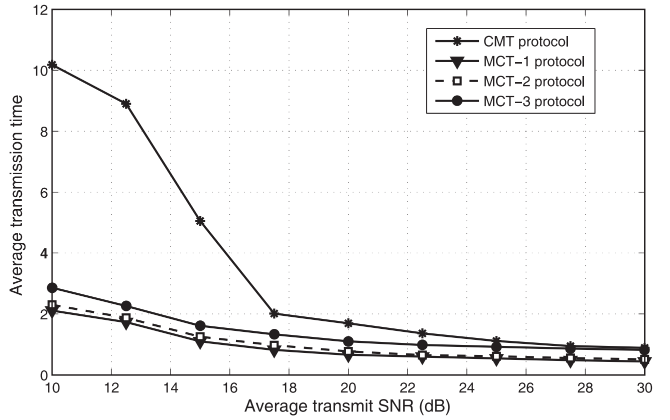

Fig. 2. Average transmission time as a function of the average transmit SNR ($\overline{\gamma}$) when $\psi = 1,\ \beta = 3,\ D = 1$ and N=6.

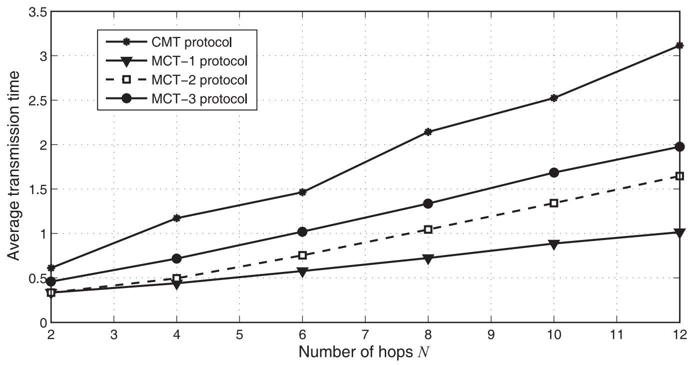

Fig. 3. Average transmission time as a function of the number of hop N when $\psi = 1,\ \beta = 3,\ D = 1$ and $\overline{\gamma} = 20\,\mathrm{dB}$.

Proof: If there is only a single stage, the channel between the source and the destination is better than that between the source and relay. Therefore, the probability for this case is $\Pr\left[\gamma_{1,N+1} = \max_{j=2,3,\cdots,N}(\gamma_{1,j})\right]$. On the other hand, if the number of stages is K and the relays chosen at each stage are $i_1, i_2, \cdots, i_{K-1}$, the probability for stage m $(1 \leq m \leq K)$ is calculated as $\Pr\left[\gamma_{i_{m-1},i_m} = \max_{k=i_{m-1}+1,\cdots,N+1}(\gamma_{i_m,i_k})\right]$. Due to the independence of each stage, the probability for each set of chosen nodes $\{i_1, i_2, \cdots, i_{K-1}\}$ is given by $\prod_{m=1}^{K}\Pr\left[\gamma_{i_{m-1},i_m} = \max_{k=i_{m-1}+1,\cdots,N+1}(\gamma_{i_m,i_k})\right]$.

Since there are 2^{N-1} possible cases for each set $\{i_1, i_2, \cdots, i_{K-1}\}$, the total probability for all cases is calculated by (16). Similarly, we can calculate the total number of stages as presented in (15). Now, dividing (15) by (16), we obtain the average number of stages and conclude the proof of Lemma 2. □

IV. SIMULATION RESULTS

In this section, we provide some Monte-Carlo simulations to evaluate and compare the performance of the considered protocols. We consider a line network, in which all nodes are placed in a straight line, and the distance between two adjacent nodes

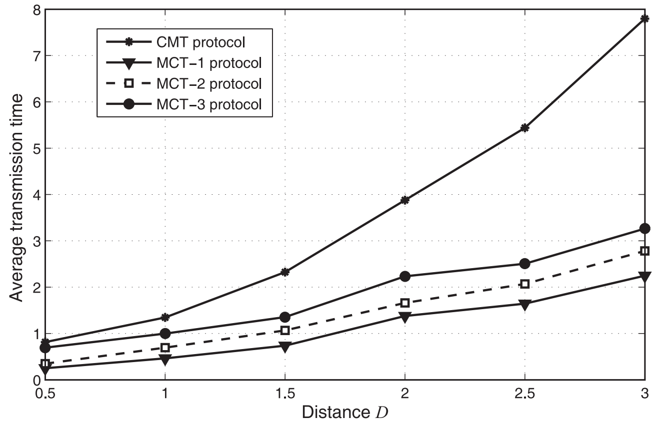

Fig. 4. Average transmission time as a function of the distance D when $\psi = 1,\ \beta = 3,\ N = 8$, and $\overline{\gamma} = 25\,\mathrm{dB}$.

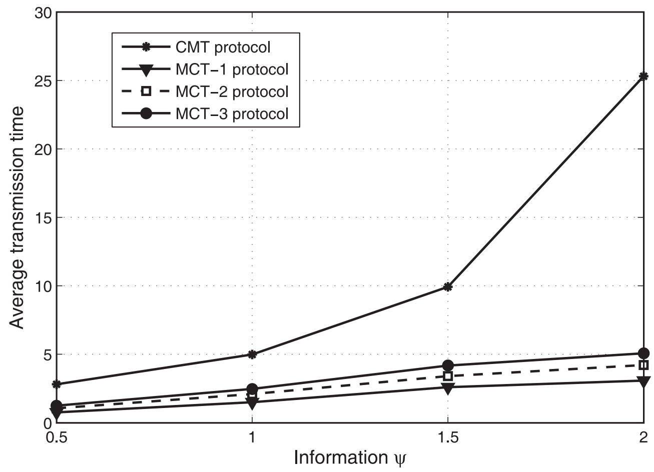

Fig. 5. Average transmission time as a function of the information ψ when $D = 1,\ \beta = 3,\ N = 10$, and $\overline{\gamma} = 15\,\mathrm{dB}$.

equals to D. In all simulations, we perform 10^5 trials, and the time delay and the number of stages are average calculations.

In Figs. 2–5, we present the average transmission time of the CMT, MCT-1, MCT-2, and MCT-3 protocols as a function of the parameters $\overline{\gamma}$, N, D, and ψ, respectively. From all of the presented results, it can be observed that the average transmission time of the CMT protocol is always higher than those of the MCT-1, MCT-2, and MCT-3 protocols. In addition, among the proposed protocols, the MCT-1 protocol has the shortest delay time and the MCT-1 has the longest.

The average number of stages for the studied protocols is presented in Figs. 6 and 7. In Fig. 6, we fix the values of ψ, β, D, and N to 1, 3, 1, and 7, respectively, while varying the value of the average transmit SNR $\overline{\gamma}$ from 0 to 40 dB. As shown, the average number of stages in the CMT protocol equals the number of hops N, while those in the proposed protocols are less than N. It can be also seen that the MCT-2 protocol uses fewer stages than do MCT-1 or MCT-3. In addition, the average number of stages for the MCT-3 protocol is the same for all values of $\overline{\gamma}$, that of the MCT-1 protocol varies slightly with changing $\overline{\gamma}$, while that of the MCT-2 protocols decreases rapidly

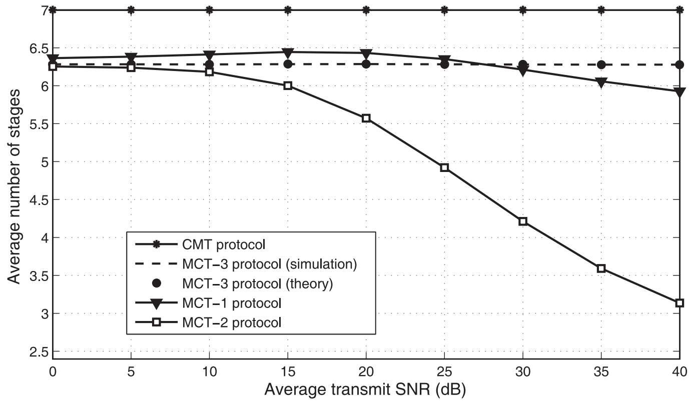

Fig. 6. Average number of stages as a function of average transmit SNR ($\overline{\gamma}$) when $\psi = 1,\ \beta = 3,\ D = 1$, and $N = 7$.

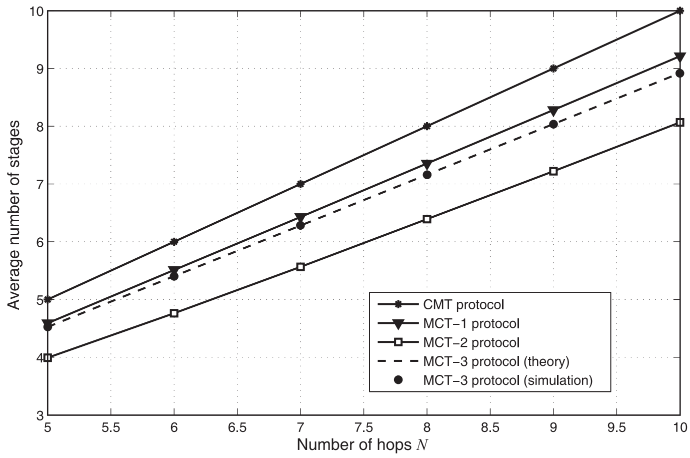

Fig. 7. Average number of stages as a function of the number of hop N when $\psi = 1,\ \beta = 3,\ D = 1$, and $\overline{\gamma} = 20\,\mathrm{dB}$.

at high value of $\overline{\gamma}$. In Fig. 7, we illustrate impact of hop number N on the average number of stages. When the value of N increases, the average number of stages in all protocols also increases. However, that of the CMT protocol is always equal to N, while the proposed protocols use fewer than N. We should note that the number of stages is equal to the number of nodes which transmit the encoded data to the destination. Hence, reducing the number of stages results in a reduced delay time for decoding and re-encoding, which is not considered in the end-to-end delay time. Finally, in Figs. 6 and 7, the performance of the MCT-3 protocol is presented by simulation and theory. It can be seen that the simulation and theoretical results are in an excellent agreement. This verifies the proof of Lemma 2.

V. CONCLUSIONS

In this paper, we proposed and evaluated three multi-hop cooperative transmission protocols using fountain codes. The proposed protocols reduced the transmission time as well as the average number of stages, compared to those in conventional multi-hop transmission. Monte-Carlo simulations are also presented to compare the performance of the considered protocols and to verify the theoretical analyses.

APPENDIX

Consider a set of random variables $S = \{X_0, X_1, \cdots, X_M\}$. Assume that all $X_j \, (0 \leq j \leq M)$ are independent exponential random variables with parameters λ_j which are different from each other. Now, we derive the probability $\Pr\left[X_0 = \max_{j=0,1,\cdots,M}(X_j)\right]$.

At first, let us denote a random variable Y as $Y = \max_{j=1,2,\cdots,M}(X_j)$. Because the cumulative density function (CDF) of X_j is $F_{X_j}(x) = 1 - e^{-\lambda_j x}$, the CDF of Y can be formulated as

$$\begin{aligned} F_Y(y) &= \Pr[Y < y] = \Pr\left[\max_{j=1,2,\cdots,M}(X_j) < y\right] \\ &= \prod_{j=1}^{M}\left(1 - e^{-\lambda_j y}\right) \\ &= 1 + \sum_{i=1}^{M}(-1)^j \sum_{\substack{j_1,j_2,\cdots,j_i=1 \\ j_1<j_2<\cdots<j_i}}^{M} e^{-\sum_{m=1}^{i}\lambda_{j_m} y} \end{aligned} \quad (17)$$

In addition, $\Pr\left[X_0 = \max_{j=0,1,\cdots,M}(X_j)\right] = \Pr[X_0 \geq Y]$; hence, we have

$$\begin{aligned} &\Pr\left[X_0 = \max_{j=0,1,\cdots,M}(X_j)\right] \\ &\quad = \Pr[X_0 \geq Y] \\ &\quad = \int_0^{+\infty}\lambda_0 e^{-\lambda_0 x}dx \int_0^{x} f_Y(y)\,dy \\ &\quad = \int_0^{+\infty}\lambda_0 e^{-\lambda_0 x}\left(1 - F_Y(x)\right)dx \end{aligned} \quad (18)$$

where $f_Y(y)$ is the probability density function (PDF) of Y.

Substituting (17) into (18), and after some manipulation, we obtain

$$\begin{aligned} &\Pr\left[X_0 = \max_{j=0,1,\cdots,M}(X_j)\right] \\ &\quad = \Pr[X_0 \geq Y] \\ &\quad = 1 + \sum_{i=1}^{M}(-1)^j \sum_{\substack{j_1,j_2,\cdots,j_i=1 \\ j_1<j_2<\cdots<j_i}}^{M} \frac{\sum_{m=1}^{i}\lambda_{j_m} y}{\lambda_0 + \sum_{m=1}^{i}\lambda_{j_m} y} \end{aligned} \quad (19)$$

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